9

Transformers

9.1 INTRODUCTION

Economical and technologically feasible voltage levels at which large chunks of electric power can be generated are typically 11-37 kV, while the most convenient utilization voltages are 230/400 V for industrial, commercial and domestic purposes. Large industrial motors may be run at 3.3, 6.6 or 11 kV. It is impossible to transmit directly, even over modest distances, the electric power as it is generated (11-37 kV). Unacceptably large power losses and voltage drops would result. As a rule of thumb economical transmission voltage is 0.625 kV/km line-to-line, e.g. 400 kV for a line of about 640 km. It is therefore essential to *step-up voltages* at the sending (generating) end and to *step-down* at the receiving end. Usually more than one step of step-down may be necessary. Step-up and step-down of voltage levels is accomplished by means of static electromagnetic devices called transformers.

It was seen in Sec. 8.8 that alternating flux is set up in a core by a coil excited with ac voltage (Fig. 8.20), which in turn induces coil emf* of excitation frequency proportional to the number of coil turns (Eq. (8.56)). If another coil is wound on the same core, the *mutual flux* (alternating) would induce emf in it also of the same frequency and of magnitude proportional to its coil turns. The ratio of the voltage of the two coils can be easily adjusted by means of their *turn-ratio*. Such a device, which indeed is a mutually coupled circuit, is called a transformer and is exhibited digrammatically in Fig. 9.1. The coil excited from the ac source is called the *primary* and receives electric power from the source. The other coil is called the *secondary* and the voltage induced in it could be used to feed a load. The subscript '1' will be associated with the primary and '2' with the secondary. Primary and

* The magnitude of flux is determined by the fact that the coil emf must equal the excitation voltage (KVL).

secondary roles in a transformer are easily reversed by the prevailing electrical conditions at the two ports. To avoid confusion in practice the two transformer coils are known as *HV* (*high-voltage*) and *LV* (*low-voltage*) windings.

Also shown in Fig. 9.1 are the mutual and leakage flux paths. Since a significant part of the leakage flux paths is through air, leakage fluxes ϕ_{11} and ϕ_{12} are less than the mutual flux ϕ.

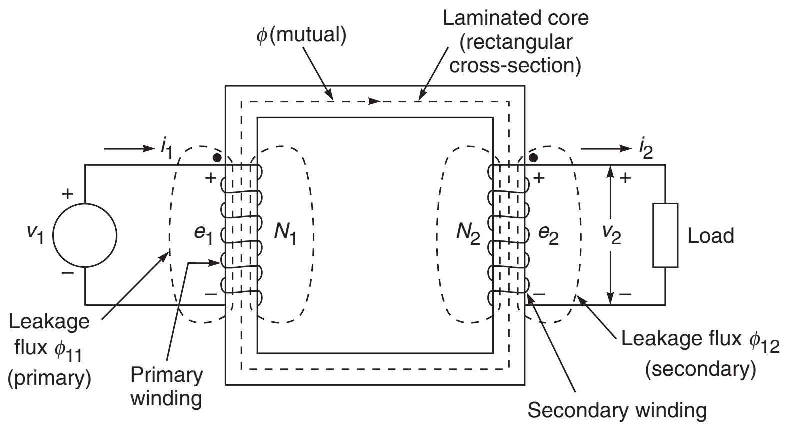


Fig. 9.1 *A simple transformer*

The *dots* indicate on the two coils (windings) are the polarity marks. As the mutual flux alternates, these coil ends simultaneously acquire the same polarity. Also current into the dot in one coil and out of the dot in the other coil would tend to produce core flux in the opposite direction.

The transformer shown in Fig. 9.1 is an *iron-core* transformer. Transformers operated at 25-400 Hz are invariably of iron-core construction. In special cases (particularly at high frequencies), the core may be made of nonmagnetic material in which case it is called an *air-core* transformer. Application range for air-core transformers are radio devices and certain types of measuring and testing instruments.

Since the transformer core carries alternating flux, it is made of laminated steel (0.35 mm thickness for 50 Hz transformers). The transformer core is constructed of rectangular sheet steel strips. Two types of core constructions are adapted for single-phase transformers—*core* and *shell type* (Fig. 9.2). The core type construction has a longer mean length of flux path and a shorter mean length of coil turn.

Flux linking only one winding of the transformer (leakage flux) is detrimental to transformer performance in terms of voltage drop. To reduce leakage flux half-LV and half-HV are wound on each limb of the core type transformer as shown in Fig. 9.2(a). For economical insulation, the LV coils are placed inside (next to core) and HV coils are placed on the outside. In a

shell type transformer reduced leakage flux is achieved by sandwiching HV and LV coil packets.

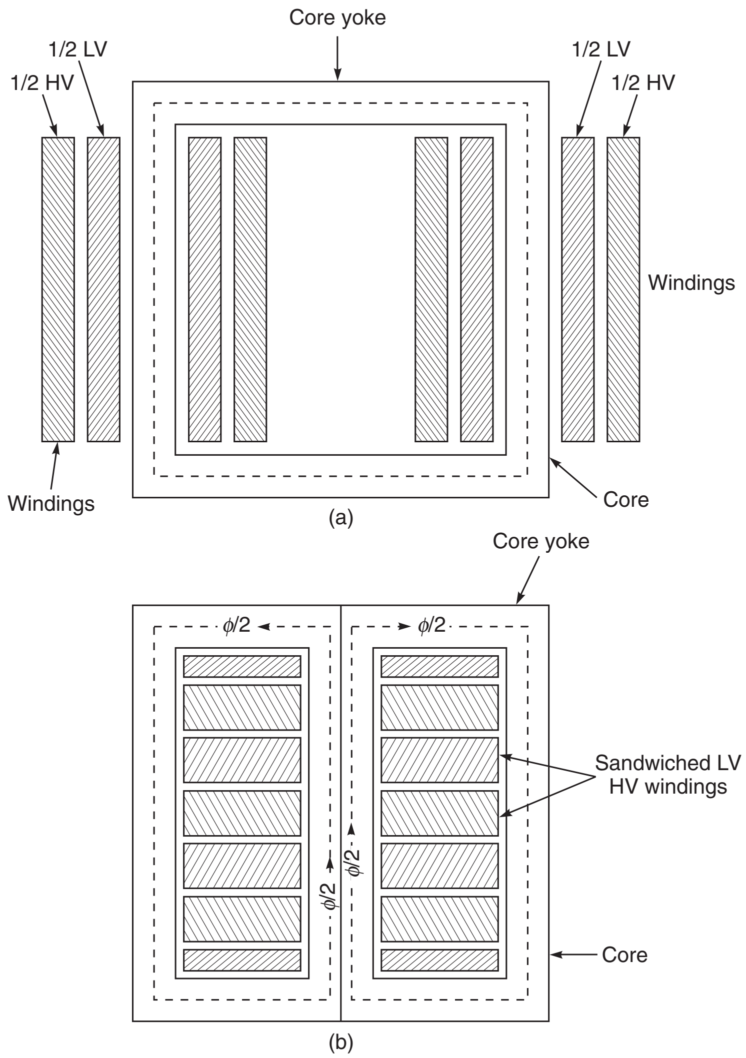


Fig. 9.2 *(a) Core type transformer, (b) Shell type transformer*

To prevent ingress of moisture and deterioration of winding insulation, the built-in core and windings are placed in a steel tank filled with *transformer oil*. Oval or circular tubes are provided on the outside surfaces of the transformer tank, aiding in natural circulations of oil, which removes the heat of core and winding (I^2R) losses and transports it to the tank surfaces for cooling purpose. Oil circulations also removes the heat generated by iron losses in the core. To prevent the coil from absorbing moisture from air and from being oxidized, the tank must be sealed and connected to the atmosphere through

a narrow passage for breathing purposes. Inside this passage is placed silica gel for drying the air that the transformer breathes in.

9.2 IDEAL TRANSFORMER (IT)

In order to develop the mathematical model of a transformer it is convenient to visualize a circuit element termed the "ideal" transformer by making certain assumptions in the realistic transformer. These assumptions only introduce insignificant model errors and are as follows:

– The transformer windings are resistanceless. This in effect means that ohmic power losses and resistance voltage drops in the actual transformer are neglected.
– The transformer core material has infinite permeability so that it requires zero mmf to create flux in the core.
– The leakage flux is negligible, i.e. no reactive voltage drops in windings.
– The transformer core losses are negligible.

Figure 9.3 is a diagrammatic representation of an ideal transformer.

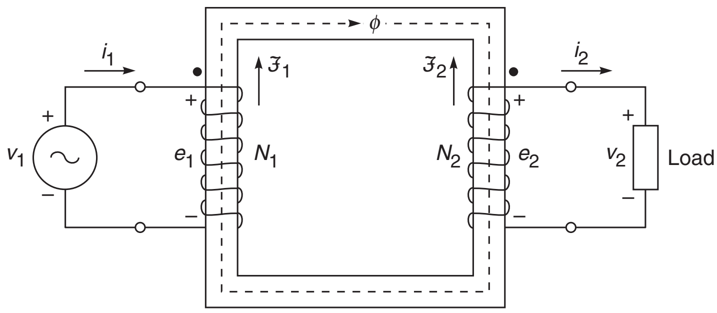


Fig. 9.3 *Ideal transformer (IT)*

Under No–Load Conditions

Let it be assumed that the ideal transformer of Fig. 9.3 is on no load, i.e. the secondary is open-circuited ($i_2 = 0$). The primary is excited from a sinusoidal source of voltage v_1 (= $\sqrt{2}\, V_1 \sin \omega t$). This requires the transformer core to carry a sinusoidal mutual flux ϕ which induces primary emf e_1 to balance the applied voltage v_1. Since the core has infinite permeability, the primary windings does not draw any exciting current from the source, i.e $i_1 = 0$. On the two sides of the transformer, we can write the circuit equations (by application of KVL)

$$v_1 = e_1 = N_1 \frac{d\phi}{dt} \tag{9.1}$$

$$v_2 = e_2 = N_2 \frac{d\phi}{dt} \tag{9.2}$$

From Eqs (9.1) and (9.2) the voltage transformation ratio is

$$\frac{v_1}{v_2} = \frac{e_1}{e_2} = \frac{N_1}{N_2} = a \tag{9.3}$$

Since a is a constant, the two voltages are in phase. In phasor form, Eq. (9.3) becomes

$$\frac{\bar{V}_1}{\bar{V}_2} = \frac{\bar{E}_1}{\bar{E}_2} = \frac{N_1}{N_2} = a \tag{9.4}$$

Also for rms values

$$\frac{V_1}{V_2} = \frac{E_1}{E_2} = \frac{N_1}{N_2} = a \tag{9.5}$$

Equations (9.4) and (9.5) will continue to hold even when the transformer windings carry current as there are no voltage drops in the windings (zero resistance and no leakage).

It is seen from Eq. (9.5) that an ideal transformer transforms voltages in the direct ratio of turns.

For sinusoidal primary excitation

$$v_1 = \sqrt{2}\, V_1 \sin \omega t \tag{9.6}$$

From Eq. (9.1)

$$v_1 = e_1 = \sqrt{2}\, V_1 \sin \omega t = N_1 \frac{\mathrm{d}\phi}{\mathrm{d}t} \tag{9.7}$$

Integrating Eq. (9.7) gives

$$\phi = -\frac{\sqrt{2}\, V_1}{\omega N_1} \cos \omega t = \frac{\sqrt{2}\, V_1}{\omega N_1} \sin (\omega t - \pi/2) \tag{9.8}$$

From Eqs (9.6) and (9.8) we observe that the flux phasor $\bar{\Phi}$ lags $\bar{V}_1$ (and $\bar{V}_2$) by 90° as illustrated in Fig. 9.4.

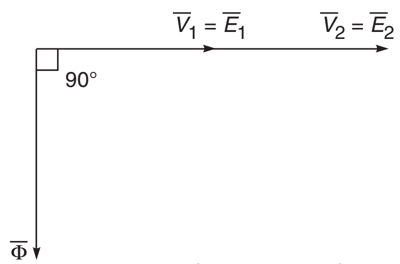


Fig. 9.4 *Phasor diagram of ideal transformer on no load*

From Eq. (9.8) the maximum value of the core flux is

$$\phi_{\max} = \frac{\sqrt{2}\, V_1}{\omega N_1} = \frac{V_1}{\sqrt{2}\, \pi f N_1} = \frac{V_2}{\sqrt{2}\, \pi f N_2} \text{ Wb} \tag{9.9}$$

It is noted from Eq. (9.9) that the maximum core flux in a transformer is dictated by the V_1/f (voltage/frequency) ratio at which the transformer is excited. In other words, for a given excitation voltage and frequency, the maximum core flux cannot change. It will be seen later that this is an important characteristic of all ac machines.

Under Loading Conditions

If the secondary load is switched on so that a current i_2 flows in the secondary winding (Fig. 9.2), the secondary mmf $\mathcal{F}_2 = i_2N_2$ tends to oppose the core flux (by Lenz's law). Since the excitation conditions of primary do not allow any change in core flux, a current i_1 must flow in the primary winding (Fig. 9.3) to prevent any change in core flux. With current directions indicated in Fig. 9.2 (out of secondary polarity dot and into primary polarity dot), the two mmfs balance such that

$$i_1N_1 = i_2N_2$$

or

$$\frac{i_1}{i_2} = \frac{N_2}{N_1} = \frac{1}{a} \tag{9.10}$$

The corresponding phasor relationship is

$$\frac{\bar{I}_1}{\bar{I}_2} = \frac{N_2}{N_1} = \frac{1}{a} \tag{9.11}$$

which also implies that $\bar{I}_1$ and $\bar{I}_2$ are in phase. In terms of rms magnitudes

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{a} \tag{9.12}$$

Equations (9.12) implies that *current transforms in an ideal transformer in the inverse ratio of turns*. Further, it is to be emphasized that the *core flux remains unchanged even under loading conditions* (for given excitation voltage and frequency).

The complete phasor diagram of the ideal transformer is drawn in Fig. 9.5 for power factor angle θ.

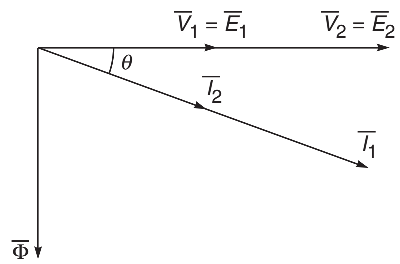


Fig. 9.5 *Phasor diagram of ideal transformer*

From Eqs (9.3) and (9.10) it follows that

$$p_1 = v_1 i_1 = v_2 i_2 = p_2 \tag{9.13}$$

i.e. all the *instantaneous power entering the ideal transformer at the primary exits from the secondary*. This is a statement of no losses in the ideal transformer.

Equation (9.13) in phasor form becomes

$$\bar{V}_1 \bar{I}_1^* = \bar{V}_2 \bar{I}_2^* \tag{9.14}$$

which indicates the balance of active and reactive powers in an ideal transformer.

The equivalent circuit of an ideal transformer is drawn in Fig. 9.6. Here

$$\bar{V}_2' = a\,\bar{V}_2 \tag{9.15}$$

and

$$\bar{I}_2' = \frac{1}{a}\,\bar{I}_2 \tag{9.16}$$

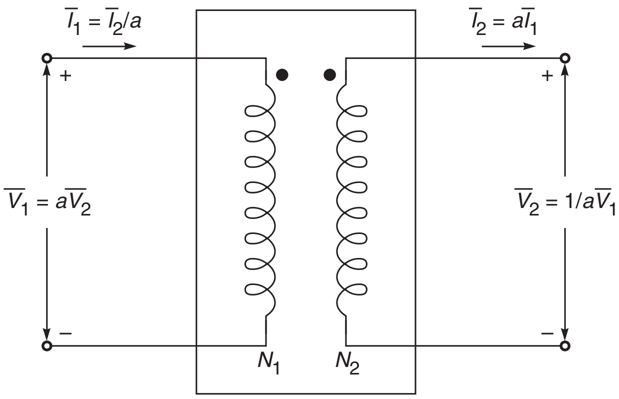


Fig. 9.6 *Equivalent circuit of IT ($a = N_1/N_2$)*

are called 'the secondary voltage and current referred to the primary'. Similarly we define

$$\bar{V}_1' = \frac{1}{a}\,\bar{V}_1 \tag{9.17}$$

$$\bar{I}_1' = a\,\bar{I}_1 \tag{9.18}$$

as 'the primary voltage and current referred to the secondary'.

It may be remarked here that V_2 applied to the secondary winding of an ideal transformer produces the same maximum core flux as V_1 applied to the primary winding (Eq. (9.9).

Impedance Transformation

On the secondary side of the ideal transformer (Fig. 9.7).

$$\frac{\bar{V}_2}{\bar{I}_2} = \bar{Z}_2 \tag{9.19}$$

or
$$\frac{(N_2/N_1)\bar{V}_1}{(N_1/N_2)\bar{I}_1} = \bar{Z}_2$$

or
$$\frac{\bar{V}_1}{\bar{I}_1} = \left(\frac{N_1}{N_2}\right)^2 \bar{Z}_2$$
$$= a^2\bar{Z}_2 = \bar{Z}_2' \tag{9.20}$$

The impedance transformation property (Eq. (9.20) is illustrated diagrammatically in Fig. 9.7. $\bar{Z}_2'$ is called 'the secondary impedance referred to the primary', vice versa applies equally, i.e. $\bar{Z}_1' = (1/a^2)\bar{Z}_1$. The *impedance transforms from one side of the ideal transformer to the other in the direct square ratio of turns.*

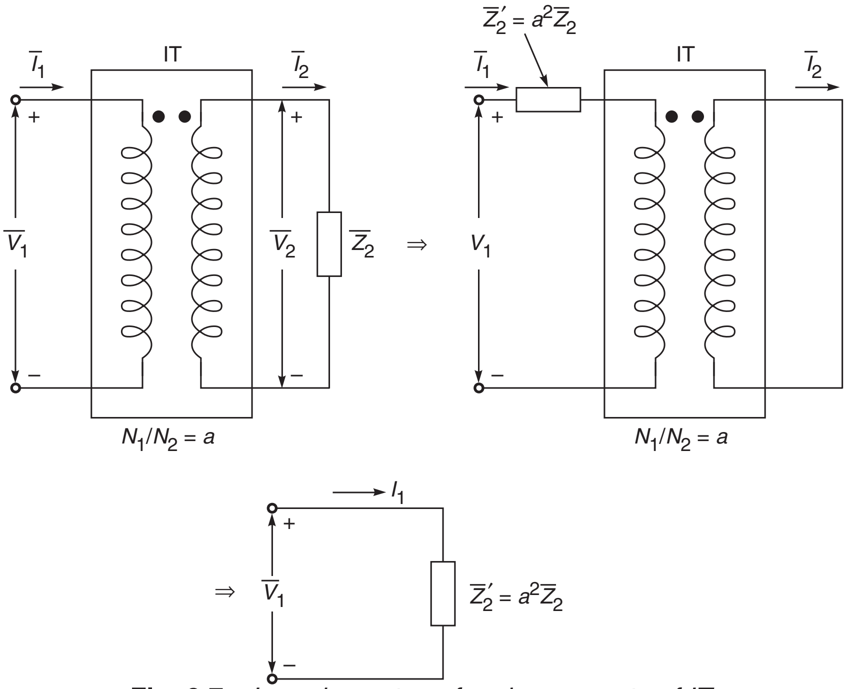


Fig. 9.7 *Impedance transforming property of IT*

Equation (9.20) can be put in the admittance form as

$$\bar{Y}_2' = \frac{1}{a^2}\bar{Y}_2 \tag{9.21}$$

i.e., the admittance transforms from one side of the ideal transformer to the other in the inverse square ratio of turns.

The impedance transforming property of the transformer is employed in impedance matching in electric circuits (Eq. (4.48)).

Example 9.1 An ideal transformer has a turn-ratio of 100/300. The LV winding is connected to a source of 3.3 kV, 50 Hz. An impedance of (100 + j 35) Ω is connected across the secondary terminals. Calculate (a) the value of maximum core flux, (b) the primary and secondary currents, (c) the real and reactive powers supplied by the source to the transformer primary, and (d) the value of impedance which connected directly across the source would draw the same real and reactive power as in (c).

Solution

(a) From Eq. (9.9)

$$\phi_{\max} = \frac{V_1}{\sqrt{2}\,\pi f N_1} = \frac{3.3 \times 1000}{\sqrt{2}\,\pi \times 50 \times 100}$$

$$= 0.149 \text{ Wb}$$

(b)

$$V_2 = 3.3 \times (300/100) = 9.9 \text{ kV}$$

$$\bar{I}_2 = \frac{9.9 \times 1000}{(100 + j\,35)} = 93.44 \;\angle -19.3° \text{ A}$$

$$\bar{I}_1 = (300/100) \times 93.44 \;\angle -19.3°$$
$$= 280.3 \;\angle -19.3° \text{ A}$$

(c)

$$\bar{S} = \bar{V}_1 \bar{I}_1^* = \bar{V}_2 \bar{I}_2^*$$
$$= 9.9 \times 93.44 \;\angle 19.3° \text{ kVA}$$
$$= (873.1 + j\;305.7)$$

Hence $P_1 = 873.1$ kW, $Q_1 = 305.7$ kVAR

(d)

$$\bar{Z}_1 = \bar{Z}_2' = a^2 \bar{Z}_2$$
$$= (100/300^2)\;(100 + j\,35) = 11.11 + j\,3.89)\;\Omega$$

9.3 ACCOUNTING FOR FINITE PERMEABILITY AND CORE LOSS

In a real transformer, the core has finite permeability and to establish flux in the core the primary winding would draw a current component called *magnetizing current* from the source over and above the load current. Assuming the core to be linear, let the core reluctance be $\mathcal{R}$. The magnetizing current is then given by

$$i_m = \frac{\mathcal{R}\phi}{N_1} \text{ A} \tag{9.22}$$

Substituting for ϕ from Eq. (9.8)

$$i_{\text{m}} = \sqrt{2}\left(\frac{\mathcal{R}V_1}{\omega N_1}\right) \sin\,(\omega t - \pi/2) \tag{9.23}$$

It is seen from Eqs (9.22) and (9.8) that the magnetizing current is in phase with the core flux and lags the induced emf by 90° as drawn in the phasor diagram of Fig. 9.8.

In phasor form Eq. (9.23) is written as

$$\bar{I}_m = \frac{\bar{V}_1}{jX_m} = -\,j B_m \bar{V}_1; \; B_m = \frac{1}{X_m} = \frac{\mathcal{R}}{\omega N_1^2} \tag{9.24}$$

In circuit model of the magnetizing current, it is the current drawn by a magnetizing reactance X_m (or magnetizing susceptance B_m) from the primary voltage source.

A real core will also have power loss (core loss) because it carries alternating flux. It can be modelled as a resistance R_i (or conductance G_i) across the primary voltage source. This is a sufficiently accurate representation for constant frequency operation. This power loss is $G_i V_1^2$ while the actual core loss has two components, viz. the eddy-current loss proportional to V_1^2 ($\phi_{\max} \propto V_1$) and hysteresis loss proportional to $V_1^{1.6}$; but square law assumption does not cause any significant error.

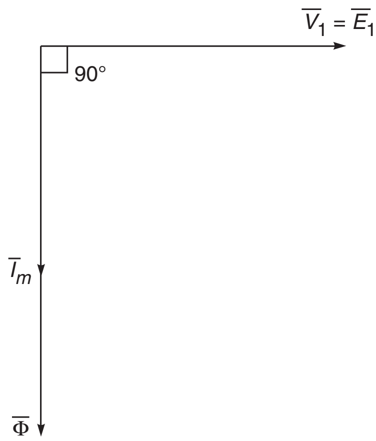


Fig. 9.8 *Magnetizing current phasor diagram*

The net exciting current* drawn by the primary to create core flux is then

$$\bar{I}_0 = \bar{I}_m + \bar{I}_i \tag{9.25}$$

where $\bar{I}_m = -\,j B_m \bar{V}_1$ = magnetizing current

$\bar{I}_m = G_i \bar{V}_1$ = core (iron) loss current

The phasor diagram of the exciting current is drawn in Fig. 9.9. The magnitude of I_i is much smaller than that of I_m and so the angle θ_0 is close to 90°.

The circuit model of the transformer at this stage of development is drawn in Fig. 9.10. The only assumption that is still made is that the windings are resistanceless and their leakage flux is negligible.

In the above development the core has been assumed to be linear. In the real core with hysteresis, to produce sinusoidal core flux, the exciting current i_0 will be periodic but nonsinusoidal with a predominant third harmonic**.

* Loosely the term magnetizing current will be used to mean exciting current.

** Taking the hysteresis curve of Fig. 8.8 and assuming B to be sinusoidal, the reader may find out by point-by-point method the wave shape of the exciting current.

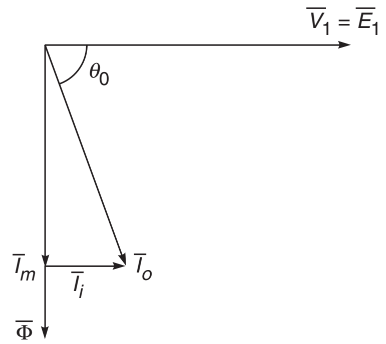


Fig. 9.9 *Exciting current phasor diagram*

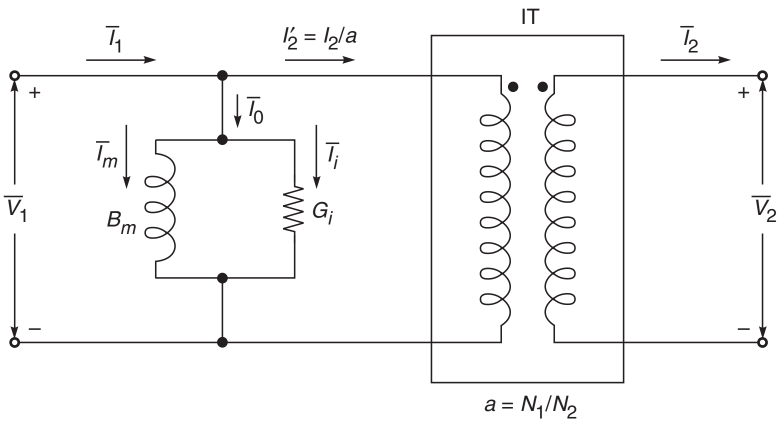


FIg. 9.10 *Circuit model of transformer (resistance and leakage neglected)*

Except for special effects, this fact is usually ignored and i_0 is taken as the equivalent sinusoidal current with the same rms value.

It is seen from Fig. 9.10 that the resultant current (under load) drawn from the primary is

$$\bar{I}_1 = \bar{I}_0 + \bar{I}_2' \tag{9.26}$$

The exciting current (also called magnetizing current) is predominantly reactive and is essential but not a load-delivering component of the primary current. It must therefore be kept as low as possible. This is why transformer cores are constructed of high-permeability sheet steel. The magnetizing current in a transformer is in the range 2–5% of the rated current. Further, it being mainly reactive (θ_0 close to 90°), rms magnitudewise

$$I_1 \approx I_2' = I_2/a \tag{9.27}$$

It is also clear from Fig. 9.9 that on no-load ($I_2' = I_2/a = 0$) the transformer primary would draw only the exciting current from the source, which therefore is synonymous with the term *no-load current* (hence the symbol I_0).

9.4 CIRCUIT MODEL OF TRANSFORMER

Both primary and secondary of a transformer have winding resistances. Apart from this the two windings have leakage flux; ϕ_{l1} linking only the primary and ϕ_{l2} linking only the secondary (see Fig. 9.1). These leakage fluxes do not contribute in the process of energy transfer, which takes place via the mutual flux ϕ_m, but these cause the primary and secondary windings to possess leakage inductances and therefore leakage reactances at steady sinusoidal operation. The winding resistances and leakage reactances can be lumped in series with the ideal windings (resistance and leakage-less) in a circuit model. The ideal primary and secondary windings along with the core (which now carries only the mutual flux ϕ_m) indeed constitute the ideal transformer. Let windings resistances be r_1, r_2 and winding reactances (inductive) be x_1 and x_2.

By the technique of impedance transformation, these can be transferred to one side of the transformer say the primary. Then equivalent series resistance and reactance of the transformer referred to the primary side are

$$\text{Equivalent resistance } R = r_1 + r_2' = r_1 + a^2 r_2 \tag{9.28}$$

$$\text{Equivalent resistance } X = x_1 + x_2' = x_1 + a^2 x_2 \tag{9.29}$$

The transformer circuit model (equivalent circuit) of Fig. 9.10 with inclusion of resistance and reactance, referred to the primary side, gets modified to the form shown in Fig. 9.11 where

$$\bar{I}_2' = \bar{I}_2/a \tag{9.30}$$

$$\bar{V}_2' = a\bar{V}_2 \tag{9.31}$$

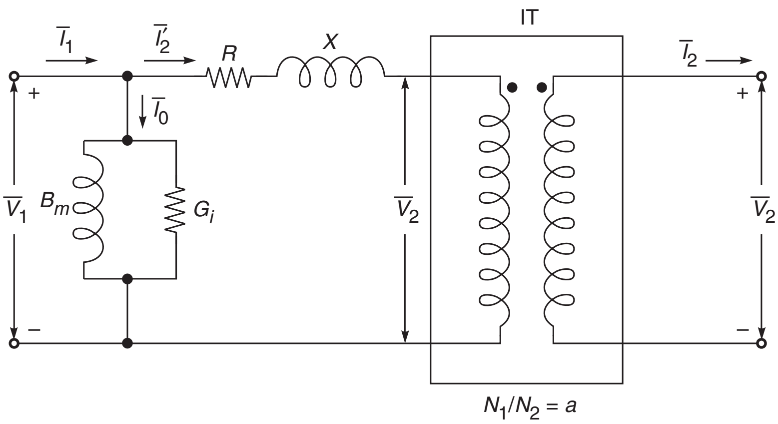


Fig. 9.11 *Complete circuit model of transformer*

In the circuit model of a transformer it is not necessary to carry the ideal transformer as these voltage and current conversions (Eqs (9.30) and (9.31) can always be carried out computationally. The transformer circuit model with

ideal transformer left out is drawn in Fig. 9.12. A similar circuit with appropriate values of circuit elements would apply on the secondary side.

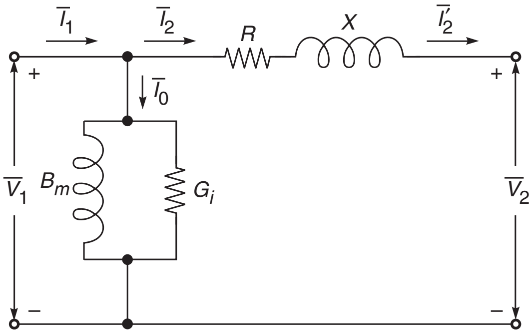


Fig. 9.12 *Circuit model of transformer (IT left out)*

The magnetizing shunt branches in the circuit model of Fig. 9.12 do not affect voltage computation and may therefore be ignored. Further, since R is much smaller in a transformer than is X, R may also be ignored. These two steps lead to the simplified circuit models of Fig. 9.13. It is also unnecessary to carry the superscript dash on current and voltage (as in Fig. 9.13).

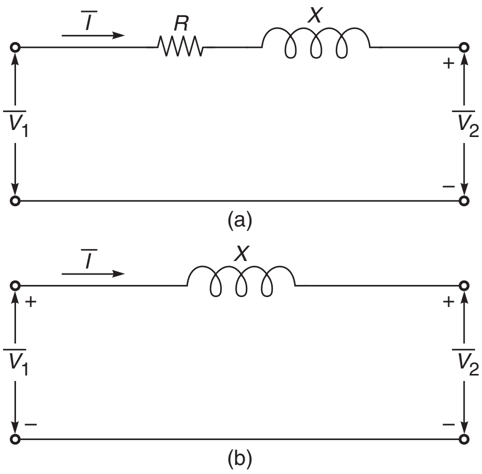


Fig. 9.13 *Simplified circuit model of transformer*

9.5 DETERMINATION OF PARAMETERS OF CIRCUIT MODEL OF TRANSFORMER

It is not practical to test a transformer for its voltage drop characteristic (voltage regulation; Sec. 9.7) and its efficiency by a direct loading test. Such a test would suffer from three disadvantages, viz. (i) loss of energy during testing, (ii) it may not be practical to arrange for load except for small size

transformers and (iii) losses as determined by direct loading would have serious error as these are determined by the difference of two power readings (i.e. input and output) which are close to each other. It is therefore standard practice in transformer testing to determine the transformer losses and the parameters of the circuit model by means of *nonloading* tests. The transformer performance is then computed from the circuit model.

Transformer parameter determination necessitates two tests, viz. open-circuit test and short-circuit test.

Open-Circuit (OC) or No-load Test

The transformer is excited at rated voltage (and frequency) from one side while the other side is kept open-circuited as shown in Fig. 9.14(a). It is usually convenient to conduct such a test from the LV side. The circuit model under open-circuit is drawn in Fig. 9.14(b); it follows from Fig. 9.12 by setting $I_2' = 0$.

Let the meter readings be

$$\begin{aligned} \text{voltage (V)} &= V_1 \\ \text{current (A)} &= I_0 \\ \text{and} \quad \text{power (W)} &= P_0 = \textit{core loss } (P_i) \end{aligned} \tag{9.32}$$

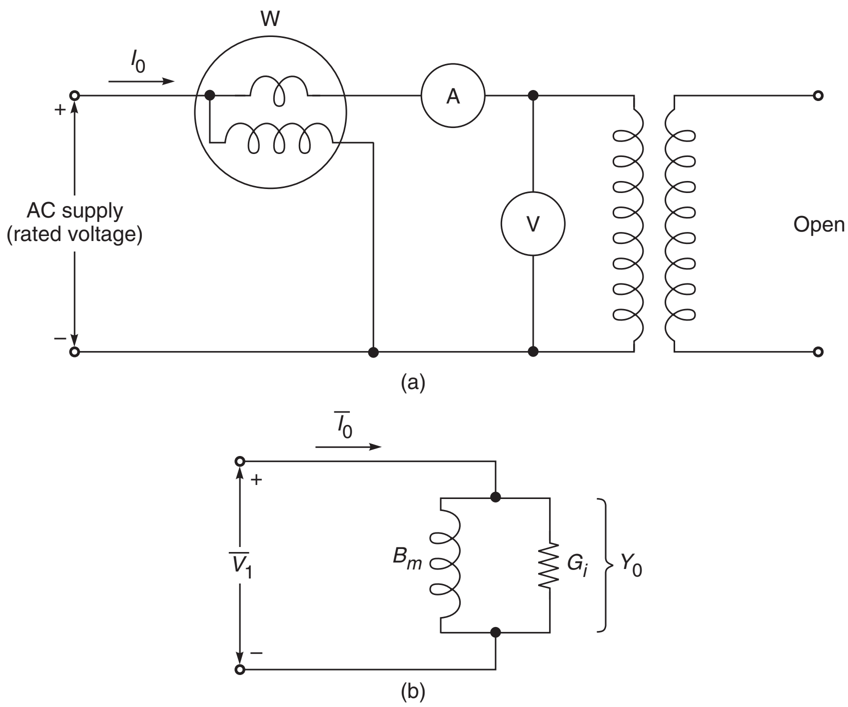


Fig. 9.14 *(a) Circuit diagram for OC test*
(b) Circuit model as seen on open-circuit

It then follows that

$$Y_0 = \frac{I_0}{V_1} \tag{9.33}$$

$$G_{\mathrm{i}} = \frac{P_0}{V_1^2} \tag{9.34}$$

and $$B_m = \sqrt{Y_0^2 - G_i^2} \tag{9.35}$$

By connecting a voltmeter on the secondary side, the OC test also yields the voltage ratio of the transformer, which is practically its turn ratio a.

The values of G_i and B_m as computed can be transferred to the other side of the transformer if so desired.

It is seen that the OC test yields (i) core loss and (ii) parameters of the shunt branch of the transformer model.

Short-Circuit (SC) Test

This test determines the series parameters of the transformer circuit model. The transformer is shorted on one side and is excited from a reduced voltage (rated frequency) source from the other side as shown in Fig. 9.15. The transformer circuit model under short-circuit conditions is drawn in Fig. 9.16(a). As the primary current is limited only by the resistance and leakage reactance of the transformer, V_{SC} needed to circulate full-load current is only of the order of 5–8% of the rated voltage. Under these conditions the exciting current of the transformer is of negligible order (0.1 – 0.5% of the rated current) as I_0 at rated voltage is 2–5% of the rated current. The magnetizing shunt branch of the circuit model can therefore be conveniently dropped resulting in the circuit of Fig. 9.16(b).

In conducting the SC test, as in Fig. 9.15, the source voltage is gradually raised till the transformer draws *full-load* current. The meter readings under these conditions are

$$\begin{aligned} \text{voltage (V)} &= V_{SC} \\ \text{Current (A)} &= I_{SC} \\ \text{power input (W)} &= P_{SC} = I^2R \text{ loss* or copper loss} \\ &\quad \text{(total in the two windings } P_c) \end{aligned} \tag{9.36}$$

From the circuit model of Fig. 9.16(b)

$$Z = \frac{V_{SC}}{I_{SC}} = \sqrt{R^2 + X^2} \tag{9.37}$$

* As the transformer is excited at 5–8% rated voltage, the core flux is reduced to the same percentage and the core losses proportional to square of core flux are reduced to 0.25 – 0.64% and are hence negligible. The power drawn by the transformer under SC condition is therefore wholly I^2R loss for all practical purposes.

Equivalent resistance $$R = \frac{P_{SC}}{(I_{SC})^2} \tag{9.38}$$

Equivalent reactance $$X = \sqrt{Z^2 + R^2} \tag{9.39}$$

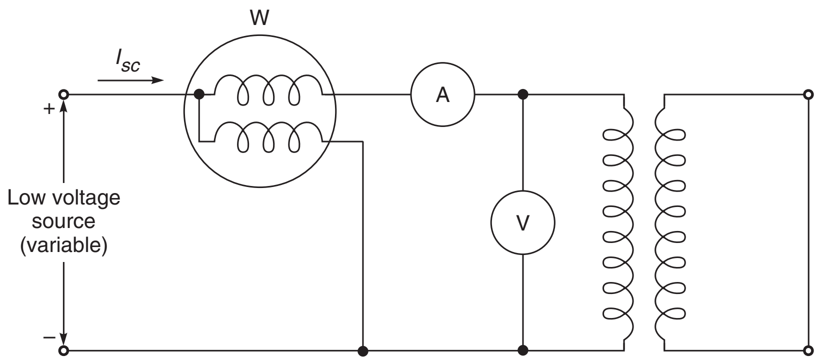


Fig. 9.15 *Short-circuit (SC) test on transformer*

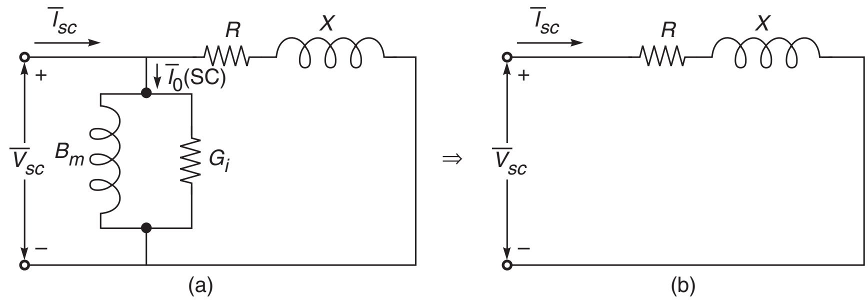


Fig. 9.16 *Circuit model under SC conditions*

It is thus seen that the SC test yields information about (i) full-load copper loss and (ii) equivalent resistance and reactance of the transformer.

Together OC and SC tests determine all the four parameters of the transformer circuit model of Fig. 9.12—two shunt parameters (G_i, B_m) and two series parameters (R, X).

9.6 PER UNIT SYSTEM

It is often convenient to scale electrical quantities in per unit of the base or reference values of these quantities. The basic per unit (pu) scaling equation is

$$\text{Per unit value} = \frac{\text{Actual value}}{\text{Base value}} \tag{9.40}$$

The pu system offers the advantage that the device parameters tend to fall in a relatively narrow range, making the erroneous values conspicuous. Also in

computations, one does not have to deal with very small and very large numbers. In a power system (with many transformers of different voltage ratio) ideal transformers are no longer necessary in the model.

Base values are related to each other by the usual electrical laws. For a single-phase system

$$P_B,\ Q_B,\ (\mathrm{VA})_B = V_B I_B \tag{9.41a}$$

$$R_B,\ X_B,\ Z_B = \frac{V_B}{I_B} \tag{9.41b}$$

$$G_B,\ B_B,\ Y_B = \frac{I_B}{V_B} \tag{9.41c}$$

$(VA)_B$ and V_B are first to be selected, then it follows from Eq. (9.41) that

$$Z_B = \frac{V_B^2}{(\mathrm{VA})_B} \tag{9.42}$$

Then
$$Z\,(\mathrm{pu}) = \frac{Z(\Omega)\times(\mathrm{VA})_B}{V_B^2} \tag{9.43}$$

In large devices and systems it is more practical to use base values in kVA/MVA and kV. Equation (9.43) can then be written as

$$Z(\mathrm{pu}) = \frac{Z(\Omega)\times(\mathrm{kVA})_B}{1000(\mathrm{kV})_B^2} \tag{9.44}$$

or
$$Z(\mathrm{pu}) = \frac{Z(\Omega)\times(\mathrm{MVA})_B}{(\mathrm{kV})_B^2} \tag{9.45}$$

In changing Z(pu) from one set of base values to another,

$$Z(\mathrm{pu})_{\mathrm{new}} = Z(\mathrm{pu})_{\mathrm{old}} \times \frac{(\mathrm{MVA})_{B,\,\mathrm{new}}}{(\mathrm{MVA})_{B,\,\mathrm{old}}} \times \frac{(\mathrm{kV})_{B,\,\mathrm{old}}^2}{(\mathrm{kV})_{B,\,\mathrm{new}}^2} \tag{9.46}$$

In a 3-phase star-connected system (equivalent star can always be found)

$$\begin{aligned}
(\mathrm{MVA})_{P,\,B} &= \text{MVA base per phase}\\
(\mathrm{MVA})_{3P,\,B} &= \text{MVA base 3-phase}\\
(\mathrm{kV})_{P,\,B} &= \text{kV base line-to-neutral}\\
(\mathrm{kV})_{L,\,B} &= \text{kV base line-to-line}
\end{aligned}$$

Then

$$(\mathrm{kV})_{L,\,B} = \sqrt{3}\ (\mathrm{kV})_{P,\,B} \tag{9.47}$$

$$(\mathrm{MVA})_{3P,\,B} = 3\ (\mathrm{MVA})_{P,\,B} \tag{9.48}$$

It can be easily shown that

$$Vp\ (\mathrm{pu}) = V_L(\mathrm{pu});\ \text{no factor of}\ \sqrt{3} \tag{9.49}$$

$$(\text{MVA})_P\ (\text{pu}) = (\text{MVA})_{3P}(\text{pu});\ \text{no factor of 3} \tag{9.50}$$

$$I_{P,\,B} = I_{L,\,B} = \frac{(\text{MVA})_{3P,\,B}}{\sqrt{3}\ (\text{kV})_{L,\,B}} \tag{9.51}$$

Now

$$Z_B = \frac{\left((\text{kV})_{L,\,B}\big/\sqrt{3}\right)^2}{(1/3)\ (\text{MVA})_{3P,\,B}} = \frac{(\text{kV})^2_{L,\,B}}{(\text{MVA})_{3P,\,B}} \tag{9.52}$$

$$Z(\text{pu}) = \frac{Z(\Omega) \times (\text{MVA})_{3P,\,B}}{(\text{kV})^2_{L,\,B}} \tag{9.53}$$

By definition

$$Z_{\Delta,B} = 3\ Z_{Y,B} \tag{9.54}$$

It then follows that

$$Z_Y(\text{pu}) = Z_\Delta(\text{pu}) \tag{9.55}$$

Since it is a common practice to use 3-phase MVA, and line-to-line kV bases, suffixing can be simplified as

$$\begin{aligned}(\text{MVA})_{3P,\,B} &\to (\text{MVA})_B\\ (\text{kV})_{L,\,B} &\to (\text{kV})_B\end{aligned} \tag{9.56}$$

Example 9.2 A 50 kVA, 2200/220 V transformer when tested gave the following results

OC test, measurements on LV side: 405 W, 5 A, 220 V
SC test, measurements on HV side: 805 W, 20.2 A, 95 V

(a) Draw the circuit model of the transformer referred to the HV and LV sides. Label all the parameters.
(b) Calculate the parameters of the transformer in pu referred to the HV and LV sides. Use base kVA as 50, base voltage as 2200 V on HV side and 220 V on LV side (i.e. the base voltages on the two sides of the transformer are in the ratio of transformation).

Solution OC test (LV side)

$$Y_0 = \frac{5}{220} = 0.0227\ \mho$$

$$G_i = \frac{405}{(220)^2} = 0.0084\ \mho$$

$$B_m = (0.0227 - 0.0084)^{1/2} = 0.021\ \mho$$

SC test (HV side):

$$Z = \frac{95}{20.2} = 4.7\ \Omega$$

$$R = \frac{805}{(20.0)^2} = 1.97 \ \Omega$$

$$X = [(4.7)^2 - (1.97)^2]^{1/2} = 4.27 \ \Omega$$

(a) Circuit model referred to HV side:

$$a = \frac{2200}{220} = 10$$

$$G_i = 0.0084 \times \frac{1}{(10)^2} = 0.84 \times 10^{-4} \ \mho$$

$$B_m = 0.021 \times \frac{1}{(10)^2} = 2.1 \times 10^{-4} \ \mho$$

$$R = 1.97 \ \Omega$$

$$X = 4.27 \ \Omega$$

The circuit model is drawn in Fig. 9.17.

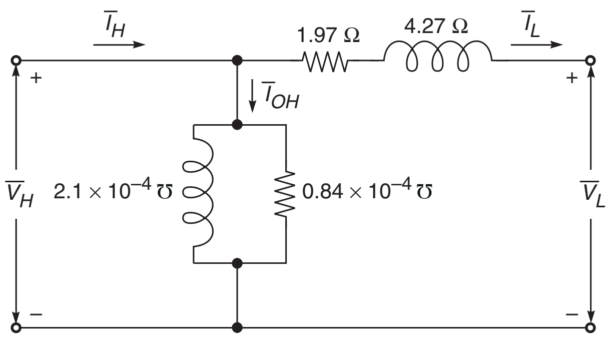


Fig. 9.17

Circuit model referred to LV side:

$$G_i = 0.84 \times 10^{-2} \ \mho$$

$$B_m = 2.1 \times 10^{-2} \ \mho$$

$$R = 1.97 \times \frac{1}{(10)^2} = 0.02 \ \Omega$$

$$X = 4.27 \times \frac{1}{(10)^2} = 0.043 \ \Omega$$

The circuit model is drawn in Fig. 9.18.

(b) $$(\text{kVA})_B = 50$$

$$V_B(\text{HV}) = 2200 \text{ V}$$

$$I_B(\text{HV}) = \frac{50 \times 1000}{2200} = 22.73 \text{ A}$$

$$Z_B\,(\text{HV}) = \frac{2200}{22.73} = 96.79\ \Omega;\ Y_B\,(\text{HV}) = 0.0103\ \mho$$

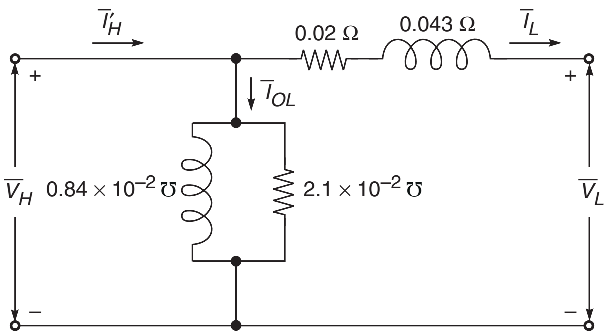


Fig. 9.18

Pu parameter values seen on HV side:

$$G_i = \frac{0.84\times10^{-4}}{0.0103} = 0.8 \times 10^{-2}\ \mho$$

$$B_m = \frac{2.1\times10^{-4}}{0.0103} = 2.04 \times 10^{-2}\ \mho$$

$$R = \frac{1.97}{96.79} = 0.02\ \Omega$$

$$X = \frac{4.27}{96.79} = 0.044\ \Omega$$

Pu parameter values seen on LV side:

$$V_B\,(\text{LV}) = 220\ \text{V}$$

$$I_B\,(\text{LV}) = \frac{50\times1000}{220} = 227.3\ \text{A}$$

$$Z_B\,(\text{LV}) = \frac{220}{227.3} = 0.9679\ \Omega;\ Y_B\,(\text{LV}) = 1.033\ \mho$$

$$G_i = \frac{0.84\times10^{-2}}{1.033} = 0.8 \times 10^{-2}\ \mho$$

$$B_m = \frac{2.1\times10^{-2}}{1.033} = 2.02\ \mho$$

$$R = \frac{0.02}{0.9679} = 0.021\ \Omega$$

$$X = \frac{0.043}{0.9679} = 0.044\ \Omega$$

The advantage of using pu system is that the transformer circuit parameter values on either side of the transformer are the same so long as voltage bases on the two sides are in direct ratio of transformation (and current bases in inverse ratio). The reason is not hard to find as shown below:

$$Z\,(\text{pu}) = \frac{I\,(\text{HV})\,Z\,(\text{HV})}{V\,(\text{HV})}$$

$$= \frac{I\,(\text{LV})}{V\,(\text{LV})} \times \left[\frac{N\,(\text{LV})}{N\,(\text{HV})}\right]^2 \times Z\,(\text{HV})$$

$$= \frac{I\,(\text{LV})\,Z\,(\text{LV})}{V\,(\text{LV})}$$

9.7 VOLTAGE REGULATION

Domestic, commercial and industrial loads demand a nearly constant voltage supply. It is, therefore, essential that the output voltage of a transformer stays within narrow limits as load and its power factor vary. The leakage reactance is the chief cause of voltage drop in a transformer and must be kept as low as possible by design and manufacturing techniques.

The voltage regulation of a transformer at a given power factor is defined as

$$\%\ \text{Voltage regulation} = \frac{V_{20} - V_{2,\,\text{fl}}}{V_{2,\,\text{fl}}} \times 100 \tag{9.57}$$

where $V_{2,\text{fl}}$ is the full-load secondary voltage (it is assumed to be adjusted to the rated secondary voltage) and V_{20} is the secondary voltage when load is thrown off.

As stated earlier the shunt branch of the circuit model can be left out for voltage computation ($I_1 \approx I_2'$ as per Eq. (9.27)). The circuit model of the transformer with this simplification and the corresponding voltage–current phasor diagram are drawn in Fig. 9.19.

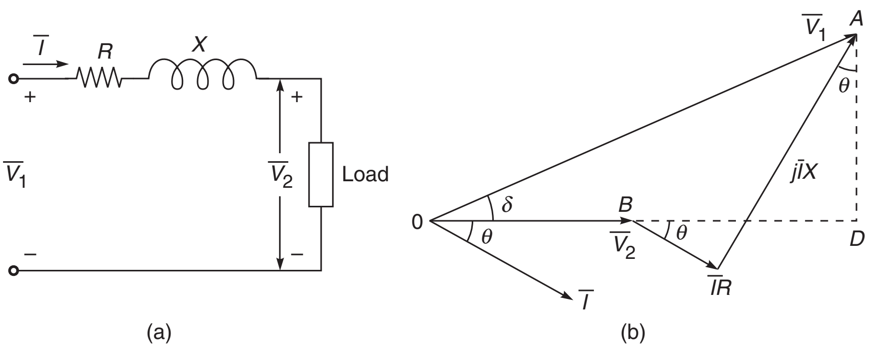


Fig. 9.19 *(a) Circuit model, (b) Phasor diagram (not proportional)*

As in a transformer IR and IX voltage drops are much smaller in magnitude compared to V_1 and V_2, the angle between $\bar{V}_1$ and $\bar{V}_2$ in Fig. 9.19(b) is only a few degrees such that

$$V_1 \approx 0D$$

$$V_1 - V_2 = BD$$

$$= I(R\cos\theta + X\sin\theta);\ \theta \text{ lagging} \tag{9.58a}$$

$$= I(R\cos\theta - X\sin\theta);\ \theta \text{ leading} \tag{9.58b}$$

When the load is thrown off

$$V_{20} = V_1$$

$$\therefore \qquad V_1 - V_2 = V_{20} - V_2$$

Then

$$\%\ \text{Voltage regulation, Reg} = \frac{V_{20} - V_2}{V_2} \times 100$$

$$= \frac{I(R\cos\theta + X\sin\theta)}{V_2} \times 100 \tag{9.59}$$

Now $$\frac{IR}{V_2} = R\,(\text{pu}),\quad \frac{IX}{V_2} = X\,(\text{pu})$$

assuming full-load (rated) current I and full-load (rated) voltage V_2 to be the base values. Therefore

$$\text{Reg (pu)} = R\,(\text{pu})\cos\theta \pm X\,(\text{pu})\sin\theta \tag{9.60}$$

For maximum voltage regulation (from Eq. (9.58a))

$$\frac{d\,(\text{Reg})}{d\theta} = 0 = -R\sin\theta + X\cos\theta$$

or $$\tan\theta = \frac{X}{R}$$

or $$\text{pf} = \cos\theta = \frac{R}{(R^2 + X^2)^{1/2}};\ \text{lagging} \tag{9.61}$$

From Eq. (9.58b) voltage regulation is zero when

$$R\cos\theta - X\sin\theta = 0;\ \theta \text{ leading}$$

or $$\tan\theta = \frac{R}{X}$$

or $$\text{pf} = \cos\theta = \frac{X}{(R^2 + X^2)^{1/2}};\ \text{leading} \tag{9.62}$$

Leading θ larger than that given in Eq. (9.62) would result in negative voltage regulation, i.e. secondary voltage on full load is higher than the no-load voltage.

Name Plate Rating

The voltage ratio of a transformer is specified as V_1 (rated)/V_2 (rated), where V_1 (rated) and V_2 (rated) are the primary and secondary voltage at full load and specified pf. Since the voltage drop in a transformer is only a few per cent, this ratio is also taken as the turns-ratio N_1/N_2 for all practical purposes, i.e.

$$\frac{V_1(\text{rated})}{V_2(\text{rated})} \approx \frac{N_1}{N_2}$$

A transformer depending upon its size can carry only a certain current, called full-load current, without overheating. The transformer rating is then

$$\text{kVA (rating)} = \frac{V(\text{rated}) \times I(\text{full-load})}{1000}$$

It could also be expressed as VA (rated) for small transformers and in MVA (rated) for very large size transformers.

The pu impedance of a transformer on its rated voltage and kVA bases is given by

$$Z\,(\text{pu}) = \frac{I(\text{full-load})\,Z(\Omega)}{V\,(\text{rated})}$$

$$= \frac{\text{kVA (rated)}\,Z(\Omega)}{1000\,(\text{kV (rated)})^2}$$

where I (full load), V (rated) and Z (Ω) pertain to any side of the transformer.

The percentage impedance is defined as

$$\%\ Z = Z\,(\text{pu}) \times 100$$

$$= \frac{I(\text{full-load})\,Z(\Omega)}{V\,(\text{rated})} \times 100$$

Obviously it has also the meaning of per cent voltage drop under full-load.

Example 9.3 The resistances and leakage reactances of a 10 kVA, 50 Hz, 2300/230 V distribution transformer are: $r_1 = 3.96\ \Omega$ and $r_2 = 0.0396\ \Omega$, $x_1 = 15.8\ \Omega$ and $x_2 = 0.158\ \Omega$; subscript 1 refers to HV and 2 to LV winding.

(a) The transformer delivers rated kVA at 0.8 pf lagging to a load on the LV side. Find the HV-side voltage necessary to maintain 230 V across load terminals. Also find the percentage voltage regulation.

(b) If a capacitor bank is connected across the load, what should be the kVA capacity of the bank to reduce the voltage regulation to zero. What should be the HV-side voltage under these circumstances?

Solution

(a) Referred to HV side,

equivalent resistance $R = 3.96 + 0.0396 \times (10)^2 = 7.92\ \Omega$

equivalent reactance $x = 15.8 + 0.158 \times (10)^2 = 31.6\ \Omega$

With reference to Fig. 9.20(a)

$$V_2 = 230 \times 10 = 2300 \text{ V}$$

$$I = \frac{10 \times 1000}{2300} = 4.35 \text{ A, } \cos\theta = 0.8 \text{ lagging}$$

$$V_1 - V_2 = I\,(R\cos\theta + X\sin\theta)$$

$$= 4.35\,(7.92 \times 0.8 + 31.6 \times 0.6) = 110 \text{ V}$$

$$V_1 = 2300 + 110 = 2410 \text{ V}$$

$$V_{20} = V_1 = 2410 \text{ V}$$

$$\text{Voltage regulation} = \frac{2410 - 2300}{2300} \times 100\%$$

$$= 4.78\%$$

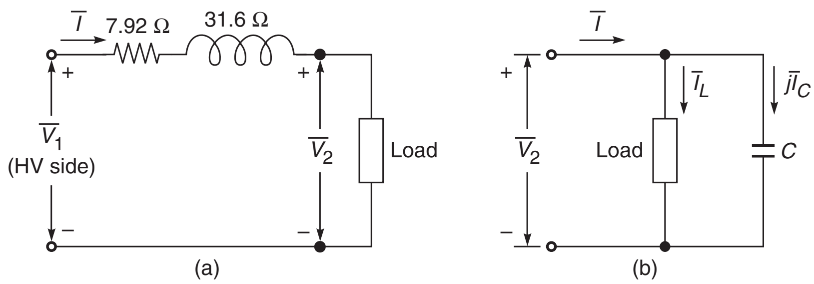


Fig. 9.20

(b) For zero voltage regulation

$$\text{pf} = \cos\theta = \frac{X}{(R^2 - X^2)^{1/2}}$$

$$= \frac{31.6}{[(7.92)^2 + (31.6)^6]^{1/2}} = 0.97 \text{ leading}$$

or $\theta = 14.1°$ leading

A capacitor C is placed in parallel with the load to improve the power factor to 0.97 leading (Fig. 9.20(b))

$$\bar{V}_2 = 2300\ \angle 0° \text{ V}$$

$$\text{Load current } \bar{I}_L = 4.35\,(0.8 - j\,0.6) = 3.48 - j\,2.61$$

$$\bar{I} = \bar{I}_L + j\,I_C = 3.48 - j\,2.61 + j\,I_C$$

$$\tan 14.1° = \frac{I_C - 2.61}{3.48}$$

or $$I_C = 3.48 \text{ A}$$

$$\text{Rating of copacitor bank} = \frac{2300 \times 3.48}{1000} = 8 \text{ kVA}$$

Since voltage regulation is zero

$$V_1 = 2300 \text{ V}$$

9.8 EFFICIENCY

The efficiency of transformer (or in fact any other device) is

$$\eta = \frac{\text{output power}}{\text{input power}} \tag{9.63}$$

or $$\eta = \frac{\text{output}}{\text{output + losses}} = 1 - \frac{\text{losses}}{\text{output + losses}} \tag{9.64}$$

The transformer has two losses:

– Core (iron) loss P_i which is a constant loss*
– Copper (I^2R) loss, P_c (both windings), a variable loss.

Other transformer losses, which are insignificant for efficiency computation, are

– Load (stray) loss which results from leakage fields inducing eddy currents in tank walls and conductors
– Dielectric loss caused by leakage current in the insulating materials

With reference to Fig. 9.21 of a transformer on load,

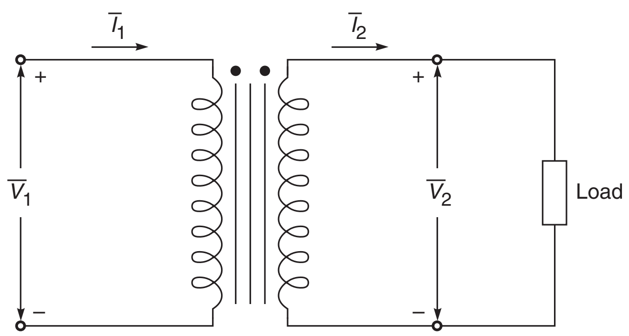


Fig. 9.21 *Transformer on load*

$$\text{output} = V_2 I_2 \cos \theta_2$$

* Operation at constant voltage and frequency.

where $\cos\ \theta_2$ = load pf

From Eq. (9.64)

$$\eta = \frac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + P_i + I_2^2 R_2} \tag{9.65}$$

where R_2 is the equivalent transformer resistance referred to secondary.

Reorganizing Eq. (9.65)

$$\eta = \frac{V_2 \cos\theta_2}{V_2 \cos\theta_2 + (P_i / I_2 + I_2 R_2)} \tag{9.66}$$

For a given pf the efficiency varies with the load current. Maximum efficiency is achieved when Eq. (9.66) has minimum denominator, i.e.

$$I_2^2 R_2 = P_i \tag{9.67}$$

or copper loss = iron loss

or variable loss = constant loss

Example 9.4 The transformer of Ex. 9.3 has a core loss of 75 W at rated voltage.

Compute its efficiency as operated in parts (a) and (b) of that example. Assume the core loss to vary as square of the primary applied voltage.

Also compute the maximum efficiency of the transformer for a load pf of 0.8. What is the value of this load?

Solution

(a) From the solution of part (a) of Ex. 9.3,

$$V_1 = 2410 \text{ V}$$

Core loss $\quad P_i = 75 \times \left(\frac{2410}{2300}\right)^2 = 82.3 \text{ W}$

$$I = 4.35 \text{ A}$$

Copper loss $\quad P_c = (4.35)^2 \times 7.92 = 150 \text{ W}$

Total power loss $\quad P_L = 82.3 + 150 = 232.3 \text{ W}$

Power output $\quad P_0 = 10 \times 0.8 = 8 \text{ kW}$

$$\eta = \frac{8}{8 + 0.232} \times 100 = 97.2\%$$

(b) $\quad V_1 = 2300 \text{ V}$

$$P_i = 75 \text{ W}$$

$$\bar{I} = 3.48 - j\,2.61 + j\,3.48$$
$$= 3.48 + j\,0.87$$

or $\quad I = 3.59 \text{ A}$

$$P_c = (3.59)^2 \times 7.92 = 102 \text{ W}$$
$$P_L = 75 + 102 = 177 \text{ W}$$
$$P_0 = 8 \text{ kW}$$
$$= \frac{8}{8.177} \times 100 = 97.8\%$$

Remarks Notice that because of pf improvement (0.8 lag to 0.97 lead), the load current has reduced in magnitude (4.35 to 3.59 A) therefore reducing the copper loss and raising the efficiency from 97.2% to 97.8%. Even this small efficiency improvement would result in significant saving in energy over a period of time, say one year. The expenditure incurred in the capacitor bank may be worthwhile.

For maximum efficiency

$$I^2R = P_i$$

or
$$I^2 \times 7.92 = 75$$

It is assumed above that $P_i = 75$ W remains constant as the voltage drop in the transformer is small and V_1 and V_2 are both close to the rated values. Now

$$I = \left(\frac{75}{7.92}\right)^{1/2} = 3.08 \text{ A}$$

$$\text{Load} = 2300 \times 3.08 = 7.08 \text{ kVA at } 0.8 \text{ pf}$$
$$= 88.5\% \text{ of full load}$$
$$P_0 = 3.08 \times 2300 \times 0.8 = 5.667 \text{ kW}$$
$$P_L = 2 \times 75 = 150 \text{ W}$$
$$\eta_{\max} = \frac{5.667}{5.817} = 97.4\%$$

Note: Efficiency in part (b) is higher than this value because the pf is 0.97.

9.9 AUTOTRANSFORMER

A two-winding transformer when electrically connected as shown in Fig. 9.22 is known as an autotransformer. Unlike a two-winding transformer the two windings of an autotransformer are not electrically isolated.

Let the two-winding transformer connected as an autotransformer be regarded as ideal. With this assumption, in Fig. 9.22 all voltages will be in phase and so will be all currents. The two-winding voltage ratio is

$$a = \frac{V_1 - V_2}{V_2} = \frac{N_1}{N_2} \tag{9.68}$$

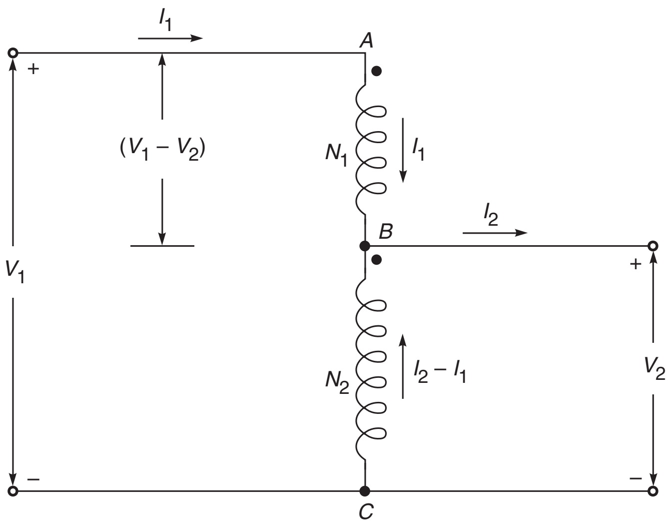


Fig. 9.22 *Autotransformer*

The autotransformer voltage ratio is

$$a' = \frac{V_1}{V_2} = \frac{(V_1 - V_2) + V_2}{V_2}$$

or $$a' = 1 + a \tag{9.69}$$

Now $$(\text{VA})_{\text{TW}} = (V_1 - V_2)I_1 = (I_2 - I_1)V_2 \tag{9.70}$$

$$(\text{VA})_{\text{Auto}} = V_1 I_1 = V_2 I_2$$

But $$\frac{I_2 - I_1}{I_1} = \frac{N_1}{N_2} = a$$

or $$I_1 = \left(\frac{1}{1+a}\right) I_2 \tag{9.71}$$

Substituting Eq. (9.71) in Eq. (9.70),

$$(\text{VA})_{\text{TW}} = \left(1 - \frac{1}{1+a}\right) V_2 I_2$$

$$= \left(1 - \frac{1}{a'}\right) (\text{VA})_{\text{Auto}}$$

or $$(\text{VA})_{\text{Auto}} = \left(\frac{1}{1 - 1/a'}\right) (\text{VA})_{\text{TW}} \tag{9.72}$$

or $$(\text{VA})_{\text{Auto}} > (\text{VA})_{\text{TW}} \tag{9.73}$$

It is easily seen from Eq. (9.72) that the nearer a' is to unity, the larger is $(\text{VA})_{\text{Auto}}$ compared to $(\text{VA})_{\text{TW}}$. An autotransformer is therefore applied for voltage ratios close to unity.

The explanation of Eq. (9.73) lies in the fact that in an autotransformer, part of VA is conducted electrically whereas in a two-winding transformer all VA is transferred magnetically.

Example 9.5 A 2500/250 V, 25 kVA transformer has a core loss of 130 W and full-load copper loss of 320 W. Calculate its efficiency at full load, 0.8 pf.

The transformer is now connected as an autotransformer to give 2500/2750 V. Calculate its kVA rating and efficiency at full load, 0.8 pf. Compare with the two-winding kVA rating and efficiency.

Solution

(i) Two-winding transformer:

$$\text{Power output} = 25 \times 0.8 = 20 \text{ kW}$$
$$\text{Losses} = 130 + 320 = 450 \text{ W}$$
$$\eta = \frac{20}{20.45} \times 100 = 97.8\%$$

(ii) Autotransformer: With reference to Fig. 9.23

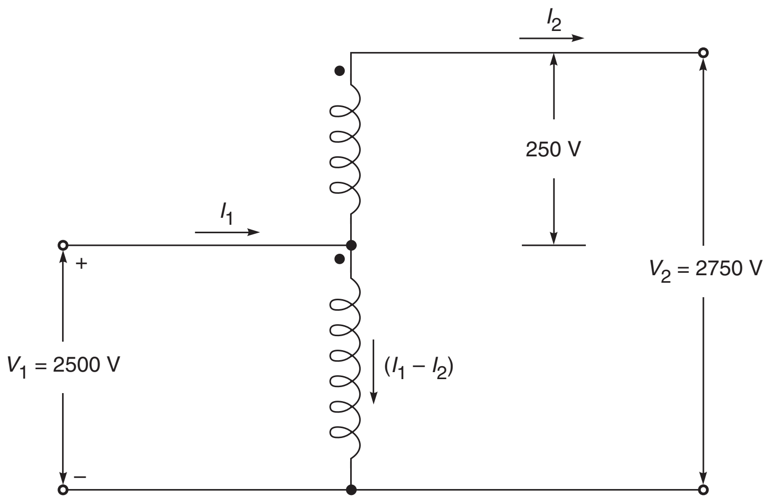


Fig. 9.23

$$I_2 = \frac{25 \times 1000}{250} = 100 \text{ A}$$
$$I_1 - I_2 = \frac{25 \times 1000}{2500} = 10 \text{ A}$$
$$\therefore \quad I_1 = 110 \text{ A}$$
$$\text{kVA rating} = \frac{2500 \times 110}{1000} = \frac{2750 \times 100}{1000} = 275$$

$$\text{Power output} = 2.75 \times 0.8 = 220 \text{ kW}$$

$$\eta = \frac{220}{220 + 0.45} = 99.8\%$$

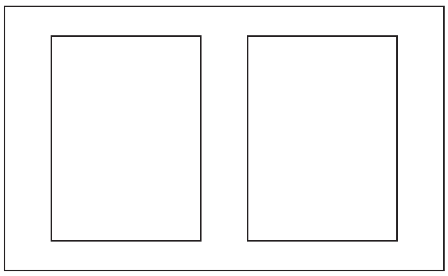
It is seen that when a two-winding transformer is connected as an autotransformer its rating goes up from 25 kVA to 275 kVA and its efficiency from 97.8% to 99.8%. This is possible because a large part of its kVA is transported conductively.

9.10 THREE-PHASE TRANSFORMERS

Three identical single-phase transformers can be connected to form a 3-phase bank. Primary and secondary sides of the bank can be connected in star/delta with various possible arrangements as

– star/star
– delta/delta
– star/delta or delta/star

Instead of three single-phase transformers, it costs about 15% less to have a single 3-limb core as shown in Fig. 9.24 with primary and secondary of a phase wound on each limb. For reasons of economy this arrangement (3-limb core) is popularly used. Of course if one phase is out, the complete transformer must be replaced.



Core type (commonly used)

Fig. 9.24 *Three-phase transformer core*

In finding voltages and currents in a 3-phase transformer along with the ratio of transformation between the coupled windings, one must employ the line and phase relationship of star/delta connections (Secs 6.3 and 6.4) with the assumption that the transformer is feeding a balanced load. Figure 9.25 shows a 3-phase transformer connected in delta on the primary side and star on the secondary side. It is a commonly used connection with low voltage on the delta side and high voltage on the star side. In this figure the coupled windings are drawn parallel to each other for ease of identification. Various line and phase voltages and currents are indicated on

the figure (these follow easily). For a phase-to-phase transformation ratio of $a : 1$ (delta/star)

$$\frac{V_{\text{line}}\,(\text{star})}{V_{\text{line}}\,(\text{delta})} = \frac{\sqrt{3}\,V/a}{V} = \frac{\sqrt{3}}{a}$$

and
$$\frac{I_{\text{line}}\,(\text{star})}{I_{\text{line}}\,(\text{delta})} = \frac{aI/\sqrt{3}}{I} = \frac{a}{\sqrt{3}}$$

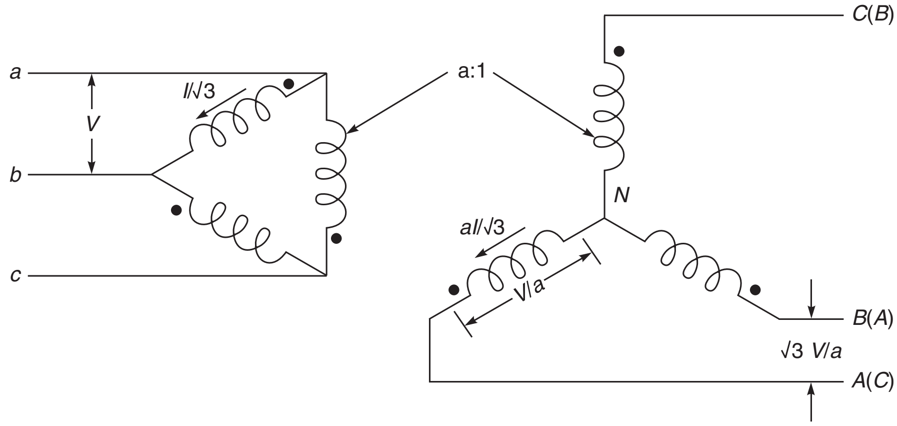


Fig. 9.25 *Delta/star transformer connection (phase shift + 30°)*

Phase Shift

In star/star and delta/delta connection the line voltages and currents are in phase on the primary and secondary sides. However, in a delta/star connection the line voltages and currents undergo a shift in phase which can be ± 30° or ± 90° depending upon the connections.

The delta/star connection of Fig. 9.25 with polarity marks indicated is a commonly used connection. The phasor diagram for voltages is shown in Fig. 9.26. The phase sequence is assumed to be *abc*/*ABC*.

It is observed from the phasor diagram that line voltages on star side lead those of delta side by + 30°. This phase shift would become –30° by changing the phase sequence to *acb*/*ACB*. Relabelling terminals on the star side would make the phase shift ± 90° (for terminal labelling shown in brackets on the star side in Fig. 9.26, phase shift is –90°). The line currents would undergo the same phase shift as voltages in balanced 3-phase loading.

In power system applications of transformers, it is standard practice to connect the transformers (Δ/Y) such that the phase shifts by + 30° in going from LV side to HV side.

Example 9.6 A 3-phase transformer consisting of three 1-phase transformers with turn ratio of 10 : 1 (primary : secondary) is used to supply a 3-phase

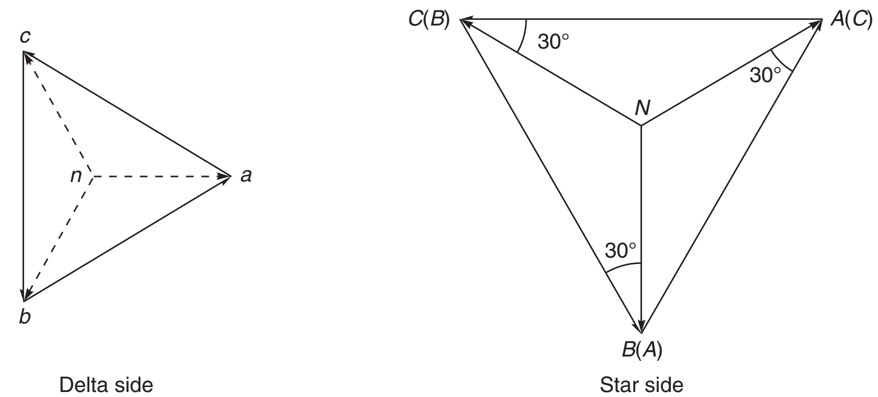


Fig. 9.26 *Voltage phasor diagram of delta/star connection of Fig. 9.25*

load of 120 kVA at 400 V on the secondary side. Calculate the primary line current and voltage if the transformer is connected (a) Δ/Y (b) *Y*/Δ. What is the line-to-line transformation ratio in each case?

Solution

(a) Δ/*Y*-connection (Fig. 9.27(a))

$$I = \frac{120 \times 1000}{\sqrt{3} \times 400} = 173.2 \text{ A}$$

$$\text{Primary line-to-line voltage} = \frac{aV}{\sqrt{3}} = 10 \times \frac{400}{\sqrt{3}} = 2309 \text{ V}$$

$$\text{Primary line current} = \frac{\sqrt{3}I}{a} = 1.732 \times 173.2 \times \frac{1}{10} = 30 \text{ A}$$

Line-to-line transformation ratio (primary/secondary)

$$= \frac{aV/\sqrt{3}}{V} = \frac{a}{\sqrt{3}} = \frac{10}{\sqrt{3}}$$

(b) *Y*/Δ-connection (Fig. 9.27(b))

$$I = 173.2 \text{ A}$$

$$\text{Primary line-to-line voltage} = \sqrt{3}\,aV = \sqrt{3} \times 10 \times 400 = 6928 \text{ V}$$

$$\text{Primary line current} = \frac{I}{a\sqrt{3}} = \frac{173.2}{10 \times 1.732} = 10 \text{ A}$$

$$\text{Line-to-line transformation ratio} = \frac{\sqrt{3}\,aV}{V} = \sqrt{3}\,a = 10\sqrt{3}$$

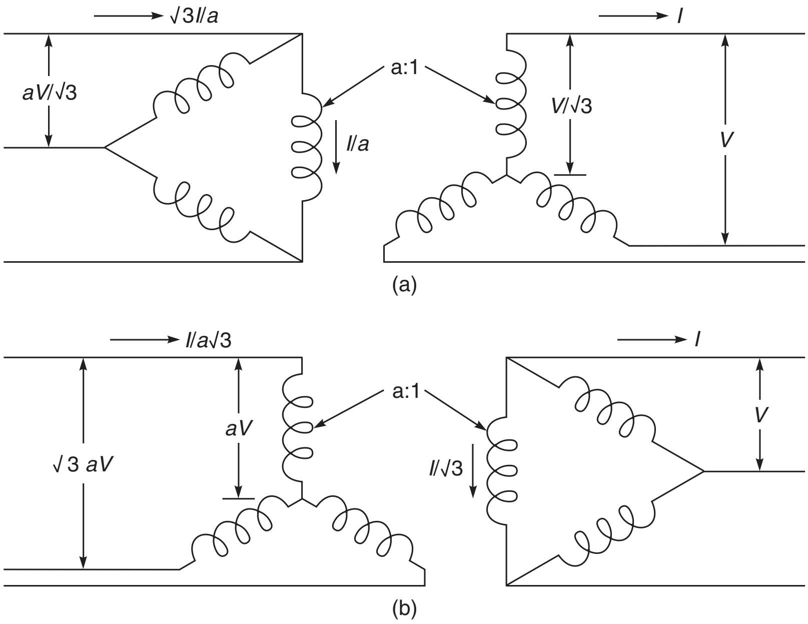


Fig. 9.27

9.11 SPECIAL TRANSFORMERS

Audio-Frequency Transformer

It is used at the output stage of audio frequency electronic amplifier for matching the load to the output impedance of the power amplifier stage. Here the load is fixed but the frequency is variable over a band (audio, 20 Hz to 20 kHz), the response being the ratio V_2/V_1 (Sec. 5.2). A flat frequency response over the frequency band of interest is most desirable. The corresponding phase angle (angle of V_2 wrt V_1) is called phase response. A small angle is acceptable.

Figure 9.28 shows the more exact circuit model of a transformer with frequency variable over a wide range. Here the magnetizing shunt branch is drawn between primary and secondary impedances (resistance and leakage reactance). Also represented is the shunting effect of transformer windings' stray capacitance C_s. In the intermediate frequency (IF) range the shunt branch acts like an open circuit and series impedance drop is also negligibly small such that V_2/V_1 remains fixed (flat response) as in Fig. 9.29.

In the LF (low frequency) region the magnetizing susceptance is low and draws a large current with a consequent large voltage drop in $(r_1 + j\omega L_1)$. As a result V_2/V_1 drops sharply to zero (at dc $B_m = 0$) (Fig. 9.29). In the HF (high frequency) region $B_s = 1/\omega C_s$ (stray capacitance susceptance) has a strong shunting effect and V_2/V_1 drops off as in Fig. 9.29, which shows the complete frequency response of a transformer on logarithmic frequency scale.

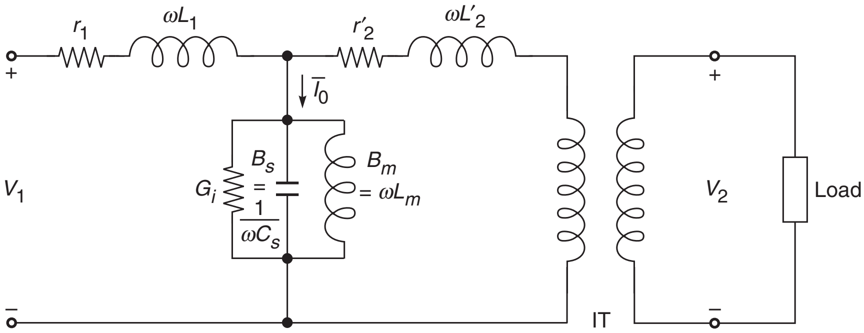


Fig. 9.28

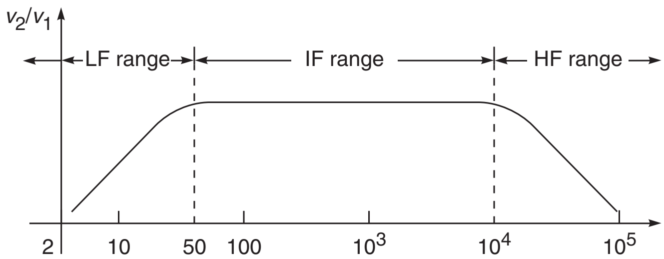


Fig. 9.29 *Frequency response (V_2/V_1 vs log (f)) of a transformer*

Current Transformer (CT)

It is a two-winding transformer whose primary is current excited and secondary is shorted (through an ammeter or current coil of a relay) to produce current proportional to the primary current (in the inverse ratio of turns). In power system use, primary may be a single turn, i.e. the line itself. The secondary is usually rated for 1–5 A and certain VA; the VA of the load (ammeter) is known as the *burden*. The current transformer is used to step-down large currents for measurement and relaying purposes.

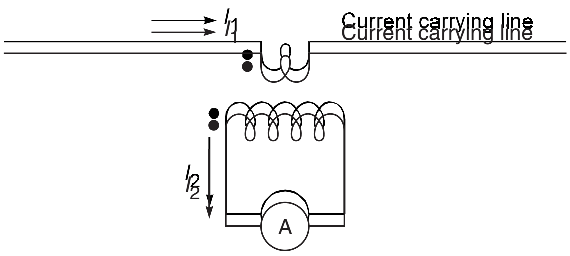


Fig. 9.30 *Current transformer*

Errors in the current measuring ratio of the CT are caused by (i) magnetizing current and (ii) voltage drops in resistances and leakage reactances. Burden should be such that the core does not get saturated otherwise it will draw an abnormally large magnetizing current introducing an intolerably large

error. It may be mentioned here that in relay applications a CT is called upon to measure the short circuit current of the power system.

The secondary of a CT should not be allowed to become open circuited (even inadvertently) otherwise the whole of the primary current acts as a magnetizing current causing extreme permanent magnetization of the core rendering the CT useless as a current ratio transducer.

Additional Solved Problems

9.7 A 50 Hz transformer has 500 turn primary which on no-load takes 60 W power at a current 0.4 A at an input voltage of 220 V. The resistance of the winding is 0.8 Ω. Calculate: (a) the core loss, (b) the magnetizing reactance X_m and (c) the core loss resistance R_i. Neglect leakage reactance.

Solution

$$P\ (\text{in}) = 60\ \text{W},\ I_0 = 0.4\ \text{A}$$

Power loss in winding resistance, $P_{ci} = (0.4)^2 \times 0.8 = 0.128$ W

(a) core loss $\quad P_i = 60 - 0.128 = 59.88\ \text{W} \approx 60\ \text{W}$

Observe that the winding resistance loss at no-load can be easily ignored.

(b) $\quad \cos\ \theta_0 = 60/(220 \times 0.4) = 0.682$ lagging. $\theta_0 = 47°$

$$I_m = 0.4 \sin\ \theta_0 = 0.293\ \text{A}$$

$$220/X_m = I_m = 0.293\ \text{A}$$

or $\quad X_m = 751\ \Omega$

(c) $\quad I_i = 0.4 \cos\ \theta = 0.273\ \text{A}$

$$220/R_i = 0.273$$

or $\quad R_i = 806\ \Omega$

9.8 A 15 kVA, 2200/220 V, 50 Hz transformer gave the following test results:

OC (LV side): 220 V, 2.72 A, 185 W
SC (HV side): 112 V, 6.3 A, 197 W

Compute the following:

(a) core loss, (b) full-load copper loss, (c) efficiency at full load, 0.85 lagging pf and (d) voltage regulation at full-load, 0.8 lagging/ leading pf.

Solution

$$\text{turn ratio} = 2200/220 = 10$$

(a) core loss $P_i = 185$ W

(b) I_{HV} (fl) = 15,000/2200 = 6.82 A

full-load copper loss

$$P_C \text{ (fl)} = (6.82/6.3)^2 \times 197 = 231 \text{ W}$$

(c) P (out) = 15 × 0.85 = 12.75 kW

$$P_L = P_i + P_c \text{ (fl)} = 185 + 231 = 416 \text{ W}$$

$$\eta = 12.75/(12.75 + 0.416) = 96.8\%$$

(d) Z(HV) = 112/6.3 = 17.78 Ω

$$R(\text{HV}) = 197/(6.3)^2 = 4.96\ \Omega$$

$$X(\text{HV}) = 17.07\ \Omega$$

$$\text{voltage drop} = 6.82\ (4.96 \times 0.8 \pm 17.07 \times 0.6)$$

$$= 96.92\text{V}, -42.76\text{ V}$$

$$\text{Voltage regulation} = +96.92/2200 = +4.41\%\ (0.8 \text{ lagging pf})$$

$$= -42.76/2200 = -1.94\%\ (0.8 \text{ leading pf})$$

9.9 The maximum efficiency of a 50 kVA transformer is 97.4% and occurs at 90% of the full load. Calculate the efficiency of the transformer at (a) full-load, 0.8 pf, and (b) (1/2) full load at 0.9 pf.

Solution

(a) $$\sqrt{P_i / P_c(\text{fl})} = 0.9$$

$$P_i/P_c(\text{fl}) = 0.81$$

$$P_L = \frac{1 - 0.974}{0.974} \times 45 = 1.33 \text{ kW} = 2P_i = 2P_c \text{ (0.9 fl)}$$

$$\therefore \quad P_i = 0.665 \text{ kW}$$

$$2 \times (0.9)^2\ P_c \text{ (fl)} = 1.33 \text{ or } P_c \text{ (fl)} = 0.82 \text{ kW}$$

$$\eta = 40/41.485 = 96.4\%$$

(b) (1/2) full-load, 0.9 pf

$$p \text{ (out)} = 25 \times 0.9 = 22.5 \text{ kW}$$

$$P_L = 0.665 + 0.82/4 = 0.87 \text{ kW}$$

$$\eta = 22.5/(22.5 + 0.87) = 96.3\%$$

9.10 A 500 kVA transformer has 95% efficiency at full load and also at 60% of full load both at upf.

(a) Separate out the transformer losses.

(b) Determine the transformer efficiency at 75% full load, upf.

Solution

(a) $$\frac{500}{500 + P_i + P_c} = 0.95 = \frac{300}{300 + P_i + 0.36\ P_c}$$

which gives the following two equations.

$$P_i + P_c = 26.32$$
$$P_i + 0.36\ P_c = 15.79$$

Solving, we get

$$P_c = 16.45 \text{ kW}, \qquad P_i = 9.87 \text{ kW}$$

(b)
$$P_L = 9.87 + (0.75)^2 \times 16.45 = 19.12 \text{ kW}$$
$$\eta = (500 \times 0.75)/(500 \times 0.75 + 19.12) = 95.15\%$$

Problems

9.1 A single-phase transformer is rated 600/200 V, 25 kVA, 50 Hz.

(a) Calculate the magnitude of primary and secondary currents when the transformer is fully loaded (use IT model).

(b) What should be the impedance of the load in ohms to fully load the transformer when connected on (i) 600 V side and (ii) 200 V side (use IT model)?

(c) What would be the value of the maximum core flux when the transformer is excited at rated voltage on either side, given $N_1 = 60$ turns?

(d) If the transformer is operated from a 60 Hz, source, what should be its voltage rating for the maximum core flux to stay at the same value as in part (c).

(e) If the 600 V side is excited at 600 V, 40 Hz, what would be the core flux and the secondary voltage? What effect do you expect to observe in the core under these conditions?

9.2 A 25 kVA, 600/200 V transformer is subjected to an SC test. The voltage applied on one side, with the other shorted, is 5.2% of the rated voltage. The transformer draws rated current and a power of 242 W during the test.

(a) Compute equivalent resistance and leakage reactance of the transformer in ohms on either side and in pu.

(b) Compute the core flux as a percentage of core flux at rated voltage.

(c) From part (b) justify that all the 242 W constitute ohmic losses.

9.3 The transformer of Prob. 9.2 is fed from a 600 V source. A load impedance of $\bar{Z}_L = 1.48 + j\,1.04\ \Omega$ is connected across the secondary.

(a) Find currents in both windings assuming transformer to be ideal.

(b) Solve part (a) again by taking the transformer impedance into account.

(c) Calculate voltage regulation of the transformer.

9.4 The transformer of Prob. 9.2 is OC tested from a 600 V source with the secondary open. The transformer draws a power of 195 W.

Based upon the SC and OC test data compute the efficiency of the transformer when loaded as in Prob. 9.3.

9.5 A 50 kVA, 1100/220 V, 50 Hz transformer has an HV winding resistance of 0.125 Ω and a leakage reactance of 0.625 Ω. The LV winding has corresponding values of 0.005 Ω and 0.025 Ω respectively. Find the equivalent impedance of the transformer referred to HV and LV sides. Find the pu impedance of the transformer.

9.6 Consider the transformer of Prob. 9.5 to give its rated output at (a) 0.8 lagging pf and (b) 0.8 leading pf on the LV side. Find the HV terminal voltage and % regulation. Use pu system.

9.7 The transformer of Prob. 9.5 has a core loss of 580 W. Find its efficiency at 3/4th full load, 0.8 lagging pf.

9.8 The transformer or Prob. 9.5 when operating as in part (a) of Prob. 9.6 gets shorted at LV terminals. Find the steady-state current which would be drawn by the HV if the source voltage is assumed to remain constant.

9.9 The transformer of Prob. 9.5 is fully loaded on the secondary side at (a) 0.8 lagging, (b) 0.8 leading pf while it is fed on the primary side at 1100 V. Calculate the voltage at the secondary terminals.

9.10 The circuit model of a 5 kVA, 200/400 V, single-phase transformer, referred to the LV side, is shown in Fig. P9.10.

(a) An OC test is conducted from the HV side at 400 V. Calculate the power input, power factor and current (magnetizing) drawn by the transformer.

(b) An SC test is conducted from the LV side by allowing full-load current to flow. Calculate the voltage required to be applied, the power input and power factor.

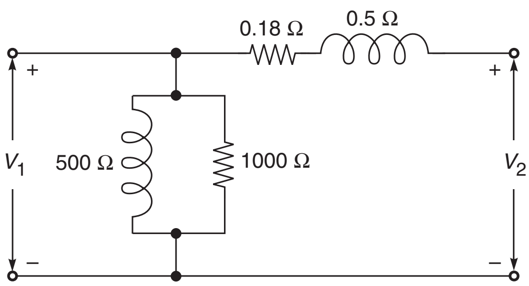


Fig. P9.10

9.11 The following test results were obtained on a 20 kVA, 2200/220 V transformer:

OC test (LV): 220 V, 1.1 A, 125 W

SC test (HV): 52.7 V, 8.4 A, 287 W

(a) The transformer is loaded at unity pf on secondary side with a voltage of 220 V. Determine the maximum efficiency and the load at which it occurs.

(b) The transformer is fully loaded. Determine the load pf for zero voltage regulation.

9.12 A 1000/200 V, 25 kVA transformer is connected as an autotransformer to yield a transformation ratio of 1000/1200 V. Calculate its kVA rating. Calculate also the currents in the two windings when the autotransformer is fully loaded.

9.13 A variable-voltage laboratory transformer is shown in Fig. P9.13. With the transformer fully loaded, compute the ratio of I_1/I_2 when

(a) the sliding contact is adjusted to 50% of input voltage and.

(b) when it is placed at 10% voltage.

(c) Is it permissible to place the sliding contact at the extreme positions?

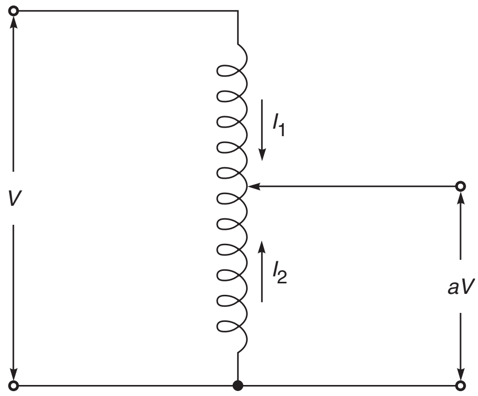


Fig. P9.13

9.14 A 20 kVA, 2400/240 V, two-winding transformer has an efficiency of 97.5% at full load, 0.8 pf. It is connected as a 2400/2640 V autotransformer. At full load calculate the kVA output, kVA transformed and kVA conducted. Find also the efficiency at full load, unity power factor.

9.15 A Δ/Y connected 3-phase transformer as shown in Fig. 9.25 has a voltage ratio of 22 kV (Δ)/345 kV (Y) (line-to-line). The transformer is feeding 500 MW and 100 MVAR to the grid (345 kV). Determine the kVA and voltage rating of each unit and compute all currents and voltages in both magnitude and phase in lines and all the windings (3 primaries and 3 secondaries). Assume the transformer units to be ideal.

9.16 Three identical transformers each rated 6.6/22 kV, 3 MVA, are connected in Y/Y. The transformer bank is fed from a source of line voltage

$6.6\sqrt{3}$ kV. The secondary side feeds a delta-connected load composed of three equal impedances. Assuming the individual transformers to be ideal find

(a) the value of Z in ohms to fully load the bank (i.e. 9 MVA),
(b) the current in each leg of the load (Δ connected) and
(c) the current in each transformer primary and secondary.

9.17 The three transformers of Prob. 9.16 are connected in Δ/Y and are fed from 6.6 kV (line-to-line) source on Δ side. The load comprises three Δ-connected impedances. Assuming all the three transformers to be ideal solve for all parts of Prob. 9.16. Also find the primary and secondary side line currents.