Asymptotes:

The coeff. of highest power should not be constt.

1 Procedure to find asymptotes

1) Substitute $y = mx + c$ in the eqn of the curve and equate the coefficients of the two highest powers of x to zero.

Determine the values of m and c from these eqns. If m_1, c_1, m_2, c_2 --- are the values of m and c, the asymptotes are

$$y = m_1x + c_1, \quad y = m_2x + c_2 \text{ etc.}$$

2) Shorter Method:

If we put $x = 1$ and $y = m$ in the highest degree terms of the eqn, we get $\phi_n(m)$ and $\phi_{n-1}(m)$ is obtained by putting $x = 1$, $y = m$ in the $(n-1)^{th}$ degree term and equate this to zero.

The corresponding values of c say c_1, c_2, c_3 ---- c_n are obtained by substituting in the formula.

$$\boxed{c = -\frac{\phi_{n-1}(m)}{\phi'_n(m)}}$$

Asymptotes are $y = m_1x + c_1$ and

$y_2 = m_2x + c_2$

classmate
Date
Page

Ex - Find the asymptotes of the curve

$$x^3 + 2x^2y - xy^2 - 2y^3 + xy - y^2 + 1 = 0$$

degree 3   degree 3   degree 3   degree 3   degree 2   degree 2   degree 1

Shorter method

$$\phi_3(m) = 1 + 2m - m^2 - 2m^3$$ equate this = 0

$$\phi_2(m) = m - m^2$$

Solving
$$1 + 2m - m^2 - 2m^3 = 0$$
$$(1+2m)(1-m^2) = 0$$
$$\boxed{m = 1, -1, -\frac{1}{2}}$$

~~m(1-m)=0~~
m=0
k=1

and putting these values of m in the formula we get ;

We have $m = 1, c = 0$
$m = -1, c = -1$
$m = -\frac{1}{2}, c = \frac{1}{2}$

asymptotes: $y - x = 0$
$y + x + 1 = 0$
$2y + x - 1 = 0$

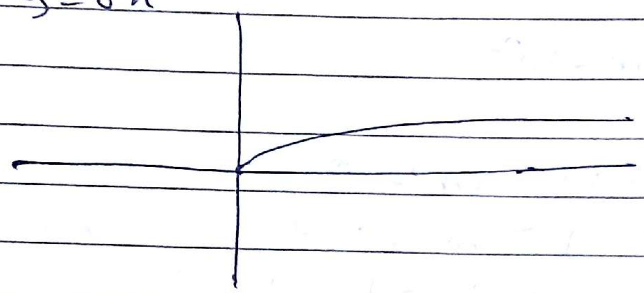
$$\boxed{\phi_1(m) + c\,\phi_2'(m) + \frac{c^2}{2!}\phi_3''(m) = 0}$$

for highest degree 3

Curve tracing

(1) find the value of x at which function is not defined, then it would be the endlines. and they were used in finding asymptotes.

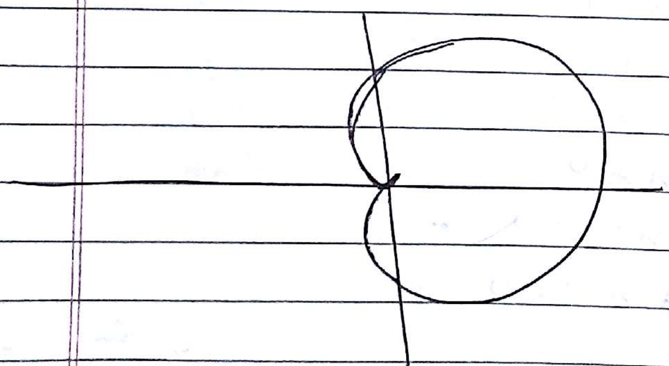
i, $y = \sqrt{x}$



INTEGRATION

Area of Sector

$$\boxed{\text{Area} = \frac{1}{2}\int_{\alpha}^{\beta} r^2 \cdot d\theta}$$



$r = a(1+\cos\theta)$

Cardioid

Volume of solids

$$V = \int_a^b A(x) \cdot dx$$

$A(x)$ ↓ Area of base

$$V = \int_c^d A(y) \cdot dy$$

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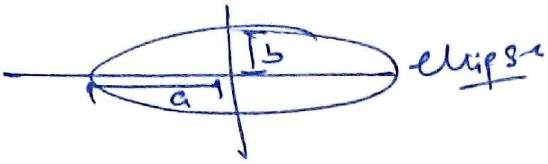


Volume of Solid of Revolution

→ Volume generated by revolving area about x axis

$$\boxed{V = \int_a^b \pi y^2 \cdot dx}$$

To find 'a' and 'b' put $y=0$ and find intersection on x axis

→ about y axis

$$\boxed{V = \int_c^d \pi x^2 \cdot dy}$$

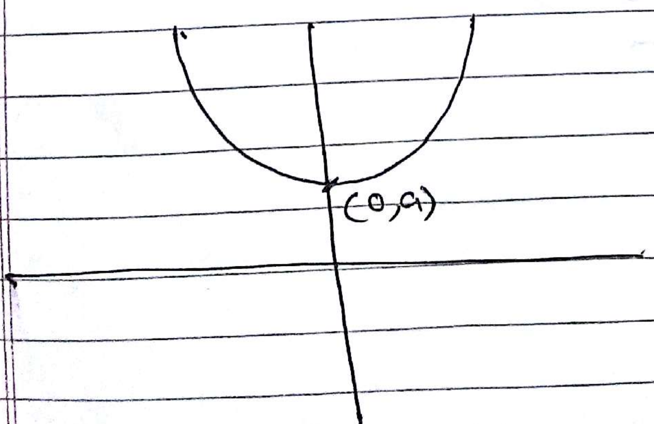
→ about line $y=p$

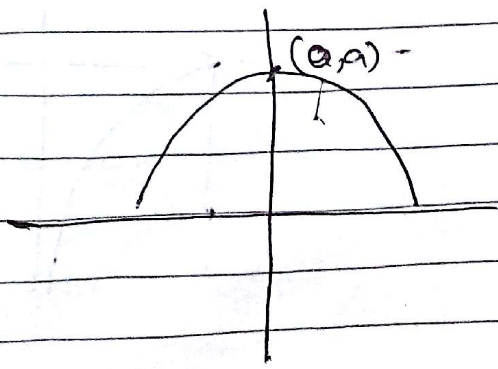
$$\boxed{V = \pi \int_a^b (y-p)^2 \cdot dx}$$

→ about line $x=q$

$$\boxed{V = \pi \int_c^d (x-q)^2 \cdot dy}$$

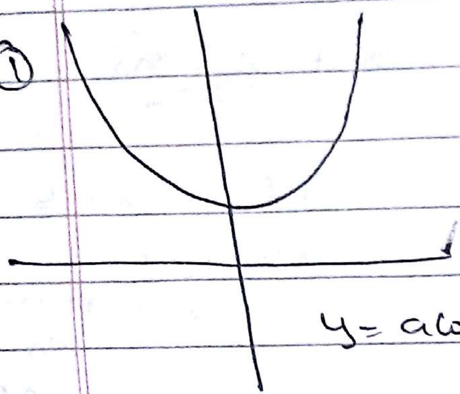
\# $y = x^2 + a$ $\qquad$ $y = a - x^2$





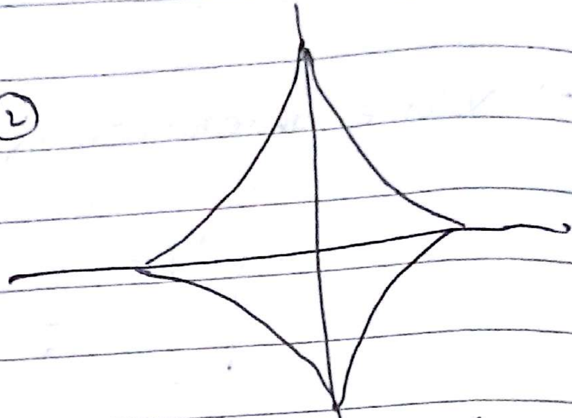
Important Graphs

① Catenary



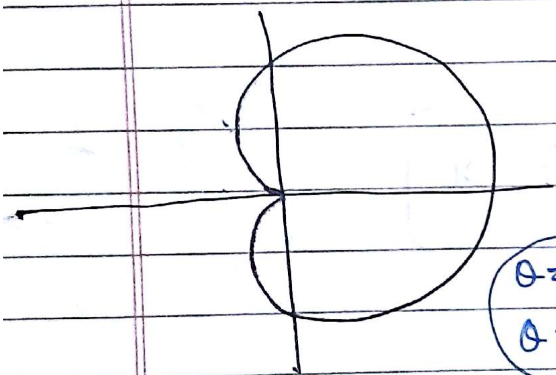
$y = a\cosh\left(\frac{x}{a}\right)$

② Astroid



$x^{2/3} + y^{2/3} = a^{2/3}$

③ Cardioid

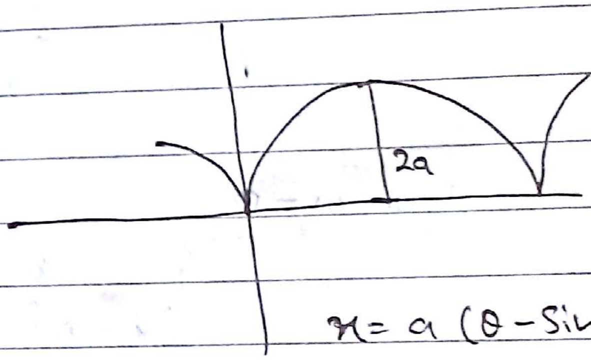


$\theta = 0$

$\theta = \pi$

$r = a(1+\cos\theta)$

④

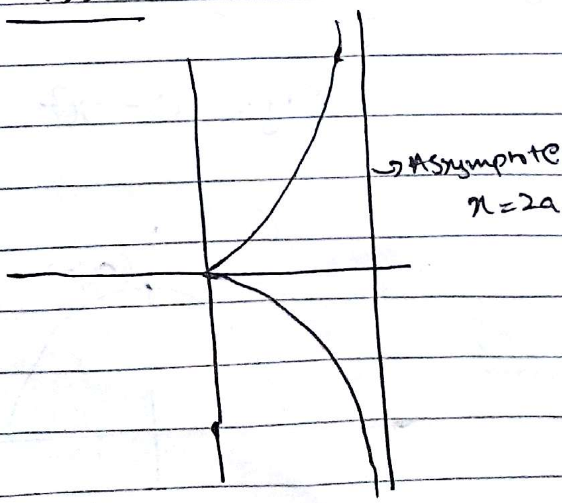


$x = a(\theta - \sin\theta)$

$y = a(1-\cos\theta)$

Cycloid

⑤ Cissoid



* Reduction formula for $\int \sin^m x \cos^n x \cdot dx$

$$= -\frac{\sin^{m-1} x \cos^{n+1} x}{m+n} + \frac{m-1}{m+n} \int \sin^{m-2} x \cos^n x \cdot dx$$

$$= \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n} \int \sin^m x \cos^{n-2} x \cdot dx$$

$$\boxed{\int_0^{\pi/2} \sin^m x \cos^n x \cdot dx = \frac{n-1}{m+n} \int_0^{\pi/2} \sin^m x \cos^{n-2} x \cdot dx}$$ ★★★

Integration of $\frac{1}{a+b\cos x}$

Write $a+b\cos x$ as

$$a\left(\cos^2\frac{x}{2} + \sin^2\frac{x}{2}\right) + b\left(\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right)$$

divide numerator and denominator by $\cos^2\frac{x}{2}$ and then put $\tan\frac{x}{2} = t$

Date
Page

INTEGRATION FORMULAES

① $\int \frac{1}{\sqrt{1-x^2}} \cdot dx = \sin^{-1} x$

② $\int \frac{1}{1+x^2} \cdot dx = \tan^{-1} x$

③ $\int \frac{1}{x\sqrt{x^2-1}} \cdot dx = \sec^{-1} x$

④ $\int \frac{1}{x^2-a^2} \cdot dx = \frac{1}{2a} \log \frac{x-a}{x+a} + c$

⑤ $\int \frac{1}{\sqrt{x^2 \pm a^2}} = \log \left\{ x + \sqrt{x^2 \pm a^2} \right\} + C$

⑥ $\int \sqrt{a^2-x^2} = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + C$

⑦ $\int \sqrt{x^2+a^2} = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2} \log \left\{ x + \sqrt{x^2+a^2} \right\}$

⑧ $\int \sqrt{x^2-a^2} = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2} \log \left\{ \left(x + \sqrt{x^2-a^2}\right) \right\}$

Area

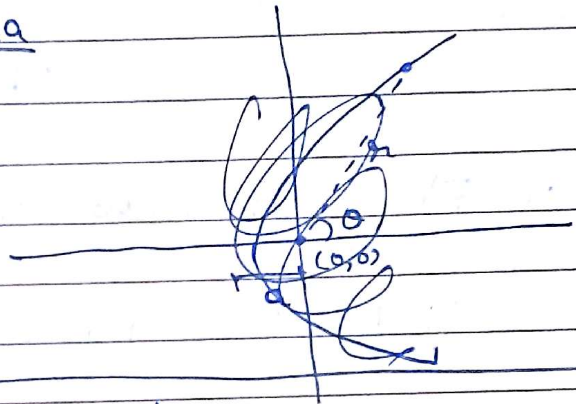
Curved Surface of solid

$$S = 2\pi \int_{x=a}^{x=b} y \cdot ds$$ length of arc

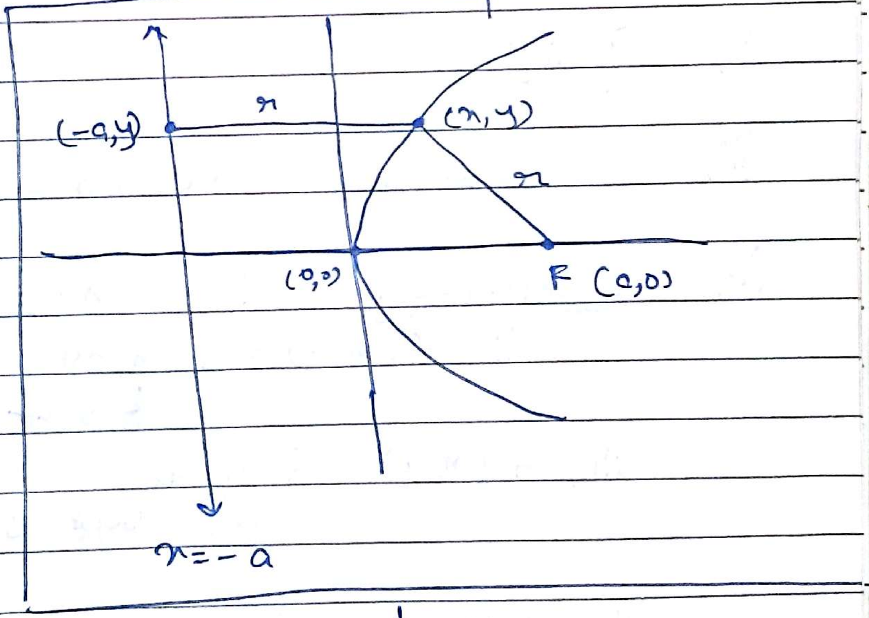
* Parametric form of parabola

$(at^2, 2at)$

Polar form of parabola



$$\boxed{r = \frac{2a}{1 - \cos\theta}}$$

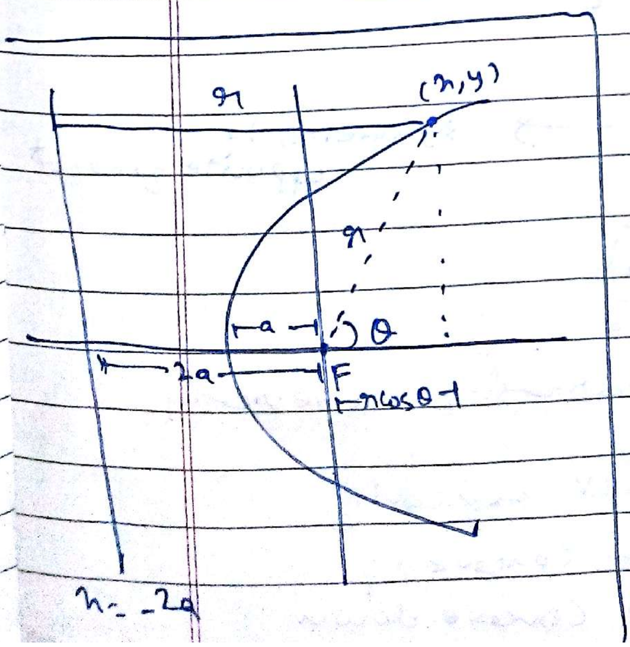


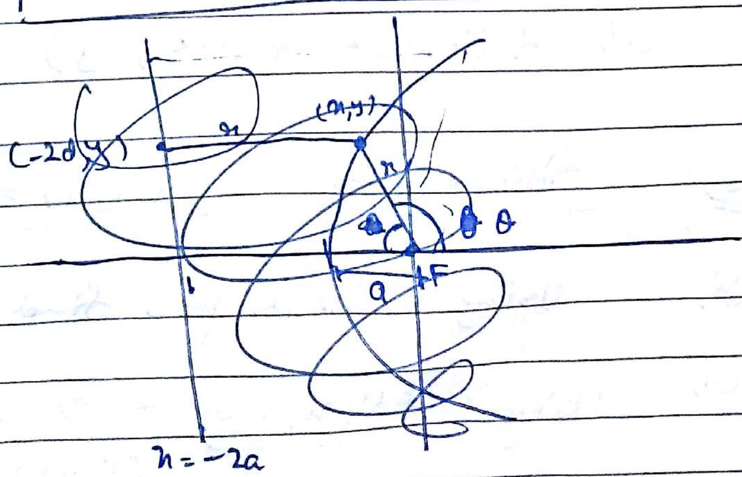
$x = r\cos\theta$

$y = r\sin\theta$

Shift the Focus to origin

$$\boxed{r = r\cos\theta + 2a}$$





If ~~Two~~ loop is given and volume is to be find

then find only intersections with co-ordination.

[if along x axis → find x coordinates
" " y axis → find y co-ordinates]

CURVE SKETCHING

i) Find domain and range

Find where the curve is not defined

ii) Origin

iii) Intersection with co-ordinate axes

iv) Symmetry = i) $f(x,y) = f(-x,y)$ (power of x is even)
→ symm. about y axis

ii) $f(x,y) = f(x,-y)$
→ symm about x axis

iii) $f(x,y) = f(y,x)$
→ about $y = x$

iv) $f(x,y) = f(-x,-y)$ → symmetry in opposite quadrant

v) Critical pts $\boxed{f'(x) = 0}$

vi) Using critical pts find increasing or decreasing.

vii) Using $\boxed{f''(x) = 0}$ → check concavity
$f''(x) > 0$ concave up
$f''(x) < 0$ concave down

Date
Page

vii) Asymptotes to curve

Q. $y = \frac{(x-1)(x-3)}{x^2}$ → not defined at $x=0$

$\lim_{x\to 0} y(x) = \lim_{x\to 0} \frac{(x-1)(x-3)}{x^2} = \infty$

$\boxed{x=0 \text{ is vertical asymptote}}$

$\lim_{x\to \pm\infty} y(x) = \lim_{x\to\pm\infty} \frac{(x-1)(x-3)}{x^2} = 1$

$\boxed{y=1}$ is asymptote

Q. $y^2 = \frac{x-3}{x^2-6x-7}$ → For this type of ques, no need to find critical pt.s and Concavity

→ not defined
→ intersection $\quad x^2-6x-7 = (x+1)(x-7)$
→ symmetry
→ Asymptotes

Asymptotes

i) $\lim_{x\to -1} y(x) = \infty$ $\quad x=-1$

ii) $\lim_{x\to 7} y(x) = \infty$ $\quad x=7$

iii) $\lim_{x\to\infty} y(x) = 0$ $\quad y=0$ is asymptote

point O → $\boxed{\text{pedal point}}$

Pedal equ:

$p_c = \sqrt{r^2 - p^2}$

contra pedal coordinate

[distance to the normal]

r → dist. from O to a point on C

p → is the tan dist. from O to the tangent line to C

pedal co-ordinate

Cartesian coordinate

O taken to be origen

pedal coordinates of pt. (x, y) is given by

$\boxed{f(x,y) = 0}$

$r = \sqrt{x^2 + y^2}$

$$p = \frac{x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}}{\sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}}$$

Polar co-ordinate

$r = f(\theta)$

$\boxed{p = r\sin\phi}$

$\boxed{r = \frac{dr}{d\theta}\tan\phi}$

$\boxed{p_c = \sqrt{r^2 - p^2}}$

$\boxed{\frac{dr}{d\theta} = \frac{r p_c}{p}}$

Case 1:

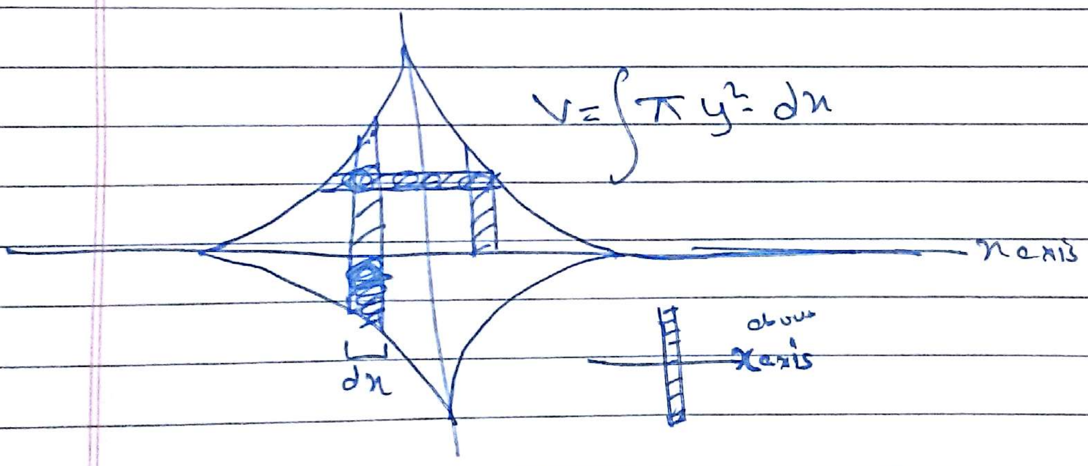
★ **For $r = f(\theta)$**

Intrinsic eqn: is $\boxed{\psi = \theta + \phi}$

$$\boxed{r = \frac{\partial r}{\partial \theta} \tan\phi}$$

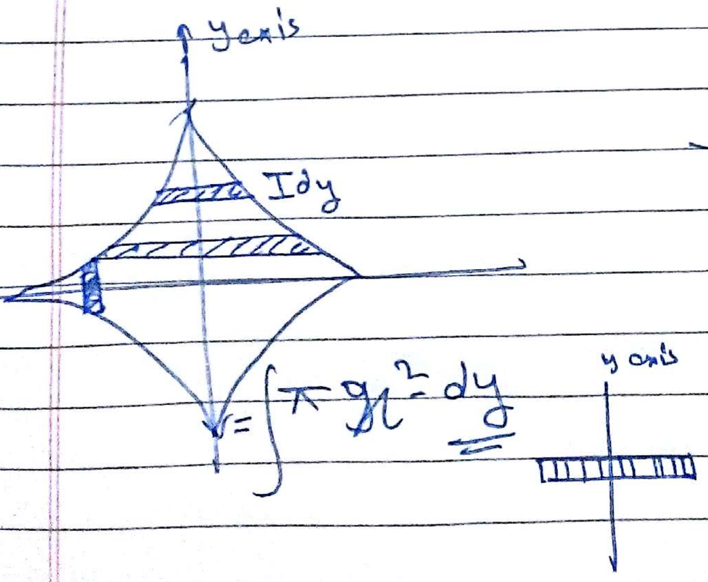
Volume

About x axis means

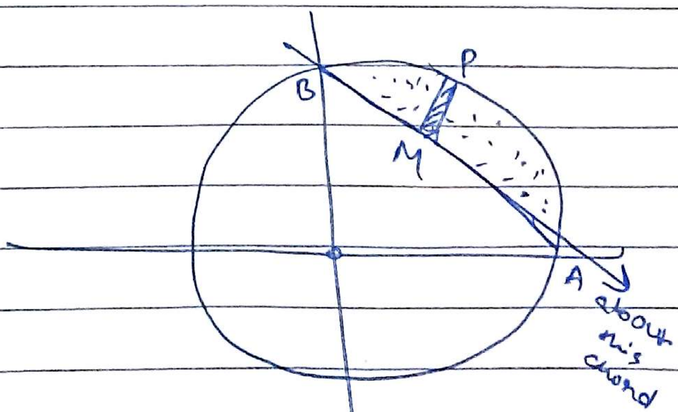


$$V = \int \pi y^2 \, dx$$

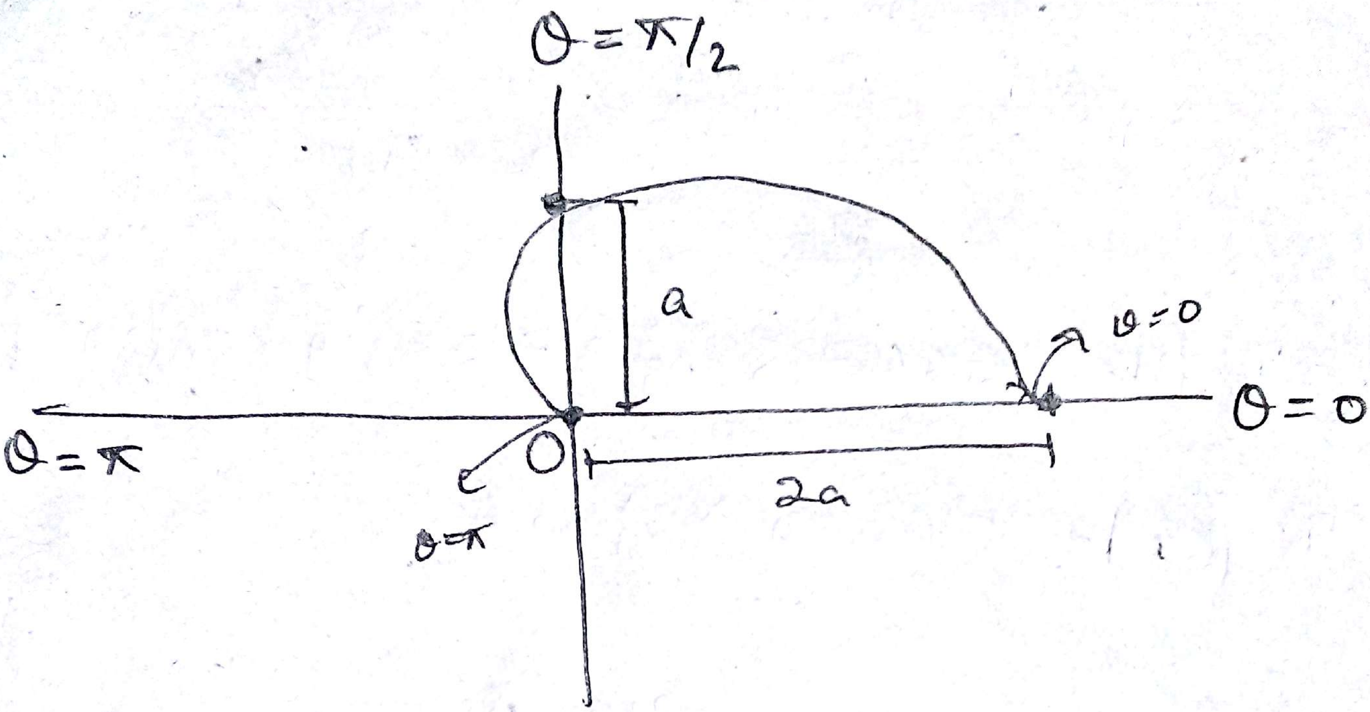
About y axis



$$V = \int \pi x^2 \, dy$$



$$V = \int_0^{AB} \pi PM^2 \, d(AM)$$



$r = a(1+\cos\theta)$

dist. from origin

For $\theta = 0$

$r = 2a$

For $\theta = \pi/2$

$r = a$

For $\theta = \pi$

$r = 0$

(1) Newton's law of cooling

$$\boxed{\frac{d\theta}{dt} = -K(\theta - \theta_s)}$$ → Find K from this eqn

$\theta = \theta(t)$ is the temp. of cooling object at time 't'

θ_s → temp. of surrounding

θ_0 = be the initial temp. of object

$$\boxed{\theta = \theta_0 \quad \text{at } t = 0}$$

$$\int \frac{d\theta}{\theta - \theta_s} = -K \int dt$$

u.

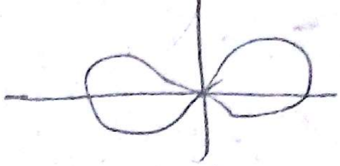
$$\ln(\theta - \theta_s) = -Kt + \ln(\theta_0 - \theta_s)$$

$$\boxed{\ln\left(\frac{\theta - \theta_s}{\theta_0 - \theta_s}\right) = -Kt}$$ → Find 'T' from this eqn

$$\frac{\theta - \theta_s}{\theta_0 - \theta_s} = e^{-Kt}$$

$$\frac{m^2}{s^2} \times 10^{-3}$$

$$\boxed{\theta - \theta_s = (\theta_0 - \theta_s) e^{-Kt}}$$ $(\theta_0 - \theta_s)$



(12-) $\boxed{r^2 = a^2 \cos 2\theta}$

Required volume = $\boxed{\frac{2}{3}\pi \int_0^{\pi/4} r^3 \sin\theta \cdot d\theta}$

$$= \frac{2\pi}{3} \int_0^{\pi/4} (\cancel{a^2 \cos 2\theta})(a\sqrt{\cos 2\theta})^3 \sin\theta \cdot d\theta$$

$$= \frac{2\pi}{3} \int_0^{\pi/4} a^3 (\cos 2\theta)^{3/2} \cdot \sin\theta \cdot d\theta$$

$$= \frac{2\pi a^3}{3} \int_0^{\pi/4} (2\cos^2\theta - 1)^{3/2} \cdot \sin\theta \cdot d\theta$$

$$= \cancel{\frac{2\pi a^3}{3} \int_0^{\pi/4}}$$

put $(\sqrt{2}\cos\theta) = \sec\phi$

$\boxed{\begin{array}{l} \theta = 0 \\ \phi = \pi/4 \\ \theta = \pi/4 \\ \phi = 0 \end{array}}$

$$-\sqrt{2}\sin\theta \cdot d\theta = \sec\phi \tan\phi \cdot d\phi$$

$$= -\frac{\sqrt{2}\,\cancel{2}\pi a^3}{3 \times \cancel{\sqrt{2}}} \int^{0} (-\sqrt{2}) (\sec^2\phi - 1)^{3/2} \cdot \sin\theta \cdot d\theta$$

$$= \frac{-\sqrt{2}\pi a^3}{3} \int_{\pi/4}^{0} \tan^3\phi \cdot \sec\phi \tan\phi \cdot d\phi$$

$$= +\frac{\sqrt{2}\pi a^3}{3} \int_0^{\pi/4} \tan^4\phi \cdot \sec\phi \cdot d\phi$$

$$= +\frac{\sqrt{2}\pi a^3}{3} \int_0^{\pi/4} (\sec^2\phi - 1)^2 \cdot \sec\phi \cdot d\phi$$

$$= \frac{\sqrt{2}\pi a^3}{3} \int_0^{\pi/4} \sec^5\phi - 2\sec^3\phi + \sec\phi \cdot d\phi$$

$-2+\frac{3}{4}-2$ $\quad -\frac{5}{4}$ $\quad \sec\phi\,(1-25$ $\quad \frac{1}{\sqrt{3}}$ $\quad (\sqrt{2})^3$ $\quad \frac{12\sqrt{2}}{4^2}$

Use

$$\boxed{\int \sec^n\phi \cdot d\phi = \frac{\sec^{n-2}\phi\tan\phi}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2}\phi \cdot d\phi}$$

$$\therefore \int_0^{\pi/4} \sec^5\phi = \left[\frac{\sec^3\phi\tan\phi}{4}\right]_0^{\pi/4} + \frac{3}{4}\int_0^{\pi/4} \sec^3\phi \cdot d\phi$$

$$= \frac{\sqrt{2}}{2} + \frac{3}{4}$$

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* Equate lowest degree term to zero for Eq. of tangent.

PAGE NO
DATE: / 201

$y = x(x^2-1)$.

Every line has certain slope.

1. Symmetry : no symmetry abt axis
2. Passes through Origin : Yes
3. Eq. of tangent at $(0,0)$ is

$$-x-y=0$$
$$y=-x$$

3. Intersection with axes

i) x-axis

Put $y=0$ $\quad x(x^2-1)=0$.

$x=0,\ x=+1,\ x=-1$

ii) y-axis

Put $x=0,\ y=0$

4. $y = x(x^2-1)$

$x \to +\infty,\ y \to +\infty$

$x^3-x-y=0$ $\quad$ & if $x \to -\infty$

$y = x^3-x$ $\quad$ $y \to -\infty$

5. Asymptote :

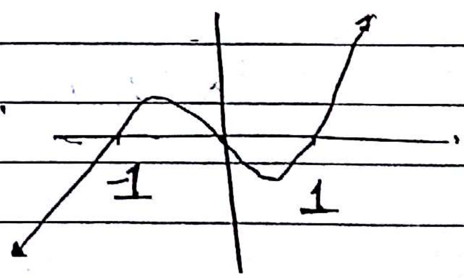
$\phi_3(m)=1$

This curve has no asymptote.

6.

x	0	1	-1	2	-2
y	0	0	0	6	-6

7.



$\frac{dy}{dx} = 3x^2-1=0$

$x = \pm 1/\sqrt{3}$

$\frac{d^2y}{dx^2} = 6x$ for $-1/\sqrt{3}$

y is max. for $1/\sqrt{3}$

$y_{min} =$

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for $m = 1$, $c = 1$
$m = -1$, $c = 0$
$m = \frac{-1}{2}$, $c = 1/2$

eq.n of asymp.

$y = x+1 \Rightarrow x-y+1=0$
$y = -x \Rightarrow x+y=0$
$y = \frac{-1}{2}x + \frac{1}{2} \Rightarrow x+2y-1=0$

Q. ① $2x^3+3x^2y-3xy^2-2y^3+3x^2-3y^2+y=0$
② $x^3+3x^2y-4y^3-x+y+3=0$

nd Sem

Curve Tracing :

① Symmetry

If all the powers of 'x' are even then the curve is asymmetric about y-axis & vice versa.

If all powers of both x & y are even then the curve is asymmetric about both the axes.

② Passing through (0,0)