# Tutorial 2

# Fluid pressure

1. A cylinder contains a fluid at a gauge pressure of 360 $KN/m^2$. Express this pressure in terms of a head of (a) water, and (b) mercury of sp gr = 13.6
What would be the absolute pressure in the cylinder if atmospheric pressure is 760mm Hg.

Solution:
Pressure (P) = 360 $KN/m^2$ = $360x10^3$ $N/m^2$
Head (h) = ?
$P = \rho g h$ where ρ = Density of fluid

$$h = \frac{P}{\rho g}$$

a) Head in terms of water (ρ = 1000 $kg/m^3$)

$h = \frac{360x10^3}{1000x9.81}$ = 36.7m

b) Head in terms of mercury
ρ = sp gr x density of water = 13.6x1000 = 13600 $kg/m^3$

$h = \frac{360x10^3}{13600x9.81}$ = 2.7m

Atmospheric pressure (h) = 760mmhg = 0.76m hg
Atmospheric pressure $(Patm) = \rho_{mercury} gh = 13600x9.81x0.76$ = 101396$N/m^2$ = 101.3$KN/m^2$
Absolute pressure (Pabs) = ?
Pabs = Pgauge + Patm = 360+101.3 = 461.3$KN/m^2$

2. What would the pressure in $kN/m^2$ be if the equivalent head is measured as 400mm of (a) mercury (sp gr 13.6) (b) water (c) oil specific weight 7.9 $kN/m^3$ (d) a liquid of density 520 $kg/m^3$?

Solution:
Head (h) = 400mm = 0.4m
Pressure (P) =?
$P = \rho g h$ where ρ = Density of fluid

a) In terms of mercury, ρ = sp gr x density of water = 13.6x1000 = 13600 $kg/m^3$
$P = \rho g h$ = 13600x9.81x0.4 = 53366 $N/m^2$ = 53.366 $KN/m^2$
b) In terms of water, ρ = 1000 $kg/m^3$
$P = \rho g h$ = 1000x9.81x0.4 = 3924 $N/m^2$ = 3.924 $KN/m^2$
c) In terms of oil of sp. wt. ($\gamma_{oil}$) = 7.9 $kN/m^3$

$P = \rho gh = \gamma_{oil} h$ = 7.9x.4 = 3.16 KN/m$^2$
d) In terms of liquid with ρ = 520 kg/m$^3$
$P = \rho gh$ = 520x9.81x0.4 = 2040 N/m$^2$ = 2.04 KN/m$^2$

3. A manometer connected to a pipe indicates a negative gauge pressure of 50mm of mercury. What is the absolute pressure in the pipe in N/m$^2$ if the atmospheric pressure is 1 bar?

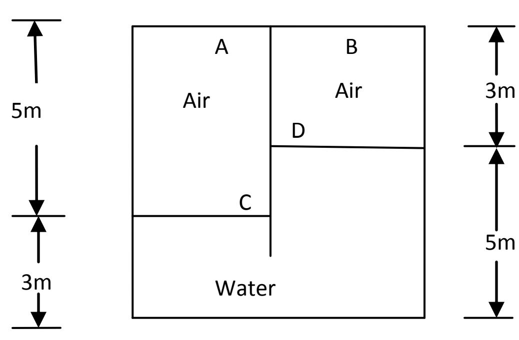
Solution:
Atmospheric pressure (Patm) = 1 bar = 1x10$^5$ N/m$^2$
Head (h) = -50mmhg = -0.05m hg
Absolute pressure (pabs) = ?

ρ of mercury = sp gr x density of water = 13.6x1000 = 13600 kg/m$^3$
Gauge pressure (Pgauge) $= \rho gh$ = -13600x9.81x0.05 = -6671 N/m$^2$
Pabs = Pgauge + Patm = -6671+1x10$^5$ = 93329 N/m$^2$ = 93.3KN/m$^2$

4. An open tank contains 5.7m of water covered with 2.6m of kerosene (sp wt = 8KN/m$^3$). Find the pressure at the interface and at the bottom of the tank.

Solution:
Height of kerosene (h) = 2.6m
Height of water (h1) = 5.7m
Sp wt of kerosene ($\gamma$) = 8KN/m$^3$
Pressure at interface (Pint) = ?
Pressure at bottom (Pbottom) = ?
$Pint = \gamma h$ = 8x2.6 = 20.8KN/m$^2$
$Pbottom = Pint + \gamma_{water} h1$ = 20.8+9.81x5.7 = 76.7 KN/m$^2$

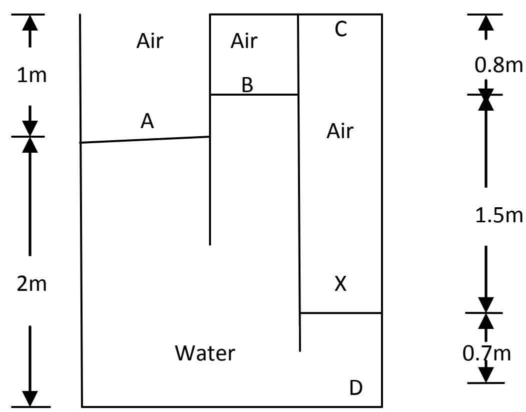
5. The closed tank in the fig. is at 20$^0$ C. If the pressure at point A is 96 Kpa absolute, what is the absolute pressure at point B? What percent error results from neglecting the specific weight of air? (Take sp wt of air = 0.0118 KN/m$^3$)




Solution:

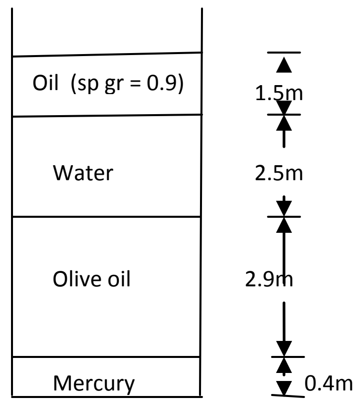
Sp wt of air ($\gamma_{air}$) = 0.0118 KN/m$^3$
Sp wt of water ($\gamma_{water}$) = 9.81 KN/m$^3$
Starting from A,
$P_A + P_{AC} - P_{CD} - P_{DB} = P_B$
$P_A + \gamma_{air} h_{AC} - \gamma_{water} h_{DC} - \gamma_{air} h_{DB} = P_B$
$P_B$ = 96 + 0.0118x5 – 9.81x2 – 0.0118x3 = 76.404 Kpa
Neglecting air,
$P_A - \gamma_{water} h_{DC} = P_B$
$P_B$ = 96 – 9.81x2 = 76.38 Kpa
Error = (76.404-76.38)/76.404 = 0.00031 = 0.031%

6. In the fig., the pressure at point A is 2900 N/m$^2$. Determine the pressures at points B, C and D. (Take density of air = 1.2 kg/m$^3$)




Solution:
Density of water (ρ) = 1000kg/m$^3$
$P_A$ = 2900 N/m$^2$
Density of air ($\rho_{air}$) = 1.2 kg/m$^3$
$P_B$ = ?, $P_C$ =?, $P_D$ = ?
Starting from A,
$P_B = P_A - \rho g h_{between\ A\ and\ B}$ = 2900 – 1000x9.81x0.2 = 938 N/m$^2$
$P_C = P_A + \rho g h_{between\ A\ and\ X} - \rho_{air} g h_{XC}$ = 2900 + 1000x9.81x1.3 -1.2x9.81x2.3 = 15626 N/m$^2$
$P_D = P_A + \rho g h_{between\ A\ and\ D}$ = 2900 + 1000x9.81x2 = 22520 N/m$^2$

7. In the fig., the absolute pressure at the bottom of the tank is 233.5 Kpa. Compute the sp gr of olive oil. Take atmospheric pressure = 101.3 Kpa.




Solution:

Absolute pressure at bottom ($P_{abs}$) =233.5 Kpa

Atmospheric pressure ($P_{atm}$) = 101.3 Kpa

Sp wt of water ($\gamma$) = 9.81 KN/$m^3$

Sp wt of oil ($\gamma_{oil}$) = 0.9x9.81 KN/$m^3$ = 8.829 KN/$m^3$

Sp wt of mercury ($\gamma_m$) = 13.6x9.81 KN/$m^3$ = 133.416 KN/$m^3$

Sp gr of olive oil (S) = ?

$P_{abs} = P_{atm} + P_{gauge}$

$P_{abs} = P_{atm} + P_{oil} + P_{water} + P_{olive\ oil} + P_{mercury}$

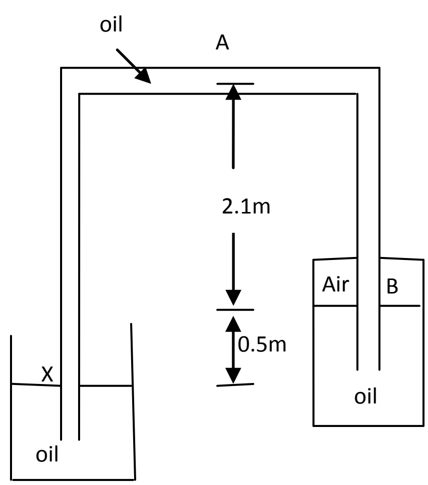
$$233.5 = 101.3 + \gamma_{oil}\, h_{oil} + \gamma\, h_{water} + \gamma_{olive\ oil}\, h_{\,olive\ oil} + \gamma_m\, h_{mercury}$$

$$233.5 = 101.3 + 8.829x1.5 + 9.81x\ 2.5 + \gamma_{olive\ oil}\, x2.9 + 133.416x0.4$$

$$\gamma_{olive\ oil} = 14.16 \text{ KN/m}^3$$

$$S = \frac{\gamma_{olive\ oil}}{\gamma} = \frac{14.16}{9.81} = 1.44$$

8. The tube shown in the fig. is filled with oil of sp gr 0.82. Determine the pressure heads at A and B in meters of water.

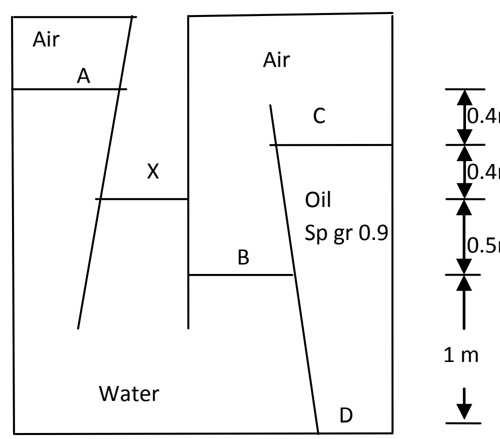


Solution:
sp gr of oil = 0.82
Sp wt of oil ($\gamma_{oil}$) = 0.82x9810 = 8044.2 N/m$^3$
Head in terms of water at a and B ($h_A$ and $h_B$) = ?
Take atmospheric pressure to be 0 for gauge pressure.
$P_A = 0 - \gamma_{oil}\, h_{between\ X\ and\ A}$ = - 8044.2x2.6 = -20914.9 Pa

$h_A = \frac{P_A}{\gamma_{water}} = -\frac{20914.9}{9810}$ = -2.132 m
$P_B = P_A + \gamma_{oil}\, h_{between\ A\ and\ B}$ = -20914.9 + 8044.2x2.1= -4022.1 Pa

$h_B = \frac{P_B}{\gamma_{water}} = -\frac{4022.1}{9810}$ = -0.41 m

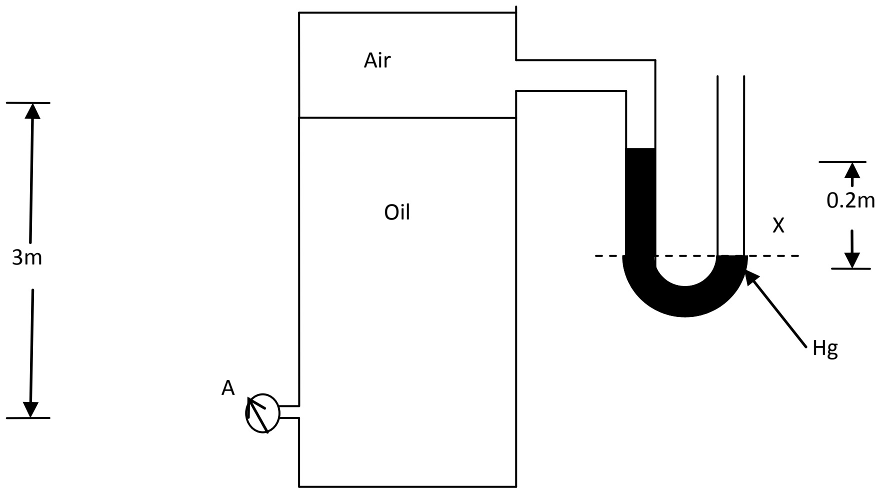
9. Calculate the pressures at A, B, C and D in the fig.




Solution:
Sp wt of water ($\gamma$) = 9810 N/m$^3$
sp gr of oil = 0.9
Sp wt of oil ($\gamma_{oil}$) = 0.9x9810 = 8829 N/m$^3$

Take atmospheric pressure to be 0 for gauge pressure.
$P_A = 0 - \gamma\, h_{between\ X\ and\ A}$ = - 9810x0.8 = 7848 Pa
$P_B = 0 + \gamma\, h_{between\ X\ and\ B}$ = 9810x0.5 = 4905 Pa
Neglecting air, $P_C$ =$P_B$ = 4905 Pa
$P_D = P_C + \gamma_{oil}\, h_{between\ C\ and\ D}$ = 4905 + 8829x1.9 = 21680 Pa

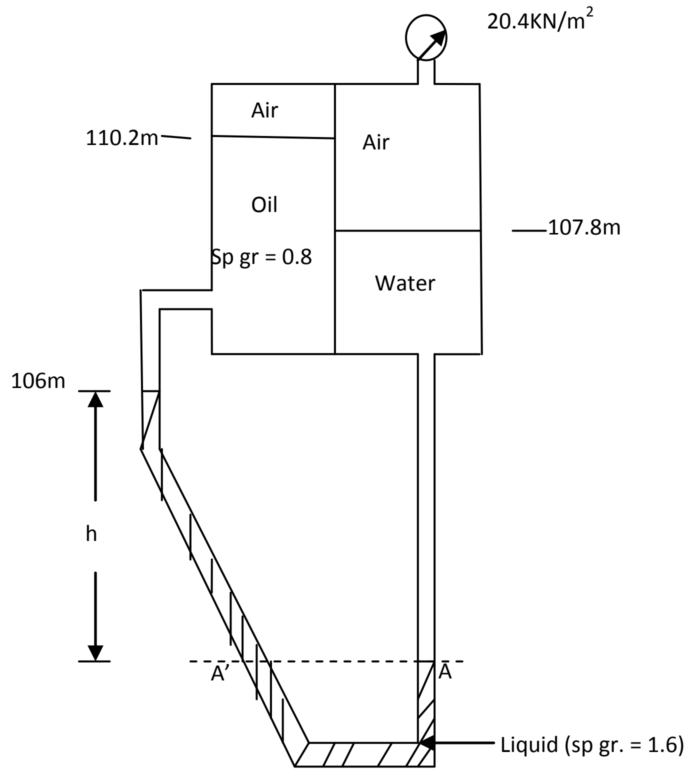
10. The tank in the fig. contains oil of sp gr 0.75. Determine the reading of gauge A in N/m$^2$.




Solution:
sp gr of oil = 0.75
Sp wt of oil ($\gamma_{oil}$) = 0.75x9810 = 7357.5 N/m$^3$
Sp wt of mercury ($\gamma_m$) = 13600x9.81 N/m$^3$ = 133416 N/m$^3$

Take atmospheric pressure to be 0 for gauge pressure.
Starting from X and neglecting air,
$0 - \gamma_m h_m + \gamma_{oil}\, h_{oil} = P_A$
PA = -133416x0.2+7357.5x3 = -4610.7 N/m$^2$

11. In the left hand of the fig., the air pressure is -225mm of Hg. Determine the elevation of the gauge liquid in the right hand column at A.




Solution:

Air pressure at the left hand tank = -225mm Hg = -0.225m Hg

Sp wt of water ($\gamma$) = 9810 N/m$^3$

Sp wt of oil ($\gamma_{oil}$) = 0.8x9810 = 7848 N/m$^3$

Sp wt of mercury ($\gamma_m$) = 13600x9.81 KN/m3 = 133416 KN/m$^3$

Sp wt of liquid ($\gamma_{liquid}$) = 1.6x9810 = 15696 N/m$^3$

$P_{air} = -0.225\gamma_m$ = -0.225x133416 = -30018.6 N/m$^2$

$P_{A'} = P_A$

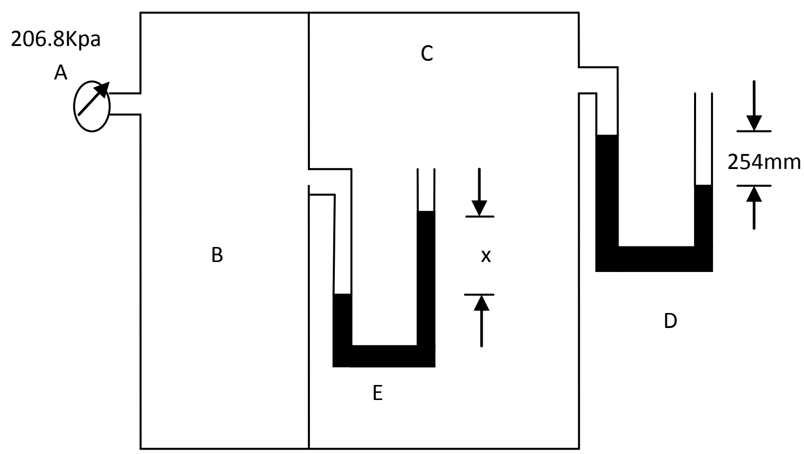
$P_{air} + \gamma_{oil}\, h_{oil} + \gamma_{liauid}\, h_{liquid} = 20400 + \gamma h_{water}$

-30018.6 + 7848x(110.2-106) + 15696 xh = 20400 + 9810 x(107.8-106+h)

h = 5.96m

Elevation at A = 106-5.96 = 100.04m

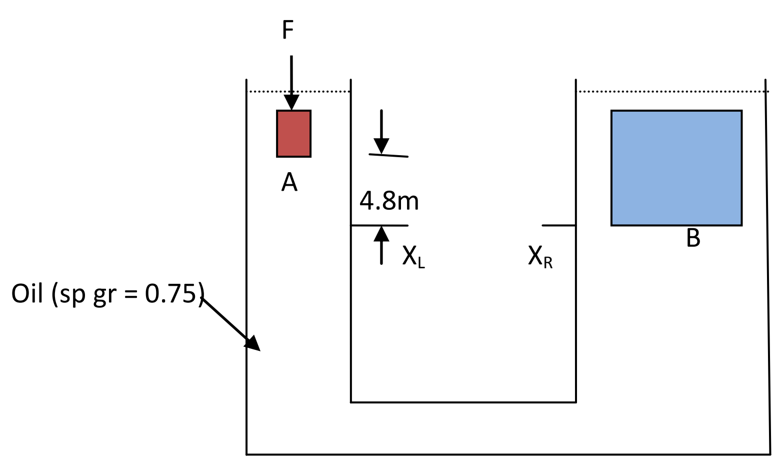
12. Compartments B and C in the fig. are closed and filled with air. The barometer reads 99.98 Kpa. When gages A and D read as indicated, what should be the value of x for gage E? (Hg in each tube)




Solution:
$P_A$ = 206.8 KPa = 206800 Pa
Sp wt of mercury ($\gamma_m$) = 13.6x9810 N/m$^3$ = 133416 N/m$^3$

Starting from A and neglecting air,
$P_A - \gamma_m\, X + \gamma_m x\; 0.254 = 0$
206800 – 133416 X + 133416x0.254 = 0
X = 1.8m

13. In the fig., the areas of the plunger A and cylinder B are 38.7 cm$^2$ and 387 cm$^2$, respectively, and the weight of B is 4500 N. The vessel and the connecting passages are filled with oil of specific gravity 0.75. What force F is required for equilibrium, neglecting the weight of A?

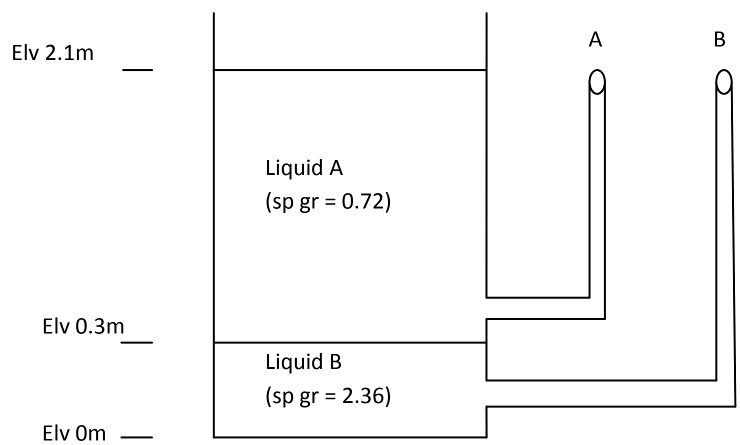


Solution:
Area of A ($A_A$) =38.7 cm$^2$, Area of B ($A_B$) =387 cm$^2$
Weight of B ($W_B$) = 4500N
Sp wt of water ($\gamma$) = 9810 N/m$^3$
Sp wt of oil ($\gamma_{oil}$) = 0.75x9810 N/m$^3$ = 7357.5 N/m$^3$

Pressure at $X_L$ = Pressure at $X_R$
$P_A + \gamma_{oil}\, h_{oil\ from\ A\ to\ X_L} = W_B/A_B$
$P_A$ + 7357.5x4.8 = 4500/387x10$^{-4}$
$P_A$ = 80963 N/m$^2$

F = $P_A\, A_A$ = 80963 x 38.7x10$^{-4}$ = 313N

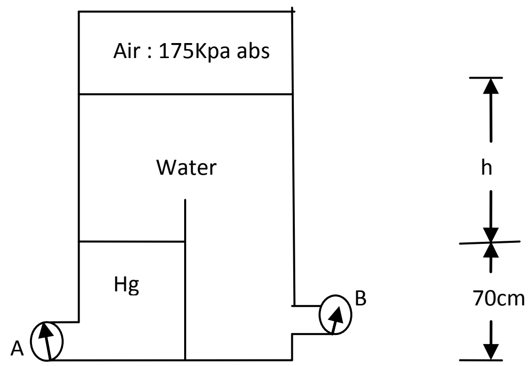
14. For the open tank, with piezometers attached on the side, containing two different immiscible liquids as shown in the fig., find (a) the elevation of liquid surface in piezometer A, (b) the elevation of liquid surface in piezometer B, and (c) the total pressure at the bottom of the tank.




Solution:
Sp wt of liquid A ($\gamma_A$) = 0.72x9810 N/m$^3$ = 7063.2 N/m$^3$
Sp wt of liquid B ($\gamma_B$) = 2.36x9810 N/m$^3$ = 23151.6 N/m$^3$

a) Elevation of liquid surface in piezometer A = elevation of liquid A in the tank = 2.1 m
b) Pressure due to A at the interface ($P_A$) = $\gamma_A\, h_{liquid\ A}$ = 7063.2 x (2.1-0.3) = 12713.8 N/m$^2$
Equivalent head for liquid B due to $P_A$ is
$h_A = P_A/\gamma_B$ = 12713.8/23151.6 = 0.55m
Elevation of liquid surface in piezometer B = Elv of liquid at B + $h_A$ = 0.3 + 0.55 = 0.85m
c) Total pressure at the bottom = $\gamma_A\, h_{liquid\ A} + \gamma_B\, h_{liquid\ B}$
= 7063.2 x (2.1-0.3) + 23151.6x0.3 = 19659 N/m$^2$

15. In the fig., gage A reads 290Kpa abs. What is the height of water h? What does gage B read?

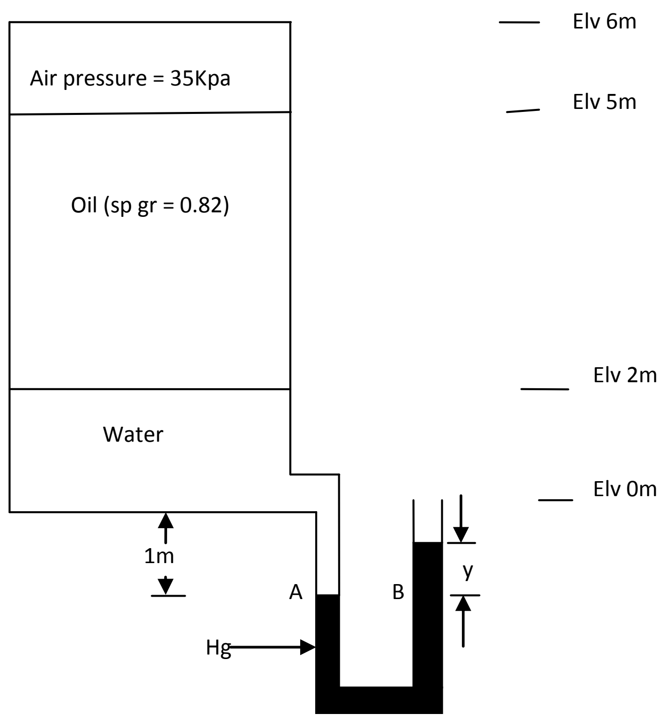



Solution:
Sp wt of water ($\gamma$) = 9.81 KN/m$^3$
Sp wt of mercury ($\gamma_m$) = 13.6x9.81 = 133.416 KN/m$^3$
$P_A$ =290Kpa abs
h = ?
$P_B$ =?

$P_A = 175 + \gamma h + \gamma_m h_m$
290 = 175 + 9.81h+133.416x0.7
h = 2.2m

$P_B = 175 + \gamma\ (h + 0.7)$
$P_B$ = 175 + 9.81 (2.2+0.7) = 203.4 KN/m$^2$

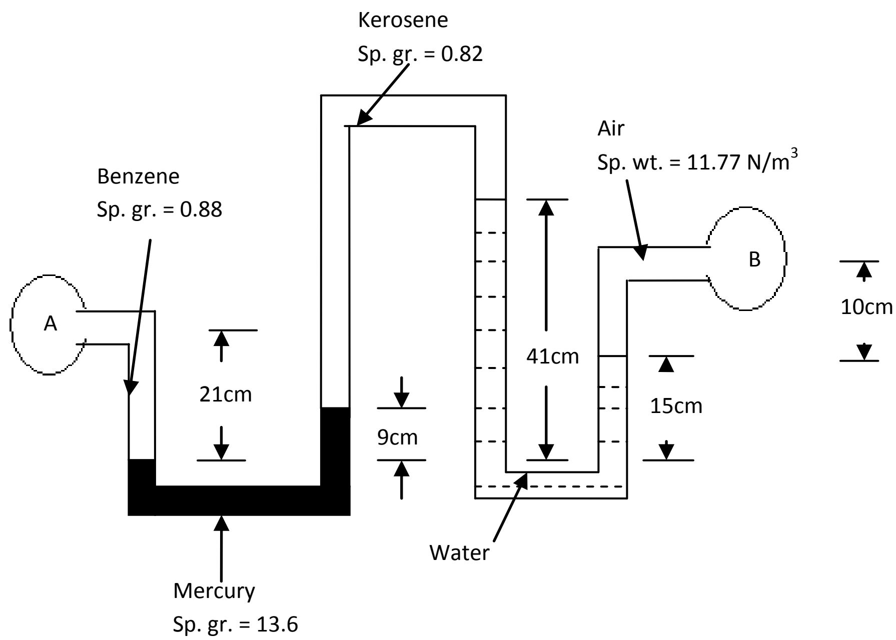
16. A manometer is attached to a tank containing three different fluids as shown in fig. What will be the difference in elevation of the mercury column in the manometer (i.e. y)?




Solution:
Sp wt of water ($\gamma$) = 9.81 KN/m$^3$
Sp wt of mercury ($\gamma_m$) = 13.6x9.81 = 133.416 KN/m$^3$
Sp wt of oil ($\gamma_{oil}$) = 0.82x9.81 = 8.0442 KN/m$^3$

$P_A = P_B$
$35 + \gamma_{oil} h_{oil} + \gamma h = \gamma_m \, y$
35+ 8.0442x3+9.81x3= 133.416y
y = 0.66m

17. Determine the pressure difference between two points A and B in the fig.



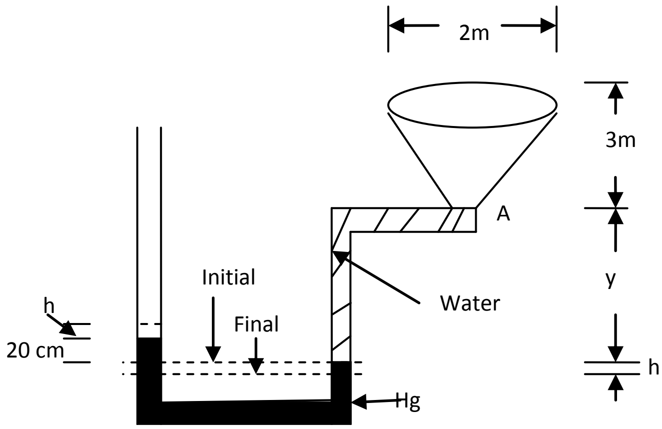

Solution:

Sp wt of water ($\gamma$) = 9.81 KN/m$^3$
Sp wt of mercury ($\gamma_m$) = 13.6x9.81 = 133.416 KN/m$^3$
Sp wt of benzene ($\gamma_B$) = 0.88x9.81 = 8.6328 KN/m$^3$
Sp wt of kerosene ($\gamma_K$) = 0.82x9.81 = 8.0442 KN/m$^3$
Sp wt of air ($\gamma_{air}$) = 0.01177 KN/m$^3$

Starting from A,

$$P_A + \gamma_B x0.21 - \gamma_m x0.09 - \gamma_K x(0.41 - 0.09) + \gamma(0.41 - 0.15) - \gamma_{air} x0.1 = P_B$$
$$P_A + 8.6328x0.21 - 133.416x0.09 - 8.0442x0.32 + 9.81x\ 0.26 - 0.01177x0.1 = P_B$$
$P_A - P_B$ = 10.22 Kpa

18. Fig. below shows a conical vessel having its outlet at A to which a U-tube manometer is connected. The reading of the manometer given in the fig. shows when the vessel is empty. Find the reading of the manometer when the vessel is completely filled with water.

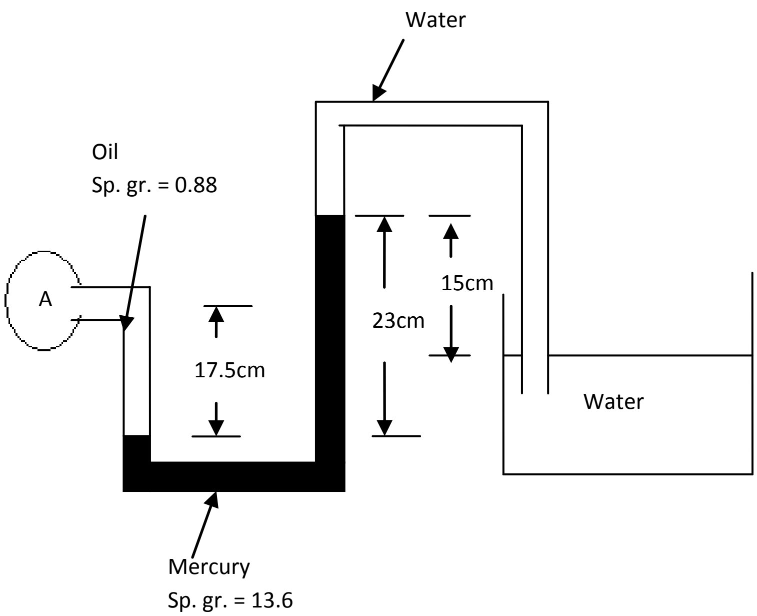



Solution:
Sp wt of water ($\gamma$) = 9.81 KN/m$^3$
Sp wt of mercury ($\gamma_m$) = 13.6x9.81 = 133.416 KN/m$^3$

When the vessel is empty, equating pressure at initial level
$\gamma y = \gamma_m x 0.2$
9.81 y = 133.416x0.2
y = 2.72m

When the vessel is filled with water, let us say the mercury moves by height h
Equating pressure at final level
$\gamma(3 + y + h) = \gamma_m(h + 0.2 + h)$
$9.81(3 + 2.72 + h) = 133.416(0.2 + 2h)$
h = 0.1145m
Deflection of mercury = 2h+0.2 =2x0.1145+0.2 = 0.429m

19. Compute the absolute pressure at point A in the fig.




Solution:

Sp wt of water ($\gamma$) = 9.81 KN/m$^3$

Sp wt of mercury ($\gamma_m$) = 13.6x9.81 = 133.416 KN/m$^3$

Sp wt of oil ($\gamma_{oil}$) = 0.88x9.81 = 8.6328 KN/m$^3$

Starting from A,

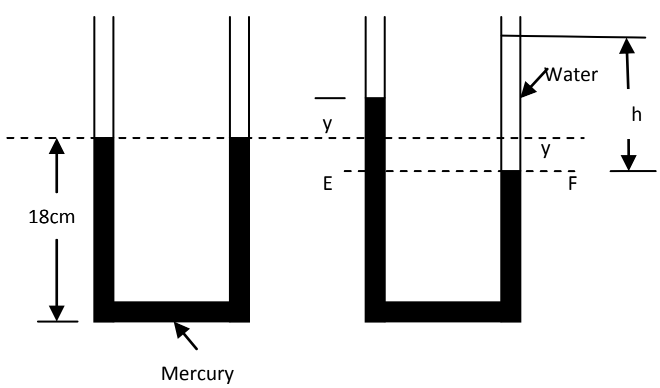
$P_A + \gamma_{oil}x0.175 - \gamma_m x0.23 + \gamma x0.15 = 0$

$P_A + 8.6328x0.175 - 133.416x0.23 + 9.81x0.15 = 0$

PA = 27.7 Kpa

Absolute pressure at A = Patm +27.7 = 101.3+27.7 = 129Kpa

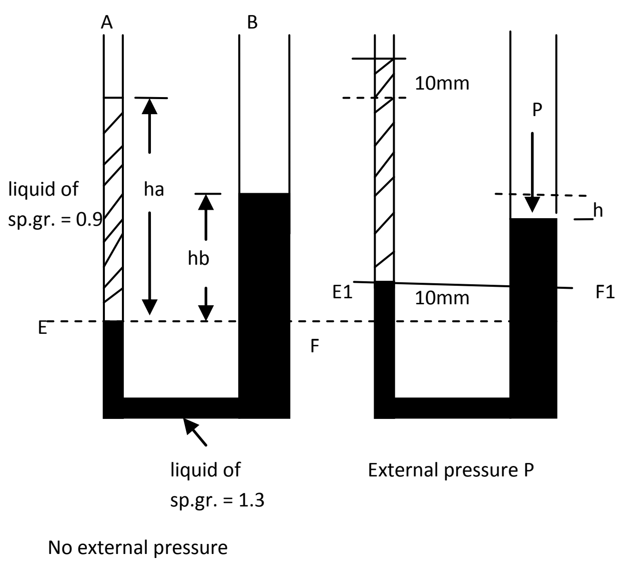
20. The fig. shows a 1cm diameter U-tube containing mercury. If now 20cc of water is poured into the right leg, find the levels of the free liquid surfaces in the two tubes.



Solution:
With the addition of 20ml of water,
Drop in mercury level in right leg = y
Rise in mercury level in left leg = y
Depth of water column = h
h= Vol of water added/Cross sectional area = $20x10^{-6}/\left(\frac{\pi}{4}x0.01^2\right)$ = 0.254m

$P_E = P_F$
$\gamma_{mercury}x2y = \gamma_{water}xh$
13.6x9810x2y = 9810x0.254
y = 0.0093m = 0.93cm
Height of free mercury level in the left leg = 18+0.93 = 18.93cm
Height of free water level in the right leg = 18-0.93+25.4 = 42.47cm

21. The diameters of the limbs A and B of a U-tube shown in fig. are 5mm and 20mm respectively. The limb A contains a liquid of sp. gr. 0.9 while the limb B contains a liquid of sp.gr. 1.3. The fig. shows the position of the liquids in the two limbs. Find what pressure should be applied on the surface of the heavier liquid in limb B so that the rise in level in the limb A is 10mm.




Solution:
Sp gr of liquid in A = 0.9
Sp. gr. of liquid in B = 1.3
Diameter of limb A = 5mm
Diameter of limb B = 20mm

a. When no external pressure is applied

$P_E = P_F$

$\gamma_A x h_a = \gamma_B x h_b$

$0.9x9810xh_a = 1.3x9810xh_b$

$h_a = 1.3h_b/0.9$ (a)

b. When external force p is applied on the surface of liquid B

Rise in limb A = 10mm

Fall in limb B = h

Level EF shifts to new position is E1F1.

Volume of liquid transferred to left limb = Volume of liquid fell in right limb

c/s area of left limbx10mm = c/s of right limbxh

$\left(\frac{\pi}{4}x(5x10^{-3})^2\right)x10x10^{-3} = \left(\frac{\pi}{4}x(20x10^{-3})^2\right)h$

h = 0.000625m

$P_{E1} = P_{F1}$

$\gamma_A x(h_a + 10x10^{-3} - 10x10^{-3}) = P + \gamma_B x(h_b - 10x10^{-3} - h)$

$0.9x9810xh_a = P + 1.3x9810x(h_b - 10x10^{-3} - 0.000625)$

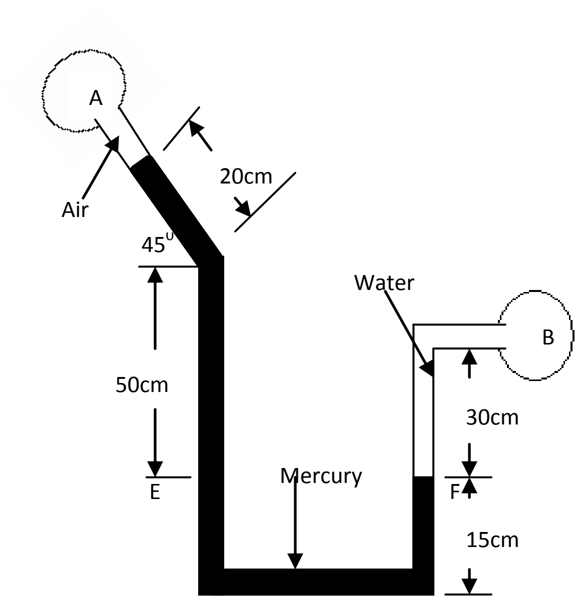
$8829h_a = P + 12753(h_b - 0.010625)$ (b)

From a and b

$8829x1.3h_b/0.9 = P + 12753(h_b - 0.010625)$

P = 135.5 N/m$^2$

22. Find the pressure difference between containers A and B shown in the figure.




Solution:

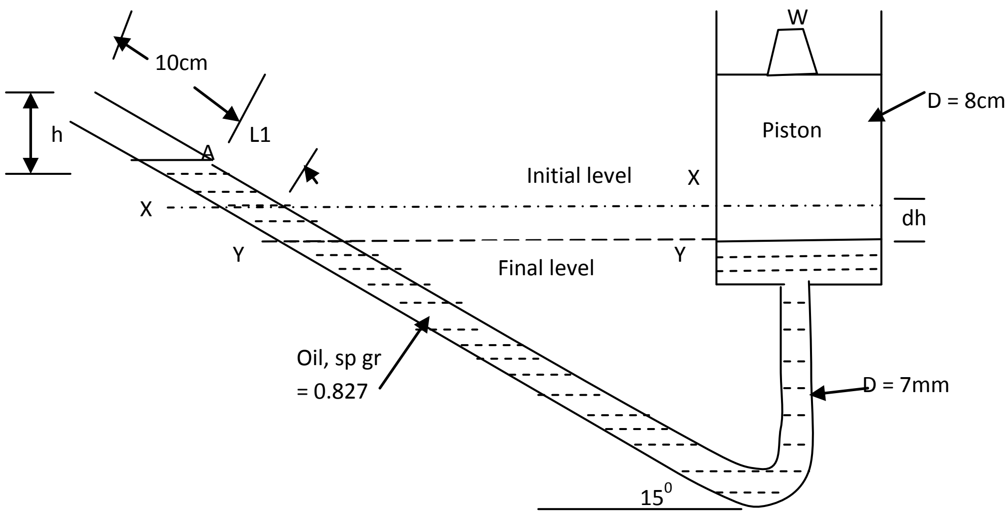
Pressure at E = pressure at F

$P_A + \gamma_{mercury} x0.20Sin45 + \gamma_{mercury} x0.5 = P_B + \gamma_{water} x0.3$

$P_A + 13.6x9810x0.20Sin45 + 13.6x9810x0.5 = P_B + 9810x0.3$

$P_A - P_B$ = -82633 N/m$^2$

23. An 8cm diameter piston compresses manometer oil into an inclined 7mm diameter tube, as shown in figure below. When a weight W is added to the top of the piston, the oil rises an additional distance of 10cm up the tube. How large is the weight, in N?




Solution:

Diameter of piston (D) = 8cm = 0.08m

Diameter of tube (d) = 7mm = 0.007m

h = 0.10Sin15 = 0.0258m

When the manometer is not connected to the container, the mercury in the reservoir is at original level and at level A in the tube.

Equating pressure at XX

$P_X = \gamma_m L1Sin15$ (a)

Due to compression, the fluid in the container moves down by dh and the fluid in the tube moves up by 10cm.

Volume of fluid fallen = Volume of fluid risen

$$\frac{\pi}{4} x0.08^2 dh = \frac{\pi}{4} x0.007^2 x0.1$$

dh = 0.000766m

Equating the pressure at new level (YY)

$\frac{W}{Area\ of\ piston} + P_X + \gamma_{air} dh = \gamma_m (h + dh) + \gamma_m L1Sin15$ (b)

From a and b

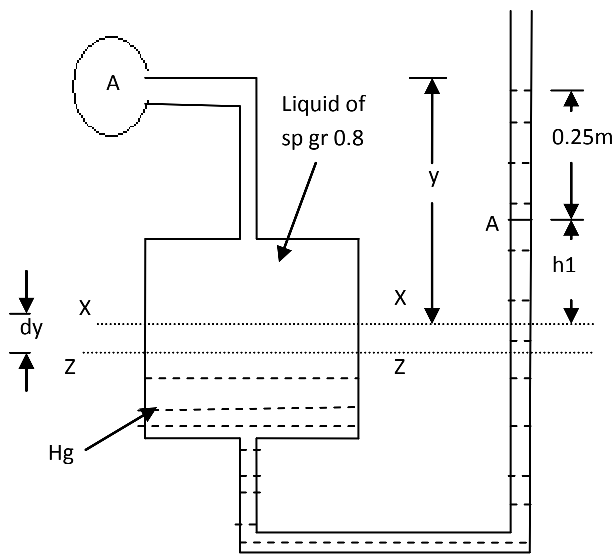
$$\frac{W}{Area\ of\ piston} = \gamma_m h + (\gamma_m - \gamma_{air})dh$$

Neglecting dh

$$\frac{W}{\frac{\pi}{4}x0.08^2} = 0.827x9810x0.0258$$

W = 1.05 N

24. Figure below shows a pipe containing a liquid of sp gr 0.8 connected to a single column micromanometer. The area of reservoir is 60 times that of the tube. The manometer liquid is mercury. Find the pressure in the pipe.




Solution:

When the manometer is not connected to the container, the mercury in the reservoir is at original level and at level A in the tube.

Equating pressure at XX

$\gamma_{oil} y = \gamma_{mercury} h1$ (a)

Due to pressure, manometric liquid in the reservoir drops by dy and it will travel a distance of 0.25m in the tube.

Volume of fluid fallen = Volume of fluid risen

Ady=ax0.25

60ady = ax0.25

dy = 0.0041m

Equating pressure at new level (ZZ)

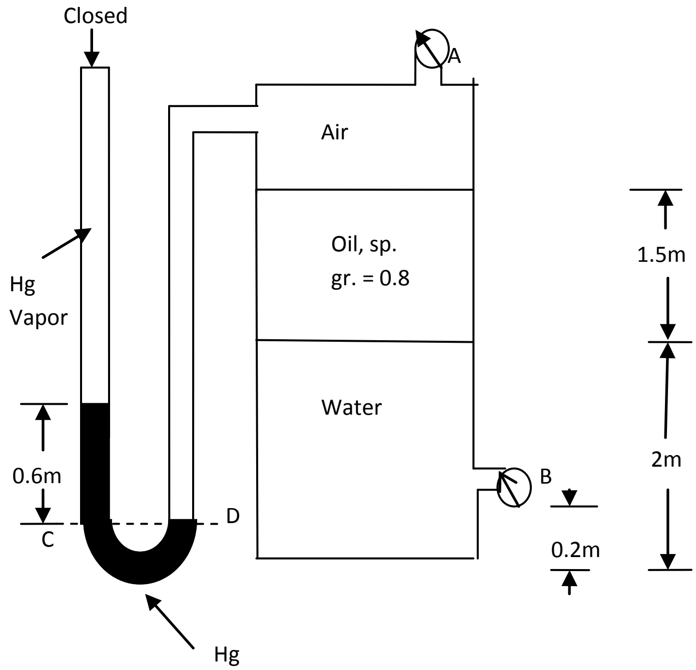
$P_A + \gamma_{oil}(y + dy) = \gamma_{mercury}(0.25 + h1 + dy)$ (b)

From a and b

$P_A + \gamma_{mercury} h1 = \gamma_{mercury} h1 + \gamma_{mercury}(0.25 + dy) - \gamma_{oil} dy$

$P_A = 13.6x9810(0.25 + 0.0041) - 0.8x9810x0.0041$ = 33868 N/m$^2$

25. Find the gauge readings at A and B if the atmospheric pressure is 755mmHg.




Solution:

Atm. pr. = $\gamma_{mercury}\ x\ 755x10^{-3} = 13.6x9810x755x10^{-3}$ = 100729 N/m$^2$

Neglect pressure due to mercury vapor and air (small)

Writing pressure equation for gauge A,

$$P_C = P_D = P_A$$
$$\gamma_{mercury}\ x\ 0.6 = P_A$$

$P_A$ = 13.6x9810x0.6 = 80049.6 N/m$^2$ (abs. pr.)

Gauge pressure at A = Abs. pr. - Atm. pr. = 80049.6-100729 = -20679.5 N/m$^2$

Writing pressure equation for gauge B starting from gauge A

$$P_A + \gamma_{oil}x1.5 + \gamma_{water}x1.8 = P_B$$
$$-20679.5 + 0.8x9810x1.5 + 9810x1.8 = P_B$$

$P_B$ = 8750.5 N/m$^2$

Gauge pressure at B = 8750.5 N/m$^2$