

*P K Nag Exercise problems - Solved*

# *Thermodynamics*

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## **Solved by**

***Er. S K Mondal***

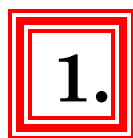
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### ***Benefits of solving Exercise (unsolved) problems of P K Nag***

- *The best way is to study thermodynamics through problems, you must know how to apply theoretical concepts through problems and to do so you must solve these problems*
- *It contains Expected Questions of IES, IAS, IFS and GATE examinations*
- *It will enable the candidates to understand thermodynamics properly*
- *It will clear all your doubts*
- *There will be no fear of thermodynamics after solving these problems*
- *Candidate will be in a comfortable position to appear for various competitive examinations*
- *Thermodynamics- “the Backbone of Mechanical Engineering” therefore Mastering Thermodynamics is most important many of the subjects which come in later like Heat and Mass Transfer, Refrigeration and Air Conditioning, Internal Combustion Engine will require fundamental knowledge of Thermodynamics*

*Every effort has been made to see that there are no errors (typographical or otherwise) in the material presented. However, it is still possible that there are a few errors (serious or otherwise). I would be thankful to the readers if they are brought to my attention at the following e-mail address: swapan\_mondal\_01@yahoo.co.in*

*S K Mondal*



# 1. Introduction

## Some Important Notes

- Microscopic thermodynamics or statistical thermodynamics
- Macroscopic thermodynamics or classical thermodynamics
- A quasi-static process is also called a reversible process

## Intensive and Extensive Properties

**Intensive property:** Whose value is independent of the size or extent i.e. mass of the system. e.g., pressure  $p$  and temperature  $T$ .

**Extensive property:** Whose value depends on the size or extent i.e. mass of the system (upper case letters as the symbols). e.g., Volume, Mass ( $V$ ,  $M$ ). If mass is increased, the value of extensive property also increases. e.g., volume  $V$ , internal energy  $U$ , enthalpy  $H$ , entropy  $S$ , etc.

**Specific property:** It is a special case of an intensive property. It is the value of an extensive property per unit mass of system. (Lower case letters as symbols) e.g: specific volume, density ( $v$ ,  $\rho$ ).

## Concept of Continuum

The concept of continuum is a kind of idealization of the continuous description of matter where the properties of the matter are considered as continuous functions of space variables. Although any matter is composed of several molecules, the concept of continuum assumes a continuous distribution of mass within the matter or system with no empty space, instead of the actual conglomeration of separate molecules.

Describing a fluid flow quantitatively makes it necessary to assume that flow variables (pressure, velocity etc.) and fluid properties vary continuously from one point to another. Mathematical descriptions of flow on this basis have proved to be reliable and treatment of fluid medium as a continuum has firmly become established.

For example density at a point is normally defined as

$$\rho = \lim_{\Delta V \rightarrow 0} \left( \frac{m}{\Delta V} \right)$$

Here  $\Delta V$  is the volume of the fluid element and  $m$  is the mass

If  $\Delta V$  is very large  $\rho$  is affected by the in-homogeneities in the fluid medium. Considering another extreme if  $\Delta V$  is very small, random movement of atoms (or molecules) would change their number at different times. In the continuum approximation point density is defined at the smallest magnitude of  $\Delta V$ , before statistical fluctuations become significant. This is called continuum limit and is denoted by  $\Delta V_c$ .

$$\rho = \lim_{\Delta V \rightarrow \Delta V_c} \left( \frac{m}{\Delta V} \right)$$

One of the factors considered important in determining the validity of continuum model is molecular density. It is the distance between the molecules which is characterized by mean free path ( $\lambda$ ). It is calculated by finding statistical average distance the molecules travel between two successive collisions. If the mean free path is very small as compared with some characteristic length in the flow domain (i.e., the molecular density is very high) then the gas can be treated as a continuous medium. If the mean free path is large in comparison to some characteristic length, the gas cannot be considered continuous and it should be analyzed by the molecular theory.

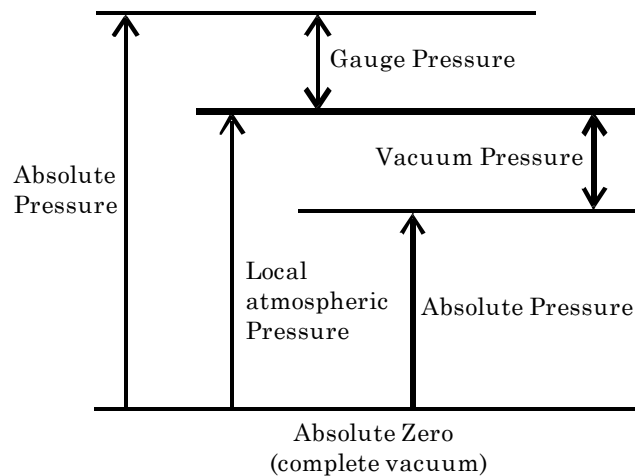
A dimensionless parameter known as Knudsen number,  $K_n = \lambda / L$ , where  $\lambda$  is the mean free path and  $L$  is the characteristic length. It describes the degree of departure from continuum. Usually when  $K_n > 0.01$ , the concept of continuum does not hold good.

In this,  $K_n$  is always less than 0.01 and it is usual to say that the fluid is a continuum.

Other factor which checks the validity of continuum is the elapsed time between collisions. The time should be small enough so that the random statistical description of molecular activity holds good.

In continuum approach, fluid properties such as density, viscosity, thermal conductivity, temperature, etc. can be expressed as continuous functions of space and time.

## The Scale of Pressure



At sea-level, the international standard atmosphere has been chosen as  $P_{\text{atm}} = 101.325 \text{ kN/m}^2$

## Some special units for Thermodynamics

$$pv = RT$$

kPa   m<sup>3</sup>/kg   kJ/kg   K

**Note:** Physicists use below units

$$pv = RT$$

Pa   m<sup>3</sup>/kg   J/kg   K

Universal gas constant,  $R_u = 8.314 \text{ kJ/kmole} \cdot \text{K}$

Characteristic gas constant,  $R_c = \frac{R_u}{M}$

$$\begin{aligned} \text{For Air } R &= \frac{8.314}{29} = \frac{\text{kJ/kmole} \cdot \text{K}}{\text{kg/kmole}} \\ &= 0.287 \text{ kJ/kg} \cdot \text{K} \end{aligned}$$

$$\begin{aligned} \text{For water } R &= \frac{8.314}{18} = \frac{\text{kJ/kmole} \cdot \text{K}}{\text{kg/kmole}} \\ &= 0.461 \text{ kJ/kg} \cdot \text{K} \end{aligned}$$

Units of heat and work is kJ

Units of pressure is kPa

$$1 \text{ atm} = 101.325 \text{ kPa}$$

$$1 \text{ bar} = 100 \text{ kPa}$$

$$1 \text{ MPa} = 1000 \text{ kPa}$$

## Questions with Solution P. K. Nag

- Q1.1** A pump discharges a liquid into a drum at the rate of  $0.032 \text{ m}^3/\text{s}$ . The drum,  $1.50 \text{ m}$  in diameter and  $4.20 \text{ m}$  in length, can hold  $3000 \text{ kg}$  of the liquid. Find the density of the liquid and the mass flow rate of the liquid handled by the pump.

(Ans.  $12.934 \text{ kg/s}$ )**Solution:**

$$\begin{aligned}
 \text{Volume of drum} &= \frac{\pi d^2}{4} \times h \\
 &= \frac{\pi \times 1.50^2}{4} \times 4.2 \text{ m}^3 \\
 &= 7.422 \text{ m}^3 \\
 \text{density} &= \frac{\text{mass}}{\text{Volume}} = \frac{3000}{7.422} \text{ kg/m}^3 = 404.203 \text{ kg/m}^3 \\
 \text{mass flow rate} &= \text{Volume flow rate} \times \text{density} \\
 &= 0.032 \times 404.203 \text{ kg/s} \\
 &= 12.9345 \text{ kg/s}
 \end{aligned}$$

- Q1.2** The acceleration of gravity is given as a function of elevation above sea level by

$$g = 980.6 - 3.086 \times 10^{-6} H$$

Where  $g$  is in  $\text{cm/s}^2$  and  $H$  is in  $\text{cm}$ . If an aeroplane weighs  $90,000 \text{ N}$  at sea level, what is the gravity force upon it at  $10,000 \text{ m}$  elevation? What is the percentage difference from the sea-level weight?

(Ans.  $89,716.4 \text{ N}$ ,  $0.315\%$ )**Solution:**

$$\begin{aligned}
 g' &= 980.6 - 3.086 \times 10^{-6} \times 10,000 \times 100 \\
 &= 977.514 \text{ cm/s}^2 = 9.77514 \text{ m/s}^2 \\
 W_{\text{sea}} &= 90,000 \text{ N} = \frac{90,000}{9.806} \text{ kgf} \\
 &= 9178.054 \text{ kgf} \\
 W_{\text{ete}} &= 9178.054 \times 9.77514 \text{ N} = 89716.765 \text{ N} \\
 \% \text{ less} &= \frac{90,000 - 89716.765}{90,000} \times 100\% \\
 &= 0.3147\% \text{ (less)}
 \end{aligned}$$

- Q1.3** Prove that the weight of a body at an elevation  $H$  above sea-level is given by

$$W = \frac{mg}{g_0} \left( \frac{d}{d + 2H} \right)^2$$

Where  $d$  is the diameter of the earth.

**Solution:** According to Newton's law of gravity it we place a man of  $m$  at an height of  $H$  then

## Introduction

**By: S K Mondal**

## Chapter 1

$$\text{Force of attraction} = \frac{GMm}{\left(\frac{d}{2} + H\right)^2} \quad \dots$$

(i)

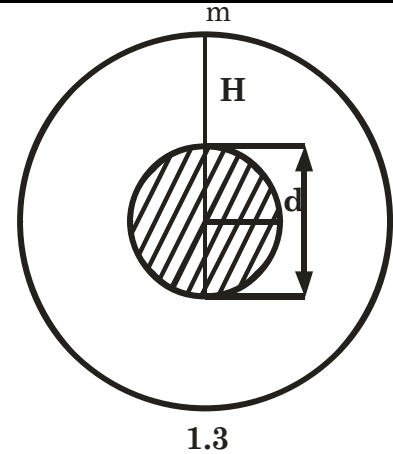
If we place it in a surface of earth then

$$\text{Force of attraction} = \frac{GMm}{\left(\frac{d}{2}\right)^2} = mg_o$$

$$\text{or} \quad g_o = \frac{GM}{\left(\frac{d}{2}\right)^2}$$

$$\begin{aligned} \therefore \text{Weight (W)} &= \frac{GMm}{\left(\frac{d}{2} + H\right)^2} \\ &= \frac{mg_o \left(\frac{d}{2}\right)^2}{\left(\frac{d}{2} + H\right)^2} \quad \text{from equation... (i)} \end{aligned}$$

$$= mg_o \left( \frac{d}{d + 2H} \right)^2 \quad \text{Proved.}$$



**Q1.4**

The first artificial earth satellite is reported to have encircled the earth at a speed of 28,840 km/h and its maximum height above the earth's surface was stated to be 916 km. Taking the mean diameter of the earth to be 12,680 km, and assuming the orbit to be circular, evaluate the value of the gravitational acceleration at this height.

The mass of the satellite is reported to have been 86 kg at sea-level. Estimate the gravitational force acting on the satellite at the operational altitude.

(Ans. 8.9 m/s<sup>2</sup>; 765 N)

**Solution:** Their force of attraction = centrifugal force

$$\begin{aligned} \text{Centrifugal force} &= \frac{mv^2}{r} \\ &= \frac{86 \times \left( \frac{28840 \times 1000}{60 \times 60} \right)^2}{\left( \frac{12680 \times 10^3}{2} + 916 \times 10^3 \right)} \text{ N} \\ &= 760.65 \text{ N (Weight)} \end{aligned}$$

**Q1.5**

Convert the following readings of pressure to kPa, assuming that the barometer reads 760 mmHg:

- |                                  |                    |
|----------------------------------|--------------------|
| (a) 90 cmHg gauge                | (b) 40 cmHg vacuum |
| (c) 1.2 m H <sub>2</sub> O gauge | (d) 3.1 bar        |

**Solution:**

$$\begin{aligned} 760 \text{ mm Hg} &= 0.760 \times 13600 \times 9.81 \text{ Pa} \\ &= 10139.16 \text{ Pa} \\ &\approx 101.4 \text{ kPa} \end{aligned}$$

- (a) 90 cm Hg gauge  
 $= 0.90 \times 13600 \times 9.81 \times 10^{-3} + 101.4 \text{ kPa}$   
 $= 221.4744 \text{ kPa}$
- (b) 40 cm Hg vacuum  
 $= (76 - 40) \text{ cm (absolute)}$   
 $= 0.36 \times 13600 \times 9.81 \text{ kPa}$   
 $= 48.03 \text{ kPa}$
- (c) 1.2 m H<sub>2</sub>O gauge  
 $= 1.2 \times 1000 \times 9.81 \times 10^{-3} + 101.4 \text{ kPa}$   
 $= 113.172 \text{ kPa}$
- (d) 3.1 bar =  $3.1 \times 100 \text{ kPa} = 310 \text{ kPa}$

**Q1.6** A 30 m high vertical column of a fluid of density 1878 kg/m<sup>3</sup> exists in a place where  $g = 9.65 \text{ m/s}^2$ . What is the pressure at the base of the column.  
 (Ans. 544 kPa)

**Solution:**

$$p = z \rho g$$

$$= 30 \times 1878 \times 9.65 \text{ Pa}$$

$$= 543.681 \text{ kPa}$$

**Q1.7** Assume that the pressure  $p$  and the specific volume  $v$  of the atmosphere are related according to the equation  $pv^{1.4} = 2.3 \times 10^5$ , where  $p$  is in N/m<sup>2</sup> abs and  $v$  is in m<sup>3</sup>/kg. The acceleration due to gravity is constant at 9.81 m/s<sup>2</sup>. What is the depth of atmosphere necessary to produce a pressure of 1.0132 bar at the earth's surface? Consider the atmosphere as a fluid column.  
 (Ans. 64.8 km)



**Solution:**  $dp = dh \rho g$

$$\text{or } dp = dh \times \frac{1}{v} \times g$$

$$\text{or } v = \frac{g dh}{dp}$$

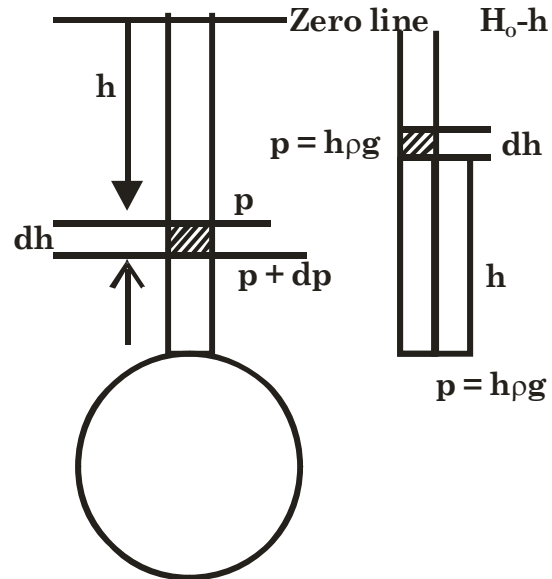
$$pv^{1.4} = 2.3 \times 10^3 = 2300$$

$$\text{or } v = \left( \frac{2300}{p} \right)^{\frac{1}{1.4}} = \left( \frac{2300}{p} \right)^n$$

$$\text{or } \frac{g dh}{dp} = \left( \frac{2300}{p} \right)^n$$

$$\text{or } g dh = \left( \frac{2300}{p} \right)^n dp$$

$$\text{where } n = \frac{1}{1.4}$$

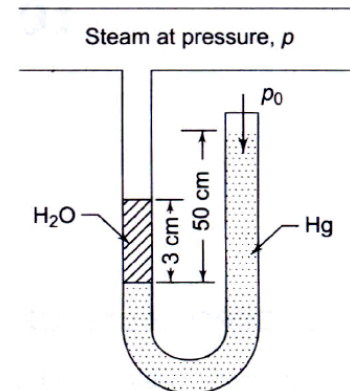


$$\text{or } \int_0^H dh = \frac{2300^n}{g} \int_0^{101320} \frac{dp}{p^n}$$

$$\text{or } h = \frac{2300^n}{g(1-n)} \left[ (101320)^{(1-n)} - 0 \right] = 2420 \text{ m} = 2.42 \text{ km}$$

**Q1.8**

The pressure of steam flowing in a pipeline is measured with a mercury manometer, shown in Figure. Some steam condenses into water. Estimate the steam pressure in kPa. Take the density of mercury as  $13.6 \times 10^3 \text{ kg/m}^3$ , density of water as  $10^3 \text{ kg/m}^3$ , the barometer reading as 76.1 cmHg, and  $g$  as  $9.806 \text{ m/s}^2$ .



**Solution:**  $p_0 + 0.50 \times \rho_{\text{Hg}} \times g = 0.03 \times \rho_{\text{H}_2\text{O}} \times g + p$

$$\text{or } p = 0.761 \times 13.6 \times 10^3 \times 9.806 + 0.5 \times 13.6 \times 10^3 \times 9.806 - 0.03 \times 1000 \times 9.806 \text{ Pa}$$

$$= 167.875 \text{ kPa}$$

**Q1.9**

A vacuum gauge mounted on a condenser reads 0.66 mHg. What is the absolute pressure in the condenser in kPa when the atmospheric pressure is 101.3 kPa?

(Ans. 13.3 kPa)

**Solution:** Absolute = atm. – vacuum

$$= 101.3 - 0.66 \times 13.6 \times 10^3 \times 9.81 \times 10^{-3} \text{ kPa}$$

$$= 13.24 \text{ kPa}$$



## 2. Temperature

### Some Important Notes

#### Comparison of Temperature scale

Boiling Point	100°C	212°F	373K	80°p	30cm
Test Temperature	C	F	K	p	x
Freezing Point	0°C	32°F	273K	0°p	10cm

$$\text{Relation: } \frac{C - 0}{100 - 0} = \frac{F - 32}{212 - 32} = \frac{K - 273}{373 - 273} = \frac{p - 0}{80 - 0} = \frac{x - 10}{30 - 10}$$

### Questions with Solution P. K. Nag

**Q2.1** The limiting value of the ratio of the pressure of gas at the steam point and at the triple point of water when the gas is kept at constant volume is found to be 1.36605. What is the ideal gas temperature of the steam point?

(Ans. 100°C)

**Solution:**

$$\frac{p}{p_t} = 1.36605$$

$$\therefore \theta_{(v)} = 273.16 \times \frac{p}{p_t}$$

$$= 273.16 \times 1.36605$$

$$= 373.15^\circ\text{C}$$

**Q2.2** In a constant volume gas thermometer the following pairs of pressure readings were taken at the boiling point of water and the boiling point of sulphur, respectively:

Water b.p.	50.0	100	200	300
Sulphur b.p.	96.4	193	387	582

The numbers are the gas pressures, mm Hg, each pair being taken with the same amount of gas in the thermometer, but the successive pairs being taken with different amounts of gas in the thermometer. Plot the ratio of  $S_{b.p.}:H_2O_{b.p.}$  against the reading at the water boiling point, and extrapolate the plot to zero pressure at the water boiling point. This

gives the ratio of  $S_{b.p.} : H_2O_{b.p.}$ . On a gas thermometer operating at zero gas pressure, i.e., an ideal gas thermometer. What is the boiling point of sulphur on the gas scale, from your plot?

(Ans. 445°C)

**Solution :**

Water b.p.	50.0	100	200	300
Sulphur b.p.	96.4	193	387	582

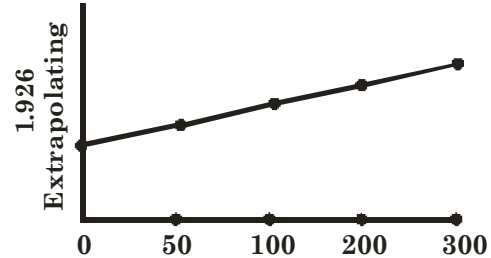
$$\text{Ratio } \frac{S_{b.p.}}{W_{b.p.}} = 1.928 \quad 1.93 \quad 1.935 \quad 1.940$$

$$\therefore T_1 = 100^\circ\text{C} = 373\text{K}$$

$$T_2 = ?$$

$$\frac{p_1}{p_2} = 1.926$$

$$\therefore T_2 = 373 \times 1.926 = 718\text{K} = 445^\circ\text{C}$$



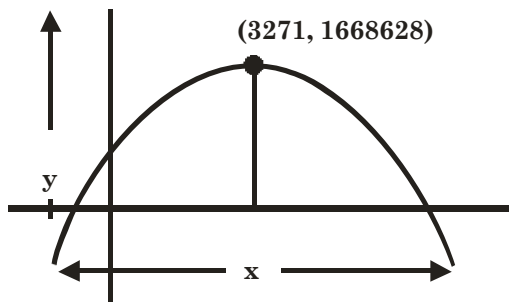
Q2.3

The resistance of a platinum wire is found to be 11,000 ohms at the ice point, 15.247 ohms at the steam point, and 28.887 ohms at the sulphur point. Find the constants A and B in the equation

$$R = R_0(1 + At + Bt^2)$$

And plot R against t in the range 0 to 660°C.

**Solution:**



$$R_0 = 11.000\Omega$$

$$R_{100} = R_0 \{1 + A \times 100 + B \times 100^2\}$$

$$\text{or } 15.247 = 11.000 + 1100A + 11 \times 10^4 B$$

$$\text{or } 3.861 \times 10^{-3} = A + 100B \quad \dots(i)$$

$$28.887 = 11.00 + 445 \times 11A + 445^2 \times 11B$$

$$3.6541 \times 10^{-3} = A + 445B \quad \dots(ii)$$

equation (ii) - (i) gives.

$$B = -6 \times 10^{-7}$$

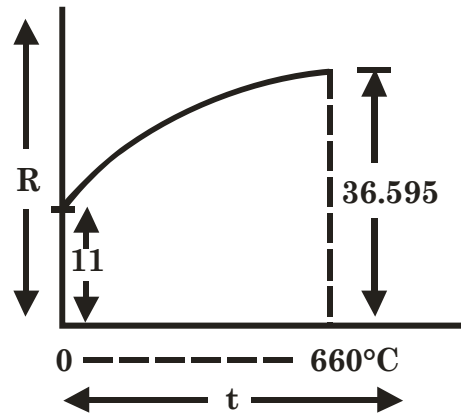
$$A = 3.921 \times 10^{-3}$$

$$R = 11 \{1 + 3.921 \times 10^{-3} t - 6 \times 10^{-7} t^2\}$$

$$\text{or } Y = 11(1 + 3.921 \times 10^{-3} t - 6 \times 10^{-7} t^2)$$

$$\text{or } (t - 3271)^2 = -4 \times 37922 \times (Y - 1668628)$$

$$R_{660} = 36.595$$



- Q2.4** when the reference junction of a thermocouple is kept at the ice point and the test junction is at the Celsius temperature  $t$ , and e.m.f.  $\varepsilon$  of the thermocouple is given by the equation

$$\varepsilon = at + bt^2$$

Where  $a = 0.20$  mV/deg, and  $b = -5.0 \times 10^{-4}$  mV/deg<sup>2</sup>

- (a) Compute the e.m.f. when  $t = -100^\circ\text{C}$ ,  $200^\circ\text{C}$ ,  $400^\circ\text{C}$ , and  $500^\circ\text{C}$ , and draw graph of  $\varepsilon$  against  $t$  in this range.  
 (b) Suppose the e.m.f.  $\varepsilon$  is taken as a thermometric property and that a temperature scale  $t^*$  is defined by the linear equation.

$$t^* = a'\varepsilon + b'$$

And that  $t^* = 0$  at the ice point and  $t^* = 100$  at the steam point. Find the numerical values of  $a'$  and  $b'$  and draw a graph of  $\varepsilon$  against  $t^*$ .

- (c) Find the values of  $t^*$  when  $t = -100^\circ\text{C}$ ,  $200^\circ\text{C}$ ,  $400^\circ\text{C}$ , and  $500^\circ\text{C}$ , and draw a graph of  $t^*$  against  $t$ .  
 (d) Compare the Celsius scale with the  $t^*$  scale.

**Solution:** Try please

- Q2.5** The temperature  $t$  on a thermometric scale is defined in terms of a property  $K$  by the relation

$$t = a \ln K + b$$

Where  $a$  and  $b$  are constants.

The values of  $K$  are found to be 1.83 and 6.78 at the ice point and the steam point, the temperatures of which are assigned the numbers 0 and 100 respectively. Determine the temperature corresponding to a reading of  $K$  equal to 2.42 on the thermometer.

(Ans.  $21.346^\circ\text{C}$ )

**Solution:**

$$t = a \ln x + b$$

$$0 = a \times \ln 1.83 + b$$

$$100 = a \times \ln 6.78 + b$$

... (i)

... (ii)

Equation {(ii) – (i)} gives

$$a \cdot \ln \left( \frac{6.78}{1.83} \right) = 100$$

or  $a = 76.35$

$\therefore b = -a \times \ln 1.83$   
 $= -46.143$

$\therefore t = 76.35 \ln k - 46.143$

$\therefore t^* = 76.35 \times \ln 2.42 - 46.143$   
 $= 21.33^\circ\text{C}$

- Q2.6** The resistance of the windings in a certain motor is found to be 80 ohms at room temperature ( $25^\circ\text{C}$ ). When operating at full load under steady state conditions, the motor is switched off and the resistance of the windings, immediately measured again, is found to be 93 ohms. The windings are made of copper whose resistance at temperature  $t^\circ\text{C}$  is given by

$$R_t = R_0 [1 + 0.00393 t]$$

Where  $R_0$  is the resistance at  $0^\circ\text{C}$ . Find the temperature attained by the coil during full load.

(Ans.  $70.41^\circ\text{C}$ )

**Solution:**

$$R_{25} = R_0 [1 + 0.00393 \times 25]$$

$$\therefore R_0 = \frac{80}{[1 + 0.00393 \times 25]} = 72.84 \, \Omega$$

$$\therefore 93 = 72.84 \{1 + 0.00393 \times t\}$$

or  $t = 70.425^\circ\text{C}$

**Q2.7** A new scale  $N$  of temperature is divided in such a way that the freezing point of ice is  $100^\circ\text{N}$  and the boiling point is  $400^\circ\text{N}$ . What is the temperature reading on this new scale when the temperature is  $150^\circ\text{C}$ ? At what temperature both the Celsius and the new temperature scale reading would be the same?

(Ans.  $550^\circ\text{N}$ ,  $-50^\circ\text{C}$ .)

**Solution:**

$$\frac{150 - 0}{100 - 0} = \frac{N - 100}{400 - 100}$$

or  $N = 550^\circ\text{N}$

let  $N = C$  for  $x^\circ$

then  $\frac{C - 0}{100 - 0} = \frac{N - 100}{400 - 100}$

or  $\frac{x}{100} = \frac{x - 100}{300}$

or  $x = \frac{x - 100}{3}$

or  $3x = x - 100$

or  $2x = -100$

or  $x = -50^\circ\text{C}$

$150^\circ\text{C}$		<b>N</b>
$100^\circ\text{C}$		$400^\circ\text{N}$
$0^\circ\text{C}$		$100^\circ\text{N}$

**Q2.8** A platinum wire is used as a resistance thermometer. The wire resistance was found to be 10 ohm and 16 ohm at ice point and steam point respectively, and 30 ohm at sulphur boiling point of  $444.6^\circ\text{C}$ . Find the resistance of the wire at  $500^\circ\text{C}$ , if the resistance varies with temperature by the relation.

$$R = R_0 (1 + \alpha t + \beta t^2)$$

(Ans. 31.3 ohm)

**Solution:**

$$10 = R_0 (1 + 0 \times \alpha + \beta \times 0^2)$$

$$16 = R_0 (1 + 100 \times \alpha + \beta \times 100^2)$$

$$30 = R_0 (1 + \alpha \times 444.6 + \beta \times 444.6^2)$$

Solve  $R_0$ ,  $\alpha$  &  $\beta$  then

$$R = R_0 (1 + 500 \times \alpha + \beta \times 500^2)$$

3.

# Work and Heat Transfer

## Some Important Notes



Our aim is to give heat to the system and gain work output from it.

So heat input  $\rightarrow$  +ive (positive)

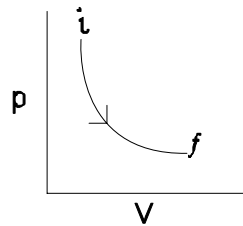
Work output  $\rightarrow$  +ive (positive)

$$W_{i-f} = \int_i^f p dV = \int_{v_i}^{v_f} p dv$$

$$\delta Q = du + \delta W$$

$$\int_i^f \delta Q = u_f - u_i + \int_i^f \delta W$$

$$Q_{i-f} = u_f - u_i + \int_{v_i}^{v_f} p dV$$



## Questions with Solution P. K. Nag

**Q3.1** (a) A pump forces  $1 \text{ m}^3/\text{min}$  of water horizontally from an open well to a closed tank where the pressure is  $0.9 \text{ MPa}$ . Compute the work the pump must do upon the water in an hour just to force the water into the tank against the pressure. Sketch the system upon which the work is done before and after the process.

(Ans.  $5400 \text{ kJ/h}$ )

(b) If the work done as above upon the water had been used solely to raise the same amount of water vertically against gravity without change of pressure, how many meters would the water have been elevated?

(Ans.  $91.74 \text{ m}$ )

(c) If the work done in (a) upon the water had been used solely to accelerate the water from zero velocity without change of pressure or elevation, what velocity would the water have reached? If the work had been used to accelerate the water from an initial velocity of  $10 \text{ m/s}$ , what would the final velocity have been?

(Ans.  $42.4 \text{ m/s}$ ;  $43.6 \text{ m/s}$ )

**Solution:** (a) Flow rate  $1 \text{ m}^3/\text{hr}$ .  
 Pressure of inlet water =  $1 \text{ atm} = 0.101325 \text{ MPa}$   
 Pressure of outlet water =  $0.9 \text{ MPa}$

$$\begin{aligned}\therefore \text{Power} &= \Delta p \dot{v} \\ &= (0.9 - 0.101325) \times 10^3 \text{ kPa} \times \frac{1}{60} \text{ m}^3/\text{s} \\ &= 13.31 \text{ kJ/s}\end{aligned}$$

(b) So that pressure will be 0.9 MPa

$$\therefore h \rho g = 0.9 \text{ MPa}$$

$$\text{or } h = \frac{0.9 \times 10^6}{1000 \times 9.81} \text{ m} = 91.743 \text{ m}$$

$$(c) \quad \frac{1}{2} \dot{m} (V_2^2 - V_1^2) = \Delta p \dot{v} \quad \text{where } \dot{m} = \dot{v} \rho$$

$$\text{or } \frac{1}{2} \rho (V_2^2 - V_1^2) = \Delta p$$

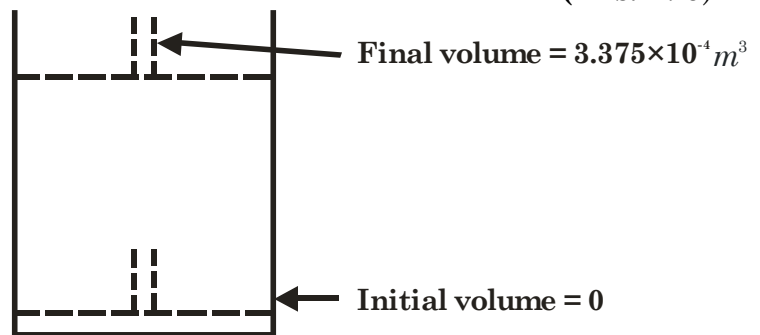
$$\text{or } V_2^2 - V_1^2 = 2 \frac{\Delta p}{\rho}$$

$$\begin{aligned}\text{or } V_2^2 &= V_1^2 + 2 \frac{\Delta p}{\rho} \\ &= 10^2 + \frac{2 \times (0.9 - 0.101325) \times 10^6}{1000} \\ V_2 &= 41.2 \text{ m/s.}\end{aligned}$$

**Q3.2** The piston of an oil engine, of area 0.0045 m<sup>2</sup>, moves downwards 75 mm, drawing in 0.00028 m<sup>3</sup> of fresh air from the atmosphere. The pressure in the cylinder is uniform during the process at 80 kPa, while the atmospheric pressure is 101.325 kPa, the difference being due to the flow resistance in the induction pipe and the inlet valve. Estimate the displacement work done by the air finally in the cylinder.

(Ans. 27 J)

**Solution :** Volume of piston stroke  
 $= 0.0045 \times 0.075 \text{ m}^3$   
 $= 0.0003375 \text{ m}^3$   
 $\therefore \Delta V = 0.0003375 \text{ m}^3$   
 as pressure is constant  
 $= 80 \text{ kPa}$   
 So work done  $= p \Delta V$   
 $= 80 \times 0.0003375 \text{ kJ}$   
 $= 0.027 \text{ kJ} = 27 \text{ J}$



**Q3.3** An engine cylinder has a piston of area 0.12 m<sup>2</sup> and contains gas at a pressure of 1.5 MPa. The gas expands according to a process which is represented by a straight line on a pressure-volume diagram. The final pressure is 0.15 MPa. Calculate the work done by the gas on the piston if the stroke is 0.30 m.

(Ans. 29.7 kJ)

**Solution:** Initial pressure ( $p_1$ ) = 1.5 MPa  
 Final volume ( $V_1$ ) = 0.12 m<sup>2</sup> × 0.3 m



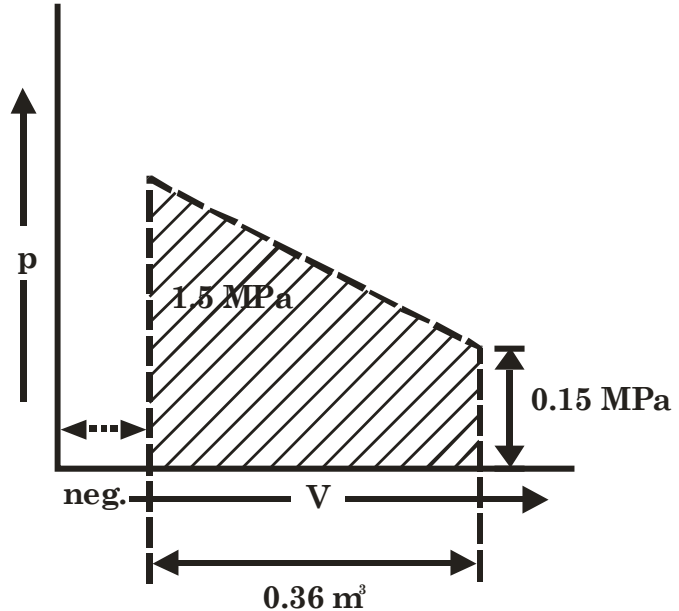
$$= 0.036 \text{ m}^3$$

Final pressure ( $p_2$ ) = 0.15 MPa

As initial pressure too high so the volume is neglected.

Work done = Area of pV diagram

$$\begin{aligned} &= \frac{1}{2}(p_1 + p_2) \times V \\ &= \frac{1}{2}(1.5 + 0.15) \times 0.036 \times 10^3 \text{ kJ} \\ &= 29.7 \text{ kJ} \end{aligned}$$



**Q3.4** A mass of 1.5 kg of air is compressed in a quasi-static process from 0.1 MPa to 0.7 MPa for which  $pv = \text{constant}$ . The initial density of air is 1.16 kg/m³. Find the work done by the piston to compress the air.

(Ans. 251.62 kJ)

**Solution:** For quasi-static process

$$\text{Work done} = \int p dV$$

$$[ \text{given } pV = C ]$$

$$= p_1 V_1 \int_{V_1}^{V_2} \frac{dV}{V}$$

$$\therefore p_1 V_1 = pV = p_2 V_2 = C$$

$$= p_1 V_1 \ln \left( \frac{V_2}{V_1} \right)$$

$$\therefore p = \frac{p_1 V_1}{V}$$

$$= p_1 V_1 \ln \left( \frac{p_1}{p_2} \right)$$

$$\therefore \frac{p_1}{p_2} = \frac{V_2}{V_1}$$

$$= 0.1 \times 1.2931 \times \ln \left| \frac{0.1}{0.7} \right| \text{ MJ}$$

$$\text{given } p_1 = 0.1 \text{ MPa}$$

$$= 251.63 \text{ kJ}$$

$$V_1 = \frac{m_1}{\rho_1} = \frac{1.5}{1.16} \text{ m}^3$$

$$p_2 = 0.7 \text{ MPa}$$

**Q3.5** A mass of gas is compressed in a quasi-static process from 80 kPa, 0.1 m³ to 0.4 MPa, 0.03 m³. Assuming that the pressure and volume are related by  $pv^n = \text{constant}$ , find the work done by the gas system.

(Ans. -11.83 kJ)

**Solution:** Given initial pressure ( $p_1$ ) = 80 kPa

$$\text{Initial volume } (V_1) = 0.1 \text{ m}^3$$

Final pressure ( $p_2$ ) = 0.4 MPa = 400 kPa

Final volume ( $V_2$ ) = 0.03 m<sup>3</sup>

As p-V relation  $pV^n = C$

$$\therefore p_1 V_1^n = p_2 V_2^n$$

taking  $\log_e$  both side

$$\ln p_1 + n \ln V_1 = \ln p_2 + n \ln V_2$$

$$\text{or } n[\ln V_1 - \ln V_2] = \ln p_2 - \ln p_1$$

$$\text{or } n \ln \left( \frac{V_1}{V_2} \right) = \ln \left( \frac{p_2}{p_1} \right)$$

$$\text{or } n = \frac{\ln \left( \frac{p_2}{p_1} \right)}{\ln \left( \frac{V_1}{V_2} \right)} = \frac{\ln \left( \frac{400}{80} \right)}{\ln \left( \frac{0.1}{0.03} \right)} = \frac{1.60944}{1.20397} \approx 1.3367 \approx 1.34$$

$$\begin{aligned} \therefore \text{Work done (W)} &= \frac{p_1 V_1 - p_2 V_2}{n - 1} \\ &= \frac{80 \times 0.1 - 400 \times 0.03}{1.34 - 1} = -11.764 \text{ kJ} \end{aligned}$$

**Q3.6**

**A single-cylinder, double-acting, reciprocating water pump has an indicator diagram which is a rectangle 0.075 m long and 0.05 m high. The indicator spring constant is 147 MPa per m. The pump runs at 50 rpm. The pump cylinder diameter is 0.15 m and the piston stroke is 0.20 m. Find the rate in kW at which the piston does work on the water.**

(Ans. 43.3 kW)

**Solution:** Area of indicated diagram ( $a_d$ ) = 0.075 × 0.05 m<sup>2</sup> = 3.75 × 10<sup>-3</sup> m<sup>2</sup>

Spring constant (k) = 147 MPa/m

$$\begin{aligned} p_m &= \frac{a_d}{l_d} \times k \\ &= \frac{0.075 \times 0.05}{0.075} \times 147 \text{ MPa} \\ &= 7.35 \text{ MPa} = 7.35 \times 10^3 \text{ kPa} \end{aligned}$$

L = 0.20 m

$$A = \frac{\pi d^2}{4} = \frac{\pi \times (0.15)^2}{4} \text{ m}^2 = 0.01767 \text{ m}^2$$

N = 50 rpm

No. of stroke per minute = 2N

$$\begin{aligned} \text{Power} &= \frac{p_m L A (2N)}{60} = \frac{7.35 \times 10^3 \times 0.20 \times 0.01767 \times 2 \times 50}{60} \text{ kW} \\ &= 43.29 \text{ kW} \end{aligned}$$

**Q3.7** A single-cylinder, single-acting, 4 stroke engine of 0.15 m bore develops an indicated power of 4 kW when running at 216 rpm. Calculate the area of the indicator diagram that would be obtained with an indicator having a spring constant of  $25 \times 10^6$  N/m<sup>3</sup>. The length of the indicator diagram is 0.1 times the length of the stroke of the engine.

(Ans. 505 mm<sup>2</sup>)

**Solution:** Given Diameter of piston (D) = 0.15 m  
 I.P = 4 kW =  $4 \times 1000$  W  
 Speed (N) = 216 rpm  
 Spring constant (k) =  $25 \times 10^6$  N/m  
 Length of indicator diagram ( $l_d$ ) = 0.1 × Stroke (L)

Let Area of indicator diagram = ( $a_d$ )

$$\therefore \text{Mean effective pressure } (p_m) = \frac{a_d}{l_d} \times k$$

$$\text{and } \text{I.P.} = \frac{p_m \text{ LAN}}{120} \quad [\text{as 4 stroke engine}]$$

$$\therefore \text{ or } \text{I.P.} = \frac{a_d \times k}{l_d} \times \frac{L \times A \times N}{120}$$

$$\text{or } a_d = \frac{\text{I.P.} \times l_d \times 120}{k \times L \times A \times N}$$

$$= \frac{\text{I.P.} \times 0.1L \times 120 \times 4}{k \times L \times \pi \times D^2 \times N} \quad \left[ \begin{array}{l} \text{area } A = \frac{\pi D^2}{4} \\ \text{and } l_d = 0.1L \end{array} \right]$$

$$= \frac{4 \times 0.1 \times 120 \times 4 \times 1000}{25 \times 10^6 \times \pi \times 0.15^2 \times 216} \text{ m}^2$$

$$= 5.03 \times 10^{-4} \text{ m}^2$$

$$= 503 \text{ mm}^2$$

**Q3.8** A six-cylinder, 4-stroke gasoline engine is run at a speed of 2520 RPM. The area of the indicator card of one cylinder is  $2.45 \times 10^3$  mm<sup>2</sup> and its length is 58.5 mm. The spring constant is  $20 \times 10^6$  N/m<sup>3</sup>. The bore of the cylinders is 140 mm and the piston stroke is 150 mm. Determine the indicated power, assuming that each cylinder contributes an equal power.

(Ans. 243.57 kW)

**Solution:**  $p_m = \frac{a_d}{l_d} \times k$

$$= \frac{2.45 \times 10^3}{58.5} \times 20 \times 10^3 \text{ Pa} \quad \therefore \frac{\text{mm}^2}{\text{mm}} \times \frac{\text{N}}{\text{m}^3} \Rightarrow \frac{\text{mm} \times \text{N}}{\text{m} \times \text{m}^2} = \left( \frac{1}{1000} \right) \text{ N/m}^2$$

$$= 837.607 \text{ kPa}$$

$$L = 0.150 \text{ m}$$

$$A = \frac{\pi D^2}{4} = \frac{\pi \times 0.14^2}{4}$$

$$N = 2520$$

$$n = 6$$

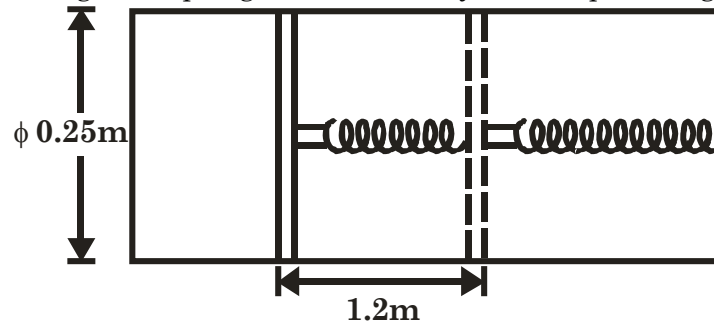
$$\begin{aligned} \therefore \text{I.P.} &= \frac{p_m LAN}{120} \times n \quad [\text{as four stroke}] \\ &= 837.607 \times 0.15 \times \frac{\pi \times 0.14^2}{4} \times \frac{2520 \times 6}{120} \text{ kW} \\ &= 243.696 \text{ kW} \end{aligned}$$

**Q3.9**

A closed cylinder of 0.25 m diameter is fitted with a light frictionless piston. The piston is retained in position by a catch in the cylinder wall and the volume on one side of the piston contains air at a pressure of 750 kN/m<sup>2</sup>. The volume on the other side of the piston is evacuated. A helical spring is mounted coaxially with the cylinder in this evacuated space to give a force of 120 N on the piston in this position. The catch is released and the piston travels along the cylinder until it comes to rest after a stroke of 1.2 m. The piston is then held in its position of maximum travel by a ratchet mechanism. The spring force increases linearly with the piston displacement to a final value of 5 kN. Calculate the work done by the compressed air on the piston.

(Ans. 3.07 kJ)

**Solution:** Work done against spring is work done by the compressed gas



$$\begin{aligned} \text{Mean force} &= \frac{120 + 5000}{2} \\ &= 2560 \text{ N} \end{aligned}$$

$$\text{Travel} = 1.2 \text{ m}$$

$$\begin{aligned} \therefore \text{Work Done} &= 2560 \times 1.2 \text{ N.m} \\ &= 3.072 \text{ kJ} \end{aligned}$$

**By Integration**

$$\text{At a travel (x) force (F}_x\text{)} = 120 + kx$$

$$\text{At 1.2 m then } 5000 = 120 + k \times 1.2$$

$$\therefore F_x = 120 + 4067 x$$

$$\begin{aligned}
 \therefore W &= \int_0^{1.2} F_x dx \\
 &= \int_0^{1.2} [120 + 4067x] dx \\
 &= \left[ 120x + 4067 \times \frac{x^2}{2} \right]_0^{1.2} \\
 &= 120 \times 1.2 + 4067 \times \frac{1.2^2}{2} \text{ J} \\
 &= 144 + 2928.24 \text{ J} \\
 &= 3072.24 \text{ J} = 3.072 \text{ kJ}
 \end{aligned}$$

**Q 3.10**      A steam turbine drives a ship's propeller through an 8: 1 reduction gear. The average resisting torque imposed by the water on the propeller is  $750 \times 10^3 \text{ mN}$  and the shaft power delivered by the turbine to the reduction gear is 15 MW. The turbine speed is 1450 rpm. Determine (a) the torque developed by the turbine, (b) the power delivered to the propeller shaft, and (c) the net rate of working of the reduction gear.

(Ans. (a)  $T = 98.84 \text{ km N}$ , (b) 14.235 MW, (c) 0.765 MW)

**Solution:**      Power of the propeller = Power on turbine shaft

$$\begin{aligned}
 \text{Torque developed by turbine} &= \frac{P}{\omega} \\
 &= \frac{15 \times 1000 \times 60}{2\pi \times 1450} \text{ mN} \\
 &= 98.786 \text{ k mN} \\
 &= 98786 \text{ mN}
 \end{aligned}$$

Power developed by propeller shaft

$$\begin{aligned}
 &= T \times \omega \\
 &= 750 \times 10^3 \times \frac{2\pi \left( \frac{1450}{60} \right)}{8} \\
 &= 14.235 \text{ MW}
 \end{aligned}$$

The net rate of working of the reduction gear

$$\begin{aligned}
 &= (15 - 14.235) \text{ MW} \\
 &= 0.7647 \text{ MW}
 \end{aligned}$$

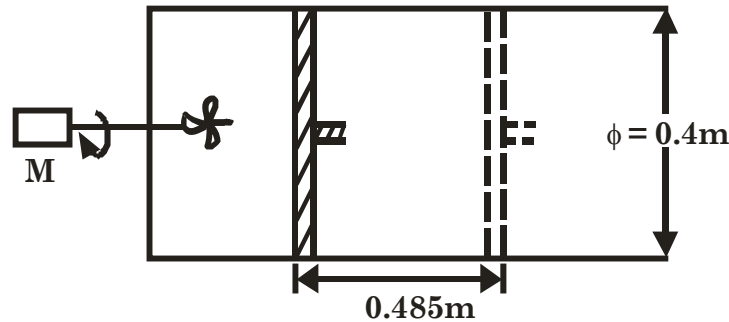
**Q 3.11**      A fluid, contained in a horizontal cylinder fitted with a frictionless leak proof piston, is continuously agitated by means of a stirrer passing through the cylinder cover. The cylinder diameter is 0.40 m. During the stirring process lasting 10 minutes, the piston slowly moves out a distance of 0.485 m against the atmosphere. The net work done by the fluid during the process is 2 kJ. The speed of the electric motor driving the stirrer is 840 rpm. Determine the torque in the shaft and the power output of the motor.

(Ans. 0.08 mN, 6.92 W)

**Solution:** Change of volume =  $A L$

$$\begin{aligned}
 &= \frac{\pi d^2}{4} \times L \\
 &= \frac{\pi \times 0.4^2}{4} \times 0.485 \text{ m}^3 \\
 &= 0.061 \text{ m}^3
 \end{aligned}$$

As piston moves against constant atmospheric pressure then work done =  $p\Delta V$



$$\begin{aligned}
 &= 101.325 \times 0.061 \text{ kJ} \\
 &= 6.1754 \text{ kJ}
 \end{aligned}$$

Net work done by the fluid = 2 kJ

$\therefore$  Net work done by the Motor = 4.1754 kJ

There for power of the motor

$$\begin{aligned}
 &= \frac{4.1754 \times 10^3}{10 \times 60} \text{ W} \\
 &= 6.96 \text{ W}
 \end{aligned}$$

Torque on the shaft =  $\frac{P}{W}$

$$= \frac{6.96 \times 60}{2\pi \times 840}$$

$$= 0.0791 \text{ mN}$$

**Q3.12**

At the beginning of the compression stroke of a two-cylinder internal combustion engine the air is at a pressure of 101.325 kPa. Compression reduces the volume to 1/5 of its original volume, and the law of compression is given by  $pv^{1.2} = \text{constant}$ . If the bore and stroke of each cylinder is 0.15 m and 0.25 m, respectively, determine the power absorbed in kW by compression strokes when the engine speed is such that each cylinder undergoes 500 compression strokes per minute.

(Ans. 17.95 kW)

**Solution:**

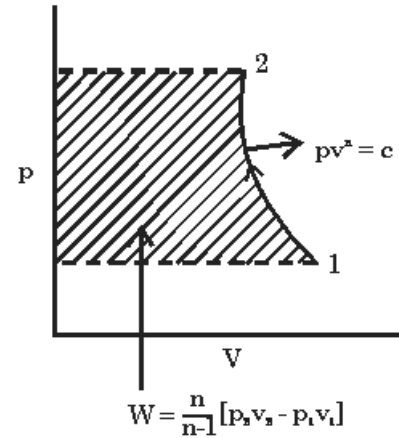
$$\begin{aligned}\text{Initial volume } (V_1) &= \frac{\pi d^2}{4} \times L \\ &= \frac{\pi \times (0.15)^2}{4} \times 0.25 \text{ m}^3 \\ &= 0.00442 \text{ m}^3\end{aligned}$$

$$\text{Initial pressure } (p_1) = 101.325 \text{ kPa.}$$

$$\text{Final volume } (V_2) = \frac{V_1}{5} = 0.000884 \text{ m}^3$$

$$p_1 V_1^{1.2} = p_2 V_2^{1.2}$$

$$\text{Or } p_2 = \frac{p_1 V_1^{1.2}}{V_2^{1.2}} = 699.41 \approx 700 \text{ kPa}$$



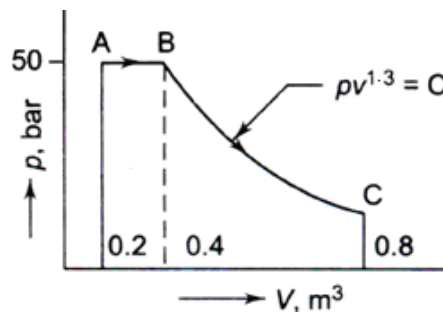
Work done/unit stroke – unit cylinder (W)

$$= \left( \frac{1.2}{1.2-1} \right) \times [p_1 V_1 - p_2 V_2]$$

$$\begin{aligned}&= \left( \frac{101.325 \times 0.00442 - 700 \times 0.000884}{1.2-1} \right) \times 1.2 \\ &\quad (\text{-ive work, as work done on the system})\end{aligned}$$

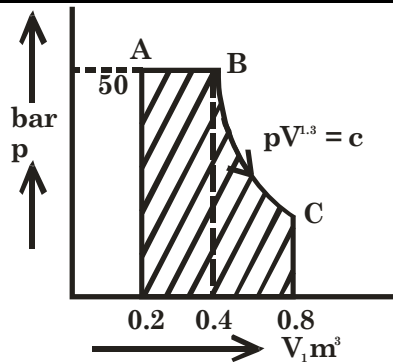
$$\begin{aligned}\text{Power} &= \frac{W \times 500 \times 2 \times 1.2}{60} \text{ kW} \\ &= 17.95 \text{ kW}\end{aligned}$$

**Q3.13** Determine the total work done by a gas system following an expansion process as shown in Figure.



(Ans. 2.253 MJ)

$$\begin{aligned}\text{Solution: Area under AB} &= (0.4 - 0.2) \times 50 \times 10^5 \text{ J} \\ &= 10^6 \text{ W} = 1 \text{ MJ}\end{aligned}$$



Area under BC

$$= \frac{p_1 V_1 - p_2 V_2}{n - 1}$$

$$= \frac{50 \times 10^5 \times 0.4 - 20.31 \times 10^5 \times 0.8}{1.3 - 1} \text{ W}$$

$$= 1.251 \text{ MJ}$$

Here  $p_B = p_A = 50 \text{ bar} = 50 \times 10^5 \text{ Pa}$

$V_B = 0.4 \text{ m}^3$

$V_C = 0.8 \text{ m}^3$

$$p_C = \frac{p_B V_B^{1.3}}{V_C^{1.3}} = \frac{50 \times 10^5 \times 0.4^{1.3}}{0.8^{1.3}}$$

$$= 20.31 \times 10^5 \text{ Pa}$$

Total work = 2.251 MJ

Q3.14

A system of volume  $V$  contains a mass  $m$  of gas at pressure  $p$  and temperature  $T$ . The macroscopic properties of the system obey the following relationship:

$$\left( p + \frac{a}{V^2} \right) (V - b) = mRT$$

Where  $a$ ,  $b$ , and  $R$  are constants.

Obtain an expression for the displacement work done by the system during a constant-temperature expansion from volume  $V_1$  to volume  $V_2$ . Calculate the work done by a system which contains 10 kg of this gas expanding from 1 m³ to 10 m³ at a temperature of 293 K. Use the values  $a = 15.7 \times 10 \text{ Nm}^4$ ,  $b = 1.07 \times 10^{-2} \text{ m}^3$ , and  $R = 0.278 \text{ kJ/kg-K}$ .

(Ans. 1742 kJ)

**Solution:** As it is constant temp-expansion then

$$\left( p + \frac{a}{V^2} \right) (V - b) = \text{constant } (mRT) \text{ (k) as } T = \text{constant}$$



$$\begin{aligned} \therefore \left(p_1 + \frac{a}{V_1^2}\right)(V_1 - b) &= \left(p_2 + \frac{a}{V_2^2}\right)(V_2 - b) = (k) \\ W &= \int_1^2 p \, dV \qquad \therefore \left(p + \frac{a}{V}\right) = \frac{\text{constant}(k)}{V - b} \\ &= \int_1^2 \left(\frac{k}{V - b} - \frac{a}{V^2}\right) dV \qquad \text{or} \quad p = \frac{k}{V - b} - \frac{a}{V^2} \\ &= \left[ k \ln(V - b) + \frac{a}{V} \right]_1^2 \qquad - \int \frac{1}{V^2} \, dv = \frac{1}{V} + c \\ &= k \ln\left(\frac{V_2 - b}{V_1 - b}\right) + a \left(\frac{1}{V_2} - \frac{1}{V_1}\right) \\ &= \left[ \left(p_1 + \frac{a}{V_1^2}\right)(V_1 - b) \ln\left|\frac{V_2 - b}{V_1 - b}\right| + a \left(\frac{1}{V_2} - \frac{1}{V_1}\right) \right] \\ \left(p + \frac{a}{V^2}\right)(V - b) &= \text{constant} \quad (mRT) \quad (k) \text{ as } T = \text{constant} \end{aligned}$$

Given  $m = 10 \text{ kg}$ ;  $T = 293 \text{ K}$ ;  $R = 0.278 \text{ kJ/kg. K}$

$$\begin{aligned} \therefore \text{Constant } k &= 10 \times 293 \times 0.278 \text{ kJ} = 814.54 \text{ kJ} \\ a &= 15.7 \times 10 \text{ Nm}^4; \quad b = 1.07 \times 10^{-2} \text{ m}^3 \\ \Rightarrow V_2 &= 10 \text{ m}^3, \quad V_1 = 1 \text{ m}^3 \end{aligned}$$

$$\begin{aligned} \therefore W &= 814.54 \ln\left(\frac{10 - 1.07 \times 10^{-2}}{1 - 1.07 \times 10^{-2}}\right) + a \left(\frac{1}{10} - \frac{1}{1}\right) \\ &= (1883.44 - a \times 0.9) \text{ kJ} \\ &= (1883.44 - 157 \times 0.9) \text{ kJ} \\ &= 1742.14 \text{ kJ} \end{aligned}$$

**Q3.15** If a gas of volume  $6000 \text{ cm}^3$  and at pressure of  $100 \text{ kPa}$  is compressed quasistatically according to  $pV^2 = \text{constant}$  until the volume becomes  $2000 \text{ cm}^3$ , determine the final pressure and the work transfer.

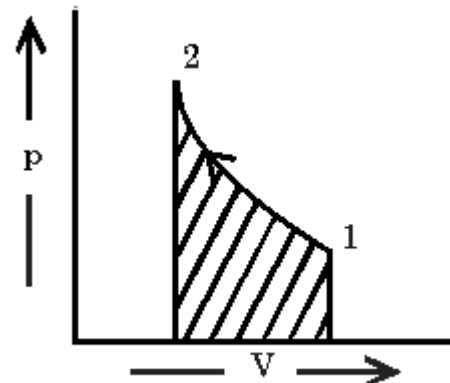
(Ans.  $900 \text{ kPa}$ ,  $-1.2 \text{ kJ}$ )

**Solution:** Initial volume ( $V_1$ ) =  $6000 \text{ cm}^3$   
 $= 0.006 \text{ m}^3$   
 Initial pressure ( $p_1$ ) =  $100 \text{ kPa}$

Final volume ( $V_2$ ) =  $2000 \text{ cm}^3$   
 $= 0.002 \text{ m}^3$

If final pressure ( $p_2$ )

$$\therefore p_2 = \frac{p_1 V_1^2}{V_2^2} = \frac{100 \times (0.006)^2}{(0.002)^2} = 900 \text{ kPa}$$



$$\begin{aligned}\text{work done on the system} &= \frac{1}{n-1} [p_2 V_2 - p_1 V_1] \\ &= \frac{1}{2-1} [900 \times 0.002 - 100 \times 0.006] \text{ kJ} \\ &= 1.2 \text{ kJ}\end{aligned}$$

**Q3.16**      **The flow energy of 0.124 m<sup>3</sup>/min of a fluid crossing a boundary to a system is 18 kW. Find the pressure at this point.**

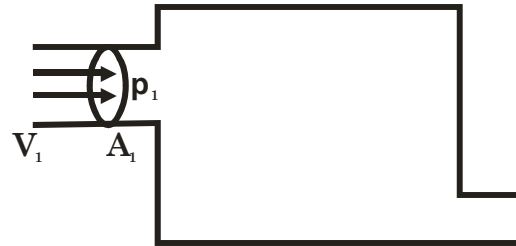
(Ans. 8709 kPa)

**Solution:**    If pressure is  $p_1$   
Area is  $A_1$   
Velocity is  $V_1$   
Volume flow rate ( $Q$ ) =  $A_1 V_1$   
 $\therefore$  Power = force  $\times$  velocity

$$\begin{aligned}&= p_1 A_1 \times V_1 \\ &= p_1 \times (Q)\end{aligned}$$

$$\therefore 18 = p_1 \times \frac{0.124}{60}$$

$$\begin{aligned}\text{or } p_1 &= \frac{18 \times 60}{0.124} \text{ kPa} \\ &= 8.71 \text{ MPa}\end{aligned}$$



**Q3.17**      **A milk chilling unit can remove heat from the milk at the rate of 41.87 MJ/h. Heat leaks into the milk from the surroundings at an average rate of 4.187 MJ/h. Find the time required for cooling a batch of 500 kg of milk from 45°C to 5°C. Take the  $c_p$  of milk to be 4.187 kJ/kg K.**

(Ans. 2h 13 min)

**Solution:**    Heat to be removed ( $H$ ) =  $mst$   
 $= 500 \times 4.187 \times (45-5) \text{ kJ}$   
 $= 83.740 \text{ MJ}$

Net rate of heat removal

$$\begin{aligned}&= \dot{H}_{\text{rej}} - H_{\text{leak}} \\ &= (41.87 - 4.187) \text{ MJ/h} \\ &= 37.683 \text{ MJ/h}\end{aligned}$$

$$\begin{aligned}\therefore \text{Time required} &= \frac{83.740}{37.683} \text{ hr} \\ &= 2 \text{ hr. } 13 \text{ min. } 20 \text{ sec.}\end{aligned}$$

**Q3.18**      **680 kg of fish at 5°C are to be frozen and stored at -12°C. The specific heat of fish above freezing point is 3.182, and below freezing point is 1.717 kJ/kg K. The freezing point is -2°C, and the latent heat of fusion is 234.5 kJ/kg. How much heat must be removed to cool the fish, and what per cent of this is latent heat?**

(Ans. 186.28 MJ, 85.6%)

**Solution:**    Heat to be removed above freezing point  
 $= 680 \times 3.182 \times \{5 - (-2)\} \text{ kJ}$   
 $= 15.146 \text{ MJ}$

Heat to be removed latent heat

$$= 680 \times 234.5 \text{ kJ}$$

$$= 159.460 \text{ MJ}$$

Heat to be removed below freezing point

$$= 680 \times 1.717 \times \{-2 - (-12)\} \text{ kJ}$$

$$= 11.676 \text{ MJ}$$

$\therefore$  Total Heat = 186.2816 MJ

$$\% \text{ of Latent heat} = \frac{159.460}{186.2816} \times 100 = 85.6 \%$$



## 4.

## First Law of Thermodynamics

## Some Important Notes

- $dQ$  is an inexact differential, and we write

$$\int_1^2 dQ = Q_{1-2} \quad \text{or} \quad {}_1Q_2 \neq Q_2 - Q_1$$

- $dW$  is an inexact differential, and we write

$$W_{1-2} = \int_1^2 dW = \int_1^2 p dV \neq W_2 - W_1$$

- $(\Sigma Q)_{\text{cycle}} = (\Sigma W)_{\text{cycle}} \quad \text{or} \quad \oint \delta Q = \oint \delta W$

The summations being over the entire cycle.

- $\delta Q - \delta W = dE$

E consists of



$$E = U + KE + PE$$

U - internal energy

KE - the kinetic energy

PE - the potential energy

For the whole process A



$$Q - W = E_2 - E_1$$

Similarly for the process B



$$Q - W = E_1 - E_2$$

- An isolated system which does not interact with the surroundings  $Q = 0$  and  $W = 0$ . Therefore, E remains constant for such a system.
- The Zeroth Law deals with thermal equilibrium and provides a means for measuring temperatures.
- The First Law deals with the conservation of energy and introduces the concept of internal energy.
- The Second Law of thermodynamics provides with the guidelines on the conversion heat energy of matter into work. It also introduces the concept of entropy.
- The Third Law of thermodynamics defines the absolute zero of entropy. The entropy of a pure crystalline substance at absolute zero temperature is zero.

## Summation of 3 Laws

- Firstly, there isn't a meaningful temperature of the source from which we can get the full conversion of heat to work. Only at infinite temperature one can dream of getting the full 1 kW work output.
- Secondly, more interestingly, there isn't enough work available to produce 0K. In other words, 0 K is unattainable. This is precisely the Third law.

- Because, we don't know what 0 K looks like, we haven't got a starting point for the temperature scale!! That is why all temperature scales are at best empirical.

**You can't get something for nothing:**

To get work output you must give some thermal energy.

**You can't get something for very little:**

To get some work output there is a minimum amount of thermal energy that needs to be given.

**You can't get every thing:**

However much work you are willing to give 0 K can't be reached.

**Violation of all 3 laws:**

Try to get everything for nothing.

### Questions with Solution P. K. Nag

**Q4.1** An engine is tested by means of a water brake at 1000 rpm. The measured torque of the engine is 10000 mN and the water consumption of the brake is 0.5 m<sup>3</sup>/s, its inlet temperature being 20°C. Calculate the water temperature at exit, assuming that the whole of the engine power is ultimately transformed into heat which is absorbed by the cooling water.

(Ans. 20.5°C)

**Solution:** Power =  $T \cdot \omega$

$$\begin{aligned}
 &= 10000 \times \left( \frac{2\pi \times 1000}{60} \right) \\
 &= 1.0472 \times 10^6 \text{ W} \\
 &= 1.0472 \text{ MW}
 \end{aligned}$$

Let final temperature =  $t^\circ\text{C}$

$$\begin{aligned}
 \therefore \text{Heat absorb by cooling water / unit} &= \dot{m} \cdot s \cdot \Delta t \\
 &= \dot{v} \rho s \Delta t \\
 &= 0.5 \times 1000 \times 4.2 \times (t - 20) \\
 \therefore 0.5 \times 1000 \times 4.2 \times (t - 20) &= 1.0472 \times 10^6 \\
 \therefore t - 20 &= 0.4986 \approx 0.5 \\
 \therefore t &= 20.5^\circ\text{C}
 \end{aligned}$$

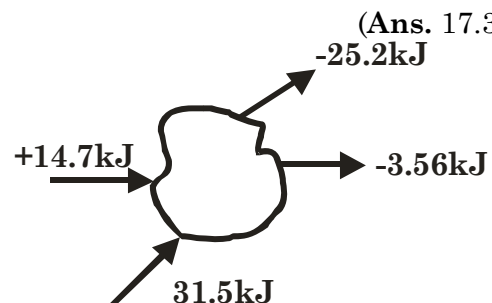
**Q4.2** In a cyclic process, heat transfers are + 14.7 kJ, - 25.2 kJ, - 3.56 kJ and + 31.5 kJ. What is the net work for this cyclic process?

(Ans. 17.34 kJ)

**Solution :**  $\sum Q = (14.7 + 31.5 - 25.2 - 3.56) \text{ kJ}$   
 $= 17.44 \text{ kJ}$

From first law of thermodynamics  
 (for a cyclic process)

$$\begin{aligned}
 \sum Q &= \sum W \\
 \therefore \sum W &= 17.44 \text{ kJ}
 \end{aligned}$$



**Q4.3** A slow chemical reaction takes place in a fluid at the constant pressure of 0.1 MPa. The fluid is surrounded by a perfect heat insulator during the reaction which begins at state 1 and ends at state 2. The insulation is then removed and 105 kJ of heat flow to the surroundings as the fluid goes to state 3. The following data are observed for the fluid at states 1, 2 and 3.

State	$v \text{ (m}^3\text{)}$	$t \text{ (}^\circ\text{C)}$
1	0.003	20
2	0.3	370
3	0.06	20

For the fluid system, calculate  $E_2$  and  $E_3$ , if  $E_1 = 0$

(Ans.  $E_2 = -29.7 \text{ kJ}$ ,  $E_3 = -110.7 \text{ kJ}$ )

**Solution:** From first law of thermodynamics

$$dQ = \Delta E + p dV$$

$$\therefore Q = \Delta E + \int p dV$$

$$\therefore Q_{1-2} = (E_2 - E_1) + \int_1^2 p dV$$

$$\text{or} \quad = (E_2 - E_1) + 0.1 \times 10^3 (0.3 - 0.003) \quad [\text{as insulated } Q_{2-3} = 0]$$

$$\text{or} \quad E_2 = -29.7 \text{ kJ}$$

$$Q_{2-3} = (E_3 - E_2) + \int_2^3 p dV$$

$$\text{or} \quad -105 = (E_3 - E_2) + 0.1 \times 10^3 (0.06 - 0.3)$$

$$\text{or} \quad -105 = E_3 + 29.7 + 0.1 \times 10^3 (0.06 - 0.3)$$

$$\text{or} \quad -105 = E_3 + 29.7 - 24$$

$$\begin{aligned} \text{or} \quad E_3 &= -105 - 29.7 + 24 \\ &= -110.7 \text{ kJ} \end{aligned}$$

**Q4.4** During one cycle the working fluid in an engine engages in two work interactions: 15 kJ to the fluid and 44 kJ from the fluid, and three heat interactions, two of which are known: 75 kJ to the fluid and 40 kJ from the fluid. Evaluate the magnitude and direction of the third heat transfer.

(Ans. – 6 kJ)

**Solution:** From first law of thermodynamics

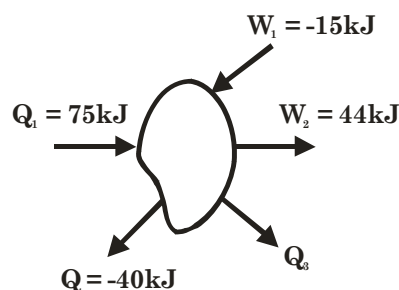
$$\sum dQ = \sum dW$$

$$\therefore Q_1 + Q_2 + Q_3 = W_1 + W_2$$

$$\text{or} \quad 75 - 40 + Q_3 = -15 + 44$$

$$Q_3 = -6 \text{ kJ}$$

i.e. 6 kJ from the system

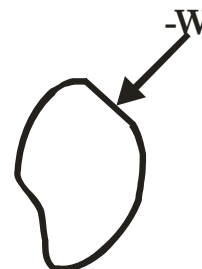


**Q4.5** A domestic refrigerator is loaded with food and the door closed. During a certain period the machine consumes 1 kWh of energy and the internal energy of the system drops by 5000 kJ. Find the net heat transfer for the system.

(Ans. – 8.6 MJ)

**Solution:**  $Q = \Delta E + W$

$$\begin{aligned} Q_{2-1} &= (E_2 - E_1) + W_{2-1} \\ &= -5000 \text{ kJ} + \frac{-1000 \times 3600}{1000} \text{ kJ} \\ &= -8.6 \text{ MJ} \end{aligned}$$





# First Law of Thermodynamics

**By: S K Mondal**

## Chapter 4

**Q4.6** 1.5 kg of liquid having a constant specific heat of 2.5 kJ/kg K is stirred in a well-insulated chamber causing the temperature to rise by 15°C. Find  $\Delta E$  and  $W$  for the process.

(Ans.  $\Delta E = 56.25$  kJ,  $W = -56.25$  kJ)

**Solution:** Heat added to the system  $= 1.5 \times 2.5 \times 15$  kJ  
 $= 56.25$  kJ

$\therefore \Delta E \text{ rise} = 56.25$  kJ

As it is insulated then  $dQ = 0$

$\therefore \Delta Q = \Delta E + W$

or  $0 = 56.25 + W$

or  $W = -56.25$  kJ

**Q4.7** The same liquid as in Problem 4.6 is stirred in a conducting chamber. During the process 1.7 kJ of heat are transferred from the liquid to the surroundings, while the temperature of the liquid is rising to 15°C. Find  $\Delta E$  and  $W$  for the process.

(Ans.  $\Delta E = 54.55$  kJ,  $W = 56.25$  kJ)

**Solution:** As temperature rise is same so internal energy is same

$\Delta E = 56.25$  kJ

As heat is transferred from the system so we have to give more work  $= 1.7$  kJ to the system

So  $W = -56.25 - 1.7$  kJ  
 $= -57.95$  kJ

**Q4.8** The properties of a certain fluid are related as follows:

$$u = 196 + 0.718t$$

$$pv = 0.287 (t + 273)$$

Where  $u$  is the specific internal energy (kJ/kg),  $t$  is in °C,  $p$  is pressure (kN/m<sup>2</sup>), and  $v$  is specific volume (m<sup>3</sup>/kg). For this fluid, find  $c_v$  and  $c_p$ .

(Ans. 0.718, 1.005 kJ/kg K)

**Solution:**

$$C_p = \left( \frac{\partial h}{\partial T} \right)_p$$

$$= \left[ \frac{\partial (u + pV)}{\partial T} \right]_p$$

$$= \left[ \frac{\partial \{196 + 0.718t + 0.287(t + 273)\}}{\partial T} \right]_p$$

$$= \left[ \frac{0 + 0.718 \frac{\partial t}{\partial T} + 0.287 \frac{\partial t}{\partial T} + 0}{\partial T} \right]_p$$

$$= \left[ 1.005 \frac{\partial t}{\partial T} \right]_p$$

$$= 1.005 \text{ kJ / kg - K}$$

$$\begin{bmatrix} T = t + 273 \\ \therefore \partial T = \partial t \end{bmatrix}$$

$$\begin{aligned}
 c_v &= \left( \frac{\partial u}{\partial T} \right)_v \\
 &= \left[ \frac{\partial (196 + 0.718t)}{\partial T} \right]_v \\
 &= \left[ 0 + 0.718 \frac{\partial t}{\partial T} \right]_v \\
 &= 0.718 \text{ kJ / kg - K}
 \end{aligned}$$

**Q4.9** A system composed of 2 kg of the above fluid expands in a frictionless piston and cylinder machine from an initial state of 1 MPa, 100°C to a final temperature of 30°C. If there is no heat transfer, find the net work for the process.

(Ans. 100.52 kJ)

**Solution:** Heat transfer is not there so

$$\begin{aligned}
 Q &= \Delta E + W \\
 W &= -\Delta E \\
 &= -\Delta U \\
 &= -\int_1^2 C_v dT \\
 &= -0.718(T_2 - T_1) \\
 &= -0.718(100 - 30) \\
 &= -50.26 \text{ kJ / kg}
 \end{aligned}$$

$$\therefore \text{Total work (W)} = 2 \times (-50.26) = -100.52 \text{ kJ}$$

**Q 4.10** If all the work in the expansion of Problem 4.9 is done on the moving piston, show that the equation representing the path of the expansion in the  $pv$ -plane is given by  $pv^{1.4} = \text{constant}$ .

**Solution:** Let the process is  $pV^n = \text{constant}$ .

Then

$$\begin{aligned}
 \text{Work done} &= \frac{p_1 V_1 - p_2 V_2}{n - 1} & [\because pV = mRT] \\
 &= \frac{mRT_1 - mRT_2}{n - 1} & \left[ \begin{aligned} R &= (c_p - c_v) \\ &= 1.005 - 0.718 \\ &= 0.287 \text{ kJ / kg - K} \end{aligned} \right] \\
 &= \frac{mR}{n - 1} (T_1 - T_2) \\
 \text{or} &= \frac{2 \times 0.287 \times (100 - 30)}{n - 1} = 100.52 \\
 \text{or} & n - 1 = 0.39972 \\
 \text{or} & n = 1.39972 \approx 1.4
 \end{aligned}$$

**Q4.11** A stationary system consisting of 2 kg of the fluid of Problem 4.8 expands in an adiabatic process according to  $pv^{1.2} = \text{constant}$ . The initial

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## Chapter 4

conditions are 1 MPa and 200°C, and the final pressure is 0.1 MPa. Find  $W$  and  $\Delta E$  for the process. Why is the work transfer not equal to  $\int p dV$ ?

(Ans.  $W = 217.35$ ,  $\Delta E = -217.35$  kJ,  $\int p dV = 434.4$  kJ)

**Solution:**

$$\frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} = \left( \frac{0.1}{1} \right)^{\frac{1.2-1}{1.2}}$$

$$\therefore T_2 = T_1 \times (0.1)^{\frac{0.2}{1.2}}$$

$$= 322.251$$

$$= 49.25^\circ\text{C}$$

**From first law of thermodynamics**

$$dQ = \Delta E + dW$$

$$\therefore 0 = \int C_v dT + dW$$

$$\therefore dW = - \int C_v dT$$

$$= -0.718 \times \int_1^2 dT = -0.718 \times (200 - 49.25) \text{ kJ/kg}$$

$$W = -2 \times W$$

$$= -2 \times 108.2356 \text{ kJ}$$

$$= -216.5 \text{ kJ}$$

$$\therefore \Delta E = 216.5 \text{ kJ}$$

$$\int p dV = \frac{p_1 V_1 - p_2 V_2}{n-1}$$

$$= \frac{mRT_1 - mRT_2}{n-1}$$

$$= \frac{mR(T_1 - T_2)}{n-1}$$

$$= \frac{2 \times 0.287(200 - 49.25)}{(1.2-1)}$$

$$= 432.65 \text{ kJ}$$

As this is not quasi-static process so work is not  $\int p dV$ .

**Q4.12**

A mixture of gases expands at constant pressure from 1 MPa, 0.03 m<sup>3</sup> to 0.06 m<sup>3</sup> with 84 kJ positive heat transfer. There is no work other than that done on a piston. Find  $\Delta E$  for the gaseous mixture.

(Ans. 54 kJ)

The same mixture expands through the same state path while a stirring device does 21 kJ of work on the system. Find  $\Delta E$ ,  $W$ , and  $Q$  for the process.

(Ans. 54 kJ, -21 kJ, 33 kJ)

**Solution:** Work done by the gas ( $W$ ) =  $\int p dV$

$$= p(V_2 - V_1)$$

$$= 1 \times 10^3 (0.06 - 0.03) \text{ kJ}$$

$$= 30 \text{ kJ}$$

Heat added = 89 kJ

$\therefore Q = \Delta E + W$

or  $\Delta E = Q - W = 89 - 30 = 59 \text{ kJ}$

**Q4.13** A mass of 8 kg gas expands within a flexible container so that the  $p$ - $v$  relationship is of the form  $pv^{1.2} = \text{constant}$ . The initial pressure is 1000 kPa and the initial volume is 1 m<sup>3</sup>. The final pressure is 5 kPa. If specific internal energy of the gas decreases by 40 kJ/kg, find the heat transfer in magnitude and direction.

(Ans. + 2615 kJ)

**Solution:**

$$\frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} = \left( \frac{V_1}{V_2} \right)^{n-1}$$

$$\therefore \frac{p_2}{p_1} = \left( \frac{V_1}{V_2} \right)^n$$

$$\text{or } \frac{V_2}{V_1} = \left( \frac{p_1}{p_2} \right)^{\frac{1}{n}}$$

$$\text{or } V_2 = V_1 \times \left( \frac{p_1}{p_2} \right)^{\frac{1}{n}}$$

$$= 1 \times \left( \frac{1000}{5} \right)^{\frac{1}{1.2}} = 82.7 \text{ m}^3$$

$$\therefore W = \frac{p_1 V_1 - p_2 V_2}{n - 1}$$

$$= \frac{1000 \times 1 - 5 \times 82.7}{1.2 - 1} = 2932.5 \text{ kJ}$$

$$\Delta E = -8 \times 40 = -320 \text{ kJ}$$

$$\therefore Q = \Delta E + W = -320 + 2932.5 = 2612.5 \text{ kJ}$$

**Q4.14** A gas of mass 1.5 kg undergoes a quasi-static expansion which follows a relationship  $p = a + bV$ , where  $a$  and  $b$  are constants. The initial and final pressures are 1000 kPa and 200 kPa respectively and the corresponding volumes are 0.20 m<sup>3</sup> and 1.20 m<sup>3</sup>. The specific internal energy of the gas is given by the relation

$$u = 1.5 pv - 85 \text{ kJ/kg}$$

Where  $p$  is the kPa and  $v$  is in m<sup>3</sup>/kg. Calculate the net heat transfer and the maximum internal energy of the gas attained during expansion.

(Ans. 660 kJ, 503.3 kJ)

**Solution:**

$$1000 = a + b \times 0.2 \quad \dots(i)$$

$$200 = a + b \times 1.2 \quad \dots(ii)$$

(ii) – (i) gives

$$-800 = b$$

$$\therefore a = 1000 + 2 \times 800 = 1160$$

$$\therefore p = 1160 - 800V$$

$$\begin{aligned} \therefore W &= \int_{v_1}^{v_2} p dV \\ &= \int_{0.2}^{1.2} (1160 - 800V) dV \\ &= \left[ 1160V - 400V^2 \right]_{0.2}^{1.2} \\ &= 1160 \times (1.2 - 0.2) - 400(1.2^2 - 0.2^2) \text{ kJ} \\ &= 1160 - 560 \text{ kJ} = 600 \text{ kJ} \end{aligned}$$

$$u_1 = 1.5 \times 1000 \times \frac{0.2}{1.5} - 85 = 215 \text{ kJ/kg}$$

$$u_2 = 1.5 \times 200 \times \frac{1.2}{1.5} - 85 = 155 \text{ kJ/kg}$$

$$\therefore \Delta u = u_2 - u_1 = (275 - 215) = 40 \text{ kJ/kg}$$

$$\therefore \Delta U = m \Delta u = 40 \times 1.5 = 60 \text{ kJ}$$

$$\therefore Q = \Delta U + W = 60 + 600 = 660 \text{ kJ}$$

$$\Rightarrow u = 1.5pv - 85 \text{ kJ/kg}$$

$$= 1.5 \left( \frac{1160 - 800v}{1.5} \right) v - 85 \text{ kJ/kg}$$

$$= 1160v - 800v^2 - 85 \text{ kJ/kg}$$

$$\frac{\partial u}{\partial v} = 1160 - 1600v$$

$$\text{for maximum } u, \quad \frac{\partial u}{\partial v} = 0 \therefore v = \frac{1160}{1600} = 0.725$$

$$\begin{aligned} \therefore u_{\max} &= 1160 \times 0.725 - 800 \times (0.725)^2 - 85 \text{ kJ/kg} \\ &= 335.5 \text{ kJ/kg} \end{aligned}$$

$$U_{\max} = 1.5 u_{\max} = 503.25 \text{ kJ}$$

**Q4.15**

**The heat capacity at constant pressure of a certain system is a function of temperature only and may be expressed as**

$$C_p = 2.093 + \frac{41.87}{t + 100} \text{ J/}^\circ\text{C}$$

Where  $t$  is the temperature of the system in  $^\circ\text{C}$ . The system is heated while it is maintained at a pressure of 1 atmosphere until its volume increases from  $2000 \text{ cm}^3$  to  $2400 \text{ cm}^3$  and its temperature increases from  $0^\circ\text{C}$  to  $100^\circ\text{C}$ .

(a) Find the magnitude of the heat interaction.

(b) How much does the internal energy of the system increase?

(Ans. (a) 238.32 J (b) 197.79 J)

**Solution:**  $Q = \int_{273}^{373} C_p dT$

$$t = T - 273$$

$$\therefore t + 100 = T - 173$$

$$\begin{aligned} &= \int_{273}^{373} \left( 2.093 + \frac{41.87}{T - 173} \right) dT \\ &= \left[ 2.093T + 41.87 \ln |T - 173| \right]_{273}^{373} \\ &= 2.093(373 - 273) + 41.87 \ln \left( \frac{200}{100} \right) \\ &= 209.3 + 41.87 \ln 2 \\ &= 238.32 \text{ J} \end{aligned}$$

$$Q = \Delta E + \int p dV$$

$$\Delta E = Q - \int p dV$$

$$= Q - p(V_2 - V_1)$$

$$= 238.32 - 101.325(0.0024 - 0.0020) \times 1000 \text{ J}$$

$$= (238.32 - 40.53) \text{ J}$$

$$= 197.79 \text{ J}$$

**Q4.16**

An imaginary engine receives heat and does work on a slowly moving piston at such rates that the cycle of operation of 1 kg of working fluid can be represented as a circle 10 cm in diameter on a  $p$ - $v$  diagram on which 1 cm = 300 kPa and 1 cm = 0.1 m<sup>3</sup>/kg.

(a) How much work is done by each kg of working fluid for each cycle of operation?

(b) The thermal efficiency of an engine is defined as the ratio of work done and heat input in a cycle. If the heat rejected by the engine in a cycle is 1000 kJ per kg of working fluid, what would be its thermal efficiency?

(Ans. (a) 2356.19 kJ/kg, (b) 0.702)

**Solution:** Given Diameter = 10 cm

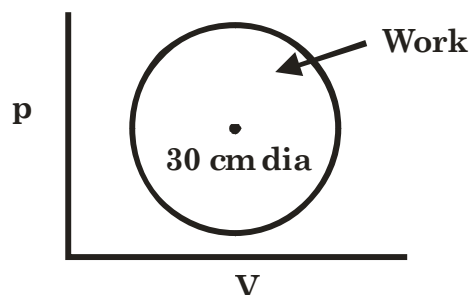
$$\therefore \text{Area} = \frac{\pi \times 10^2}{4} \text{ cm}^2 = 78.54 \text{ cm}^2$$

$$\begin{aligned} 1 \text{ cm}^2 &= 300 \text{ kPa} \times 0.1 \text{ m}^3 / \text{kg} \\ &= 30 \text{ kJ} \end{aligned}$$

$$\begin{aligned} \therefore \text{Total work done} &= 78.54 \times 30 \text{ kJ} / \text{kg} \\ &= 2356.2 \text{ kJ} / \text{kg} \end{aligned}$$

$$\text{Heat rejected} = 1000 \text{ kJ}$$

$$\begin{aligned} \text{Therefore, } \eta &= \frac{2356.2}{2356.2 + 1000} \times 100\% \\ &= 70.204\% \end{aligned}$$



# First Law of Thermodynamics

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**Q4.17** A gas undergoes a thermodynamic cycle consisting of three processes beginning at an initial state where  $p_1 = 1$  bar,  $V_1 = 1.5 \text{ m}^3$  and  $U_1 = 512 \text{ kJ}$ .

The processes are as follows:

- (i) Process 1–2: Compression with  $pV = \text{constant}$  to  $p_2 = 2$  bar,  $U_2 = 690 \text{ kJ}$
- (ii) Process 2–3:  $W_{23} = 0$ ,  $Q_{23} = -150 \text{ kJ}$ , and
- (iii) Process 3–1:  $W_{31} = +50 \text{ kJ}$ . Neglecting KE and PE changes, determine the heat interactions  $Q_{12}$  and  $Q_{31}$ .

(Ans. 74 kJ, 22 kJ)

**Solution:**  $Q_{1-2} = \Delta E + \int p dV$

$$\begin{aligned}
 Q_{1-2} &= (u_2 - u_1) + p_1 V_1 \int_{V_1}^{V_2} \frac{dV}{V} \\
 &= (690 - 512) + 100 \times 1.5 \times \ln \left( \frac{p_1}{p_2} \right) \\
 &= 178 - 103.972 \\
 &= 74.03 \text{ kJ}
 \end{aligned}$$

As  $W_{2-3}$  is ZERO so it is constant volume process. As  $W_{31}$  is +ive (positive) so expansion is done.

$$\therefore u_3 = u_2 - 150 = 540 \text{ kJ}$$

$$\begin{aligned}
 \therefore Q_{31} &= u_1 - u_3 + W \\
 &= \Delta E + W = -(540 - 512) + 50 \\
 &= -28 + 50 = 22 \text{ kJ}
 \end{aligned}$$

**Q4.18** A gas undergoes a thermodynamic cycle consisting of the following processes:

- (i) Process 1–2: Constant pressure  $p = 1.4$  bar,  $V_1 = 0.028 \text{ m}^3$ ,  $W_{12} = 10.5 \text{ kJ}$
- (ii) Process 2–3: Compression with  $pV = \text{constant}$ ,  $U_3 = U_2$
- (iii) Process 3–1: Constant volume,  $U_1 - U_3 = -26.4 \text{ kJ}$ . There are no significant changes in KE and PE.

- (a) Sketch the cycle on a  $p$ - $V$  diagram
- (b) Calculate the net work for the cycle in kJ
- (c) Calculate the heat transfer for process 1–2
- (d) Show that  $\sum_{\text{cycle}} Q = \sum_{\text{cycle}} W$ .

(Ans. (b)  $-8.28 \text{ kJ}$ , (c)  $36.9 \text{ kJ}$ )

**Solution:** (b)  $W_{12} = 10.5 \text{ kJ}$

$$W_{23} = \int_2^3 p dV$$

$$= p_2 V_2 \int_2^3 \frac{dV}{V}$$

$$= p_2 V_2 \ln \left( \frac{V_3}{V_2} \right)$$

$$= p_2 V_2 \ln \left( \frac{V_1}{V_2} \right)$$

$$= 1.4 \times 100 \times 0.103 \times \ln \left( \frac{0.028}{0.103} \right)$$

$$= -18.783 \text{ kJ} \quad \left[ \begin{array}{l} \text{as } W_{12} = p(V_2 - V_1) \\ 10.5 = 1.4 \times 100(V_2 - 0.028) \\ \therefore V_2 = 0.103 \text{ m}^3 \end{array} \right]$$

$$W_{31} = 0 \text{ as constant volume}$$

$$\therefore \text{Net work output} = -8.283 \text{ kJ} \quad \text{ans.(b)}$$

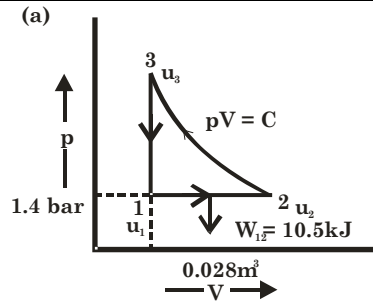
$$\begin{aligned} \text{(c) } Q_{12} &= U_2 - U_1 + W_{12} \\ &= 26.4 + 10.5 \text{ kJ} = 36.9 \text{ kJ} \end{aligned}$$

$$\begin{aligned} \text{(d) } Q_{23} &= U_3 - U_2 + W_{23} \\ &= 0 - 18.783 \text{ kJ} = -18.783 \text{ kJ} \end{aligned}$$

$$Q_{31} = U_2 - U_3 + 0 = -26.4 \text{ kJ}$$

$$\begin{aligned} \therefore \sum Q &= Q_{12} + Q_{23} + Q_{31} = 36.9 \text{ kJ} - 18.783 - 26.4 \\ &= -8.283 \text{ kJ} \end{aligned}$$

$$\therefore \sum W = \sum Q \quad \text{Proved.}$$



$$\begin{aligned} u_2 &= u_3 \\ u_1 - u_3 &= -26.4 \text{ kJ} \\ \therefore u_1 &= u_3 - 26.4 \text{ kJ} \\ &= u_2 - 26.4 \text{ kJ} \\ u_2 &= u_1 + 26.4 \text{ kJ} \end{aligned}$$



## 5. First Law Applied to Flow Process

### Some Important Notes

- **S.F.E.E. per unit mass basis**

$$h_1 + \frac{V_1^2}{2} + gZ_1 + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2} + gZ_2 + \frac{\delta W}{dm}$$

[h, W, Q should be in J/kg and C in m/s and g in m/s<sup>2</sup>]

$$h_1 + \frac{V_1^2}{2000} + \frac{gZ_1}{1000} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2000} + \frac{gZ_2}{1000} + \frac{\delta W}{dm}$$

[h, W, Q should be in kJ/kg and C in m/s and g in m/s<sup>2</sup>]

- **S.F.E.E. per unit time basis**

$$\begin{aligned} & w_1 \left( h_1 + \frac{V_1^2}{2} + Z_1 g \right) + \frac{\delta Q}{d\tau} \\ &= w_2 \left( h_2 + \frac{V_2^2}{2} + Z_2 g \right) + \frac{\delta W_x}{d\tau} \end{aligned}$$

Where, w = mass flow rate (kg/s)

- **Steady Flow Process Involving Two Fluid Streams at the Inlet and Exit of the Control Volume**

**Mass balance**

$$\frac{A_1 V_1}{v_1} + \frac{A_2 V_2}{v_2} = \frac{A_3 V_3}{v_3} + \frac{A_4 V_4}{v_4}$$

Where, v = specific volume (m<sup>3</sup>/kg)

Energy balance

$$w_1 \left( h_1 + \frac{V_1^2}{2} + Z_1 g \right) + w_2 \left( h_2 + \frac{V_2^2}{2} + Z_2 g \right) + \frac{\dot{Q}}{d\tau}$$

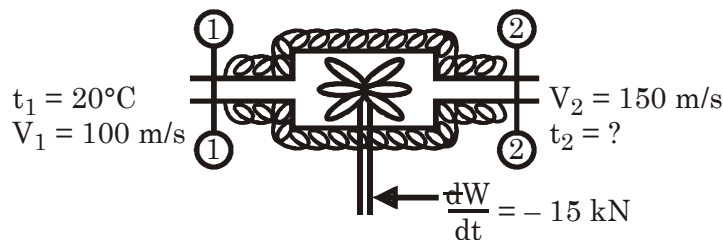
$$= w_3 \left( h_3 + \frac{V_3^2}{2} + Z_3 g \right) + w_4 \left( h_4 + \frac{V_4^2}{2} + Z_4 g \right) + \frac{\dot{W}_x}{d\tau}$$

## Questions with Solution P. K. Nag

- Q5.1** A blower handles 1 kg/s of air at 20°C and consumes a power of 15 kW. The inlet and outlet velocities of air are 100 m/s and 150 m/s respectively. Find the exit air temperature, assuming adiabatic conditions. Take  $c_p$  of air is 1.005 kJ/kg-K.

(Ans. 28.38°C)

**Solution:**



**From S.F.E.E.**

$$w_1 \left( h_1 + \frac{V_1^2}{2000} + \frac{gZ_1}{1000} \right) + \frac{\dot{Q}}{dt} = w_2 \left( h_2 + \frac{V_2^2}{2000} + \frac{gZ_2}{1000} \right) + \frac{\dot{W}}{dt}$$

Here  $w_1 = w_2 = 1 \text{ kg/s}$  ;  $Z_1 = Z_2$ ;  $\frac{\dot{Q}}{dt} = 0$ .

$$\therefore h_1 + \frac{100^2}{2000} + 0 = h_2 + \frac{150^2}{2000} - 15$$

$$\therefore h_2 - h_1 = \left( 15 + \frac{100^2}{2000} - \frac{150^2}{2000} \right)$$

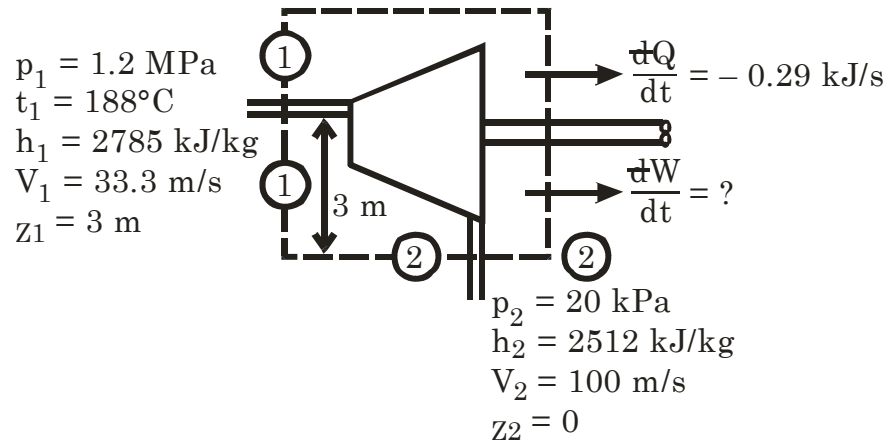
or  $C_p (t_2 - t_1) = 8.75$

or  $t_2 = 20 + \frac{8.75}{1.005} = 28.7^\circ\text{C}$

- Q5.2** A turbine operates under steady flow conditions, receiving steam at the following state: Pressure 1.2 MPa, temperature 188°C, enthalpy 2785 kJ/kg, velocity 33.3 m/s and elevation 3 m. The steam leaves the turbine at the following state: Pressure 20 kPa, enthalpy 2512 kJ/kg, velocity 100 m/s, and elevation 0 m. Heat is lost to the surroundings at the rate of 0.29 kJ/s. If the rate of steam flow through the turbine is 0.42 kg/s, what is the power output of the turbine in kW?

(Ans. 112.51 kW)

**Solution:**  $w_1 = w_2 = 0.42 \text{ kg/s}$



**By S.F.E.E.**

$$w_1 \left( h_1 + \frac{V_1^2}{2000} + \frac{gZ_1}{1000} \right) + \frac{dQ}{dt} = w_2 \left( h_2 + \frac{V_2^2}{2000} + \frac{gZ_2}{1000} \right) + \frac{dW}{dt}$$

$$\text{or } 0.42 \left\{ 2785 + \frac{33.3^2}{2000} + \frac{9.81 \times 3}{1000} \right\} - 0.29 = 0.42 \left\{ 2512 + \frac{100^2}{2000} + 0 \right\} + \frac{dW}{dt}$$

$$\text{or } 1169.655 = 1057.14 + \frac{dW}{dt}$$

$$\text{or } \frac{dW}{dt} = 112.515 \text{ kW}$$

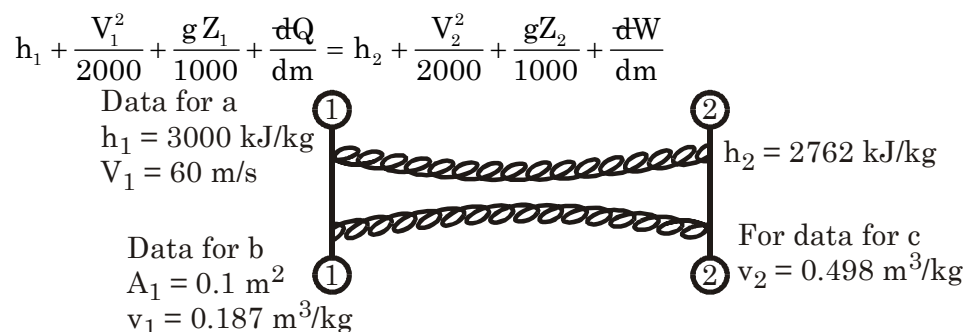
**Q5.3**

A nozzle is a device for increasing the velocity of a steadily flowing stream. At the inlet to a certain nozzle, the enthalpy of the fluid passing is 3000 kJ/kg and the velocity is 60 m/s. At the discharge end, the enthalpy is 2762 kJ/kg. The nozzle is horizontal and there is negligible heat loss from it.

- Find the velocity at exists from the nozzle.
- If the inlet area is 0.1 m<sup>2</sup> and the specific volume at inlet is 0.187 m<sup>3</sup>/kg, find the mass flow rate.
- If the specific volume at the nozzle exit is 0.498 m<sup>3</sup>/kg, find the exit area of the nozzle.

(Ans. (a) 692.5 m/s, (b) 32.08 kg/s (c) 0.023 m<sup>2</sup>)

**Solution:** (a) Find  $V_2$  i.e. Velocity at exit from S.F.E.E.



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Here  $Z_1 = Z_2$  and  $\frac{dQ}{dm} = 0$  and  $\frac{dW}{dm} = 0$

$$\therefore h_1 + \frac{V_1^2}{2000} = h_2 + \frac{V_2^2}{2000}$$

$$\text{or } \frac{V_2^2 - V_1^2}{2000} = (h_1 - h_2)$$

$$\text{or } V_2^2 = V_1^2 + 2000(h_1 - h_2)$$

$$\begin{aligned} \text{or } V_2 &= \sqrt{V_1^2 + 2000(h_1 - h_2)} \\ &= \sqrt{60^2 + 2000(3000 - 2762)} \text{ m/s} \\ &= 692.532 \text{ m/s} \end{aligned}$$

$$\begin{aligned} \text{(b) Mass flow rate (w)} &= \frac{A_1 V_1}{v_1} \\ &= \frac{0.1 \times 60}{0.187} \text{ kg/s} = 32.1 \text{ kg/s} \end{aligned}$$

(c) Mass flow rate is same so

$$32.0855613 = \frac{A_2 \times 692.532}{0.498}$$

$$\text{or } A_2 = 8.023073 \text{ m}^2$$

**Q5.4**

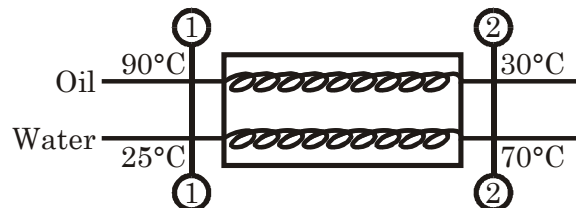
In oil cooler, oil flows steadily through a bundle of metal tubes submerged in a steady stream of cooling water. Under steady flow conditions, the oil enters at 90°C and leaves at 30°C, while the water enters at 25°C and leaves at 70°C. The enthalpy of oil at t°C is given by

$$h = 1.68 t + 10.5 \times 10^{-4} t^2 \text{ kJ/kg}$$

What is the cooling water flow required for cooling 2.78 kg/s of oil?

(Ans. 1.47 kg/s)

**Solution:**  $w_o (h_{oi} + 0 + 0) + w_{H_2O} (h_{H_2O_i} + 0 + 0) + 0 = w_o (h_{o,o} + 0 + 0) + w_{H_2O} (h_{H_2O_o} + 0 + 0) + 0$



$$\begin{aligned} \therefore w_o (h_{oi} - h_{o,o}) &= w_{H_2O} (h_{H_2O_i} - h_{H_2O_o}) \\ h_{oi} &= 1.68 \times 90 + 10.5 \times 10^{-4} \times 90^2 \text{ kJ/kg} = 159.705 \text{ kJ/kg} \\ h_{o,o} &= 1.68 \times 30 + 10.5 \times 10^{-4} \times 30^2 \text{ kJ/kg} = 51.395 \text{ kJ/kg} \end{aligned}$$

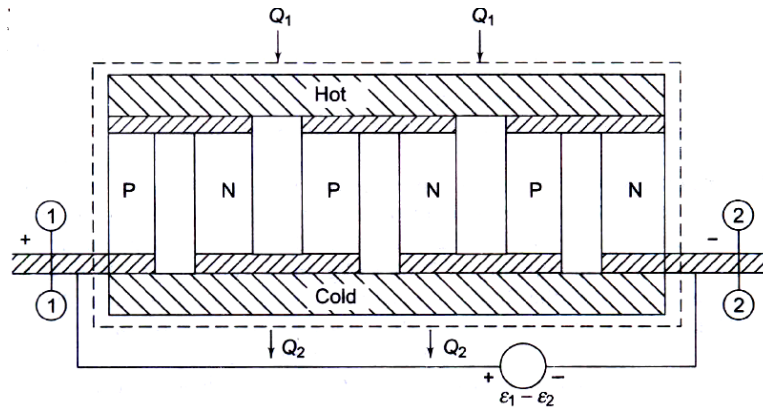
$$\begin{aligned} \therefore W_{H_2O} &= \frac{2.78 \times 108.36}{4.187 (70 - 25)} \text{ kg/s} \\ &= 1.598815 \text{ kg/s} \approx 1.6 \text{ kg/s} \end{aligned}$$

**Q5.5**

A thermoelectric generator consists of a series of semiconductor elements (Figure) heated on one side and cooled on the other. Electric current flow is produced as a result of energy transfer as heat. In a

particular experiment the current was measured to be 0.5 amp and the electrostatic potential at

- (1) Was 0.8 volt above that at
- (2) Energy transfer as heat to the hot side of the generator was taking place at a rate of 5.5 watts. Determine the rate of energy transfer as heat from the cold side and the energy conversion efficiency.



(Ans.  $Q_2 = 5.1$  watts,  $\eta = 0.073$ )

**Solution:**

$$\dot{Q}_1 = \dot{E} + \dot{Q}_2$$

or  $5.5 = 0.5 \times 0.8 + \dot{Q}_2$

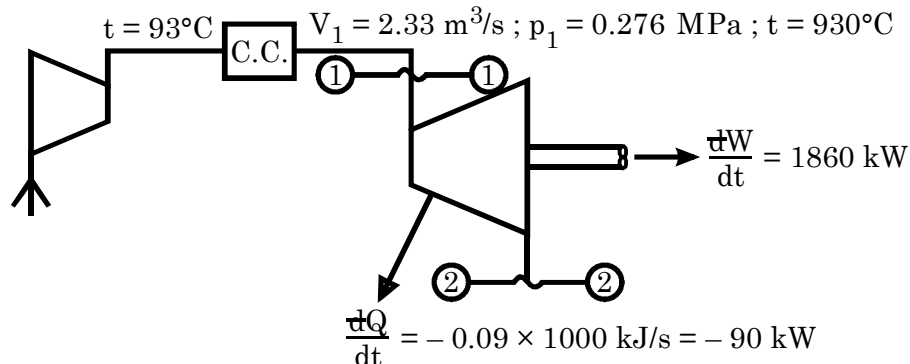
or  $\dot{Q}_2 = 5.1 \text{ watt}$

$$\eta = \frac{5.5 - 5.1}{5.5} \times 100\% = 7.273\%$$

**Q5.6** A turbo compressor delivers  $2.33 \text{ m}^3/\text{s}$  at  $0.276 \text{ MPa}$ ,  $43^\circ\text{C}$  which is heated at this pressure to  $430^\circ\text{C}$  and finally expanded in a turbine which delivers  $1860 \text{ kW}$ . During the expansion, there is a heat transfer of  $0.09 \text{ MJ/s}$  to the surroundings. Calculate the turbine exhaust temperature if changes in kinetic and potential energy are negligible.

(Ans.  $157^\circ\text{C}$ )

**Solution:**



$$w_1 h_1 + \frac{dQ}{dt} = w_2 h_2 + \frac{dW}{dt}$$

$$\therefore w_1 (h_1 - h_2) = \frac{dW}{dt} - \frac{dQ}{dt}$$

or  $= 1860 - (-90) = 1950 \text{ kW}$

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$$p_1 V_1 = m_1 R T_1$$

$$\therefore \dot{m}_1 = \frac{p_1 V_1}{R T_1} = \frac{276 \text{ kPa} \times 2.33 \text{ m}^3/\text{s}}{0.287 \text{ kJ/kg} \times 316 \text{ K}} = 7.091 \text{ kg/s}$$

Or  $h_1 - h_2 = 275$

$$\therefore C_p (t_1 - t_2) = 275$$

or  $t_1 - t_2 = \frac{275}{1.005} = 273.60$

$$\therefore t_2 = 430 - 273.60 = 156.36^\circ \text{C}$$

- Q5.7** A reciprocating air compressor takes in 2 m<sup>3</sup>/min at 0.11 MPa, 20°C which it delivers at 1.5 MPa, 111°C to an aftercooler where the air is cooled at constant pressure to 25°C. The power absorbed by the compressor is 4.15 kW. Determine the heat transfer in
- (a) The compressor
- (b) The cooler

State your assumptions.

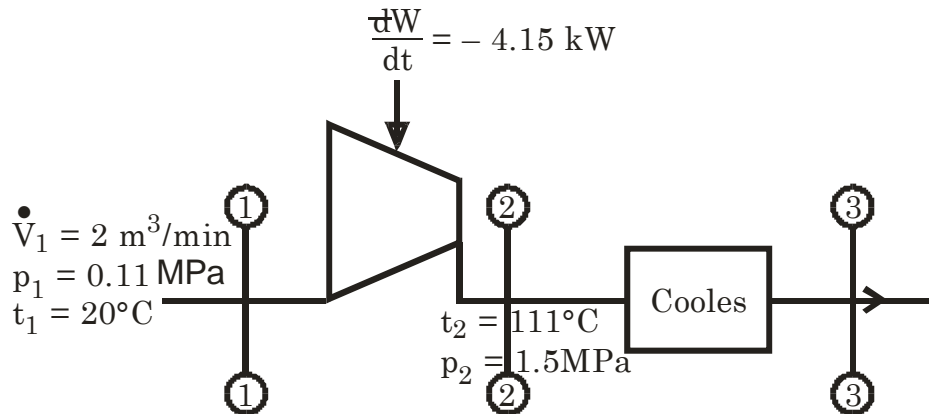
(Ans. – 0.17 kJ/s, – 3.76 kJ/s)

**Solution:**

(a)  $\therefore w_1(h_1 + 0 + 0) + \frac{dQ}{dt} = w_1 h_2 + \frac{dW}{dt}$

$$\therefore 0.0436 (111.555 - 20.1) - 4.15 = \left( \frac{dQ}{dt} \right)$$

$$\frac{dQ}{dt} = -0.1622 \text{ kW} \quad \text{i.e. 1622 kW loss by compressor}$$



$$\text{Compressor work} = \frac{n}{n-1} (p_2 V_2 - p_1 V_1) = \frac{n}{n-1} (m R T_2 - m R T_1)$$

$$= \frac{1.4}{0.4} \times 0.0436 \times 0.287 (111 - 20) \text{ kW}$$

$$= 3.9854 \text{ kW}$$

$$\therefore \frac{dQ}{dt} = 3.9854 - 4.15 = -0.165 \text{ kW}$$

(b)  $\frac{dQ}{dt}$  For cooler

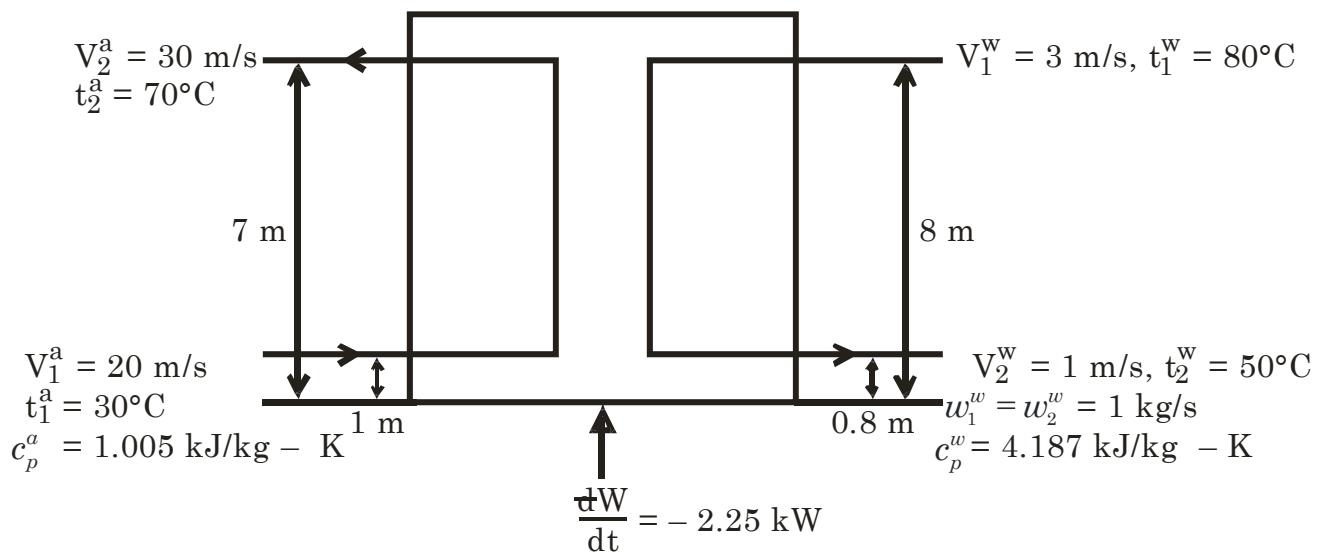
$$\begin{aligned}
 &= \dot{m} c_p (t_2 - t_1) \\
 &= 0.0436 \times 1.005 \times (111 - 25) \text{ kJ/s} \\
 &= 3.768348 \text{ kW to surroundings}
 \end{aligned}$$

**Q5.8** In water cooling tower air enters at a height of 1 m above the ground level and leaves at a height of 7 m. The inlet and outlet velocities are 20 m/s and 30 m/s respectively. Water enters at a height of 8 m and leaves at a height of 0.8 m. The velocity of water at entry and exit are 3 m/s and 1 m/s respectively. Water temperatures are 80°C and 50°C at the entry and exit respectively. Air temperatures are 30°C and 70°C at the entry and exit respectively. The cooling tower is well insulated and a fan of 2.25 kW drives the air through the cooler. Find the amount of air per second required for 1 kg/s of water flow. The values of  $c_p$  of air and water are 1.005 and 4.187 kJ/kg K respectively.

(Ans. 3.16 kg/s)

**Solution:** Let air required is  $w_1^a$  kg/s

$$\begin{aligned}
 \therefore w_1^a \left( h_1^a + \frac{V_1^{a2}}{2000} + \frac{g Z_1^a}{1000} \right) + w_1^w \left( h_1^w + \frac{V_1^{w2}}{2000} + \frac{g Z_1^w}{1000} \right) + \frac{dQ}{dt} \\
 = w_2^a \left( h_2^a + \frac{V_2^{a2}}{2000} + \frac{g Z_2^a}{1000} \right) + w_2^w \left( h_2^w + \frac{V_2^{w2}}{2000} + \frac{g Z_2^w}{1000} \right) + \frac{dW}{dt} \\
 \therefore w_1^a = w_2^a = w \text{ (say) and } \frac{dQ}{dt} = 0 \quad w_1^w = w_2^w = 1 \text{ kg/s}
 \end{aligned}$$



$$\begin{aligned}
 \therefore \left\{ (h_1^a - h_2^a) + \frac{V_1^{a2} - V_2^{a2}}{2000} + \frac{g}{1000} (Z_1^a - Z_2^a) \right\} \\
 = \left\{ (h_2^w - h_1^w) + \frac{V_2^{w2} - V_1^{w2}}{2000} + \frac{g}{1000} (Z_1^w - Z_2^w) \right\} + \frac{dW}{dt}
 \end{aligned}$$

$$\text{Or } w \left\{ 1.005 \times (30 - 70) + \frac{20^2 - 30^2}{2000} + \frac{9.81}{1000} (1 - 7) \right\}$$

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$$= 4.187 (50 - 80) + \frac{1^2 - 3^2}{2000} + \frac{9.81}{1000} \times (0.8 - 8) - 2.25$$

$$\text{or } -w \times 40.509 = -127.9346$$

$$\therefore w = \frac{127.9346}{40.509} = 3.1582 \text{ kg/s} \approx 3.16 \text{ kg/s}$$

**Q5.9**

Air at 101.325 kPa, 20°C is taken into a gas turbine power plant at a velocity of 140 m/s through an opening of 0.15 m<sup>2</sup> cross-sectional area. The air is compressed heated, expanded through a turbine, and exhausted at 0.18 MPa, 150°C through an opening of 0.10 m<sup>2</sup> cross-sectional area. The power output is 375 kW. Calculate the net amount of heat added to the air in kJ/kg. Assume that air obeys the law

$$pv = 0.287 (t + 273)$$

Where p is the pressure in kPa, v is the specific volume in m<sup>3</sup>/kg, and t is the temperature in °C. Take  $c_p = 1.005 \text{ kJ/kg K}$ .

(Ans. 150.23 kJ/kg)

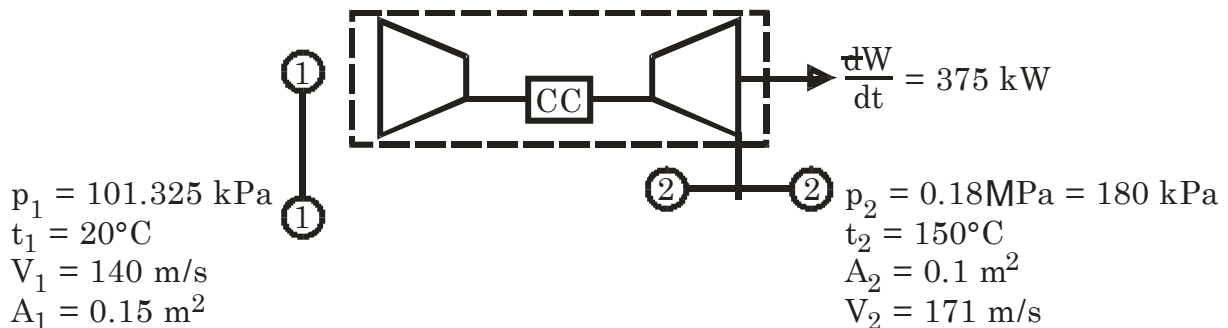
**Solution:**

$$\text{Volume flow rate at inlet } (\dot{V})_1 = V_1 A_1 \text{ m}^3/\text{s} = 21 \text{ m}^3/\text{s}$$

$$\text{Inlet mass flow rate } (w_1) = \frac{p_1 \dot{V}_1}{R T_1} = \frac{101.325 \times 21}{0.287 \times 293} = 25.304 \text{ kg/s}$$

$$\text{Volume flow rate at outlet } = (\dot{V}_2) = \frac{w_2 R T_2}{p_2}$$

$$= \frac{25.304 \times 0.287 \times 423}{180} = 17 \text{ m}^3/\text{s}$$



$$\text{Velocity at outlet} = \frac{\dot{V}_2}{A_2} = \frac{17}{0.1} = 170.66 \text{ m/s}$$

$\therefore$  Using S.F.E.E.

$$w_1 \left( h_1 + \frac{V_1^2}{2000} + 0 \right) + \frac{dQ}{dt} = w_2 \left( h_2 + \frac{V_2^2}{2000} + 0 \right) + \frac{dW}{dt}$$

$$w_1 = w_2 = w = 25.304 \text{ kg/s}$$

$$\begin{aligned} \therefore \frac{dQ}{dt} &= w \left\{ (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2000} \right\} + \frac{dW}{dt} \\ &= w \left\{ C_p (t_2 - t_1) + \frac{V_2^2 - V_1^2}{2000} \right\} + \frac{dW}{dt} \end{aligned}$$



$$\begin{aligned}
 &= 25.304 \left\{ 1.005 (150 - 20) + \frac{171^2 - 140^2}{2000} \right\} + 375 \text{ kW} \\
 &= 3802.76 \text{ kW} \\
 \frac{dQ}{dm} &= \frac{\frac{dQ}{dt}}{w} = \frac{3802.76}{25.304} = 150.28 \text{ kJ/kg}
 \end{aligned}$$

**Q5.10**

A gas flows steadily through a rotary compressor. The gas enters the compressor at a temperature of 16°C, a pressure of 100 kPa, and an enthalpy of 391.2 kJ/kg. The gas leaves the compressor at a temperature of 245°C, a pressure of 0.6 MPa, and an enthalpy of 534.5 kJ/kg. There is no heat transfer to or from the gas as it flows through the compressor.

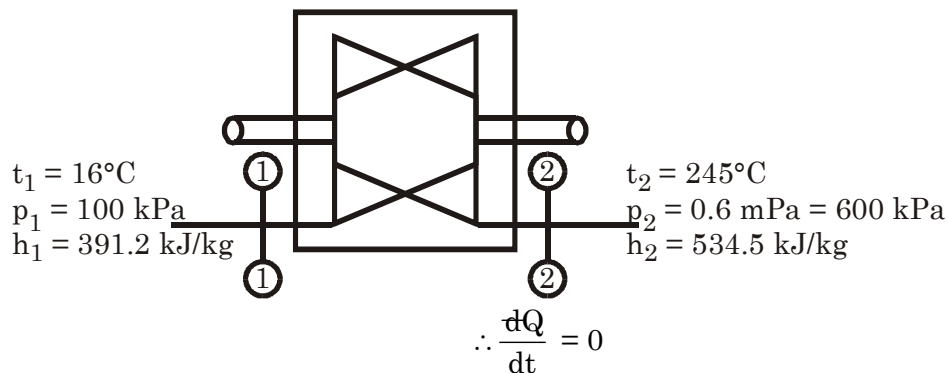
- (a) Evaluate the external work done per unit mass of gas assuming the gas velocities at entry and exit to be negligible.  
 (b) Evaluate the external work done per unit mass of gas when the gas velocity at entry is 80 m/s and that at exit is 160 m/s.

(Ans. 143.3 kJ/kg, 152.9 kJ/kg)

**Solution:** (a) 
$$h_1 + \frac{V_1^2}{2000} + \frac{gZ_1}{1000} + \frac{dQ}{dm} = h_2 + \frac{V_2^2}{2000} + \frac{gZ_2}{1000} + \frac{dW}{dm}$$

For  $V_1$  and  $V_2$  negligible and  $Z_1 = Z_2$  so

$$\begin{aligned}
 \frac{dW}{dm} &= h_1 - h_2 = (391.2 - 534.5) \text{ kJ/kg} \\
 &= -143.3 \text{ kJ/kg i.e. work have to give}
 \end{aligned}$$



(b)  $V_1 = 80 \text{ m/s}; V_2 = 160 \text{ m/s}$

$$\begin{aligned}
 \text{So } \frac{dW}{dm} &= (h_1 - h_2) + \frac{V_1^2 - V_2^2}{2000} \\
 &= -143.3 + \frac{80^2 - 160^2}{2000} \text{ kJ/kg} = (-143.3 - 9.6) \text{ kJ/kg} \\
 &= -152.9 \text{ kJ/kg i.e. work have to give}
 \end{aligned}$$

**Q5.11**

The steam supply to an engine comprises two streams which mix before entering the engine. One stream is supplied at the rate of 0.01 kg/s with an enthalpy of 2952 kJ/kg and a velocity of 20 m/s. The other stream is supplied at the rate of 0.1 kg/s with an enthalpy of 2569 kJ/kg and a velocity of 120 m/s. At the exit from the engine the fluid leaves as two

# First Law Applied to Flow Process

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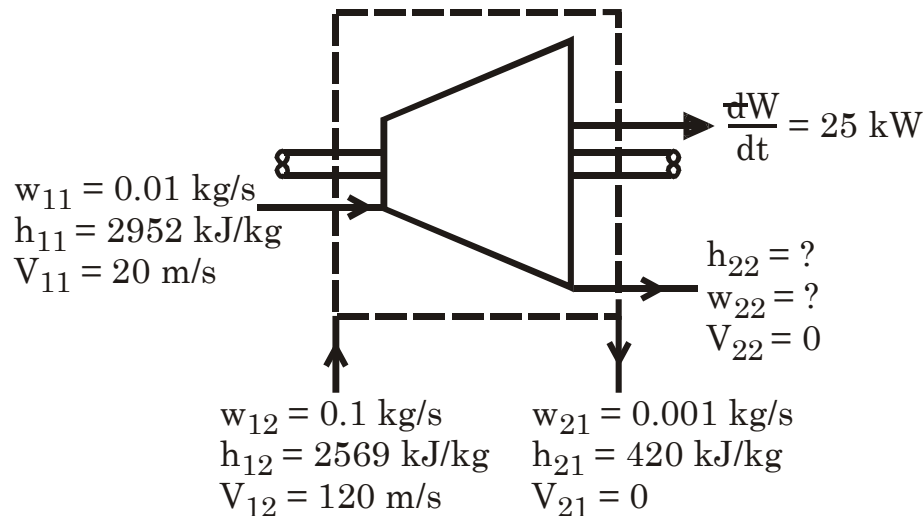
## Chapter 5

streams, one of water at the rate of 0.001 kg/s with an enthalpy of 420 kJ/kg and the other of steam; the fluid velocities at the exit are negligible. The engine develops a shaft power of 25 kW. The heat transfer is negligible. Evaluate the enthalpy of the second exit stream.

(Ans. 2402 kJ/kg)

Solution:  $\therefore \frac{dQ}{dt} = 0$

By mass balance



$$W_{11} + W_{12} = W_{21} + W_{22}$$

$$\therefore W_{22} = 0.01 + 0.1 - 0.001 \text{ kg/s} = 0.109 \text{ kg/s}$$

$$\therefore W_{11} \left( h_{11} + \frac{V_{11}^2}{2000} \right) + W_{12} \left( h_{12} + \frac{V_{12}^2}{2000} \right) + \frac{dQ}{dt} = W_{21} (h_{21}) + W_{22} \times h_{22} + \frac{dW}{dt}$$

$$\therefore 0.01 \left( 2952 + \frac{20^2}{2000} \right) + 0.1 \left( 2569 + \frac{120^2}{2000} \right) + 0$$

$$= 0.001 \times 420 + 0.109 \times h_{22} + 25$$

$$\text{or } 29.522 + 257.62 = 0.42 + 0.109 \times h_{22} + 25$$

$$\text{or } 286.722 = 0.109 \times h_{22} + 25$$

$$\text{or } h_{22} = 2401.2 \text{ kJ/kg}$$

Q5.12

The stream of air and gasoline vapour, in the ratio of 14: 1 by mass, enters a gasoline engine at a temperature of 30°C and leaves as combustion products at a temperature of 790°C. The engine has a specific fuel consumption of 0.3 kg/kWh. The net heat transfer rate from the fuel-air stream to the jacket cooling water and to the surroundings is 35 kW. The shaft power delivered by the engine is 26 kW. Compute the increase in the specific enthalpy of the fuel air stream, assuming the changes in kinetic energy and in elevation to be negligible.

(Ans. – 1877 kJ/kg mixture)

Solution: In 1 hr. this m/c will produce 26 kWh for that we need fuel

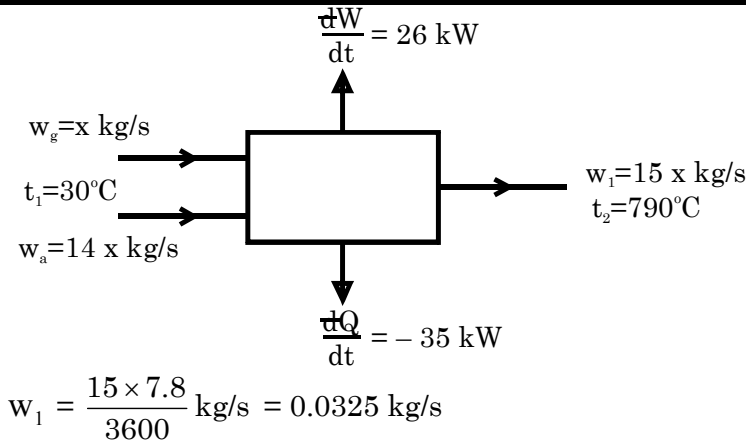
$$= 0.3 \times 26 = 7.8 \text{ kg fuel/hr.}$$

$\therefore$  Mass flow rate of fuel vapor and air mixture

# First Law Applied to Flow Process

By: S K Mondal

## Chapter 5



Applying S.F.E.E.

$$w_1 h_1 + \frac{dQ}{dt} = w_1 h_2 + \frac{dW}{dt}$$

or  $w_1 (h_2 - h_1) = \frac{dQ}{dt} - \frac{dW}{dt}$

$$\therefore h_2 - h_1 = \frac{\frac{dQ}{dt} - \frac{dW}{dt}}{w_1}$$

$$= \frac{-35 - 26}{0.0325} = -1877 \text{ kJ/kg of mixture.}$$

Q5.13

An air turbine forms part of an aircraft refrigerating plant. Air at a pressure of 295 kPa and a temperature of 58°C flows steadily into the turbine with a velocity of 45 m/s. The air leaves the turbine at a pressure of 115 kPa, a temperature of 2°C, and a velocity of 150 m/s. The shaft work delivered by the turbine is 54 kJ/kg of air. Neglecting changes in elevation, determine the magnitude and sign of the heat transfer per unit mass of air flowing. For air, take  $c_p = 1.005 \text{ kJ/kg K}$  and the enthalpy  $h = c_p t$ .

(Ans. + 7.96 kJ/kg)

Solution:

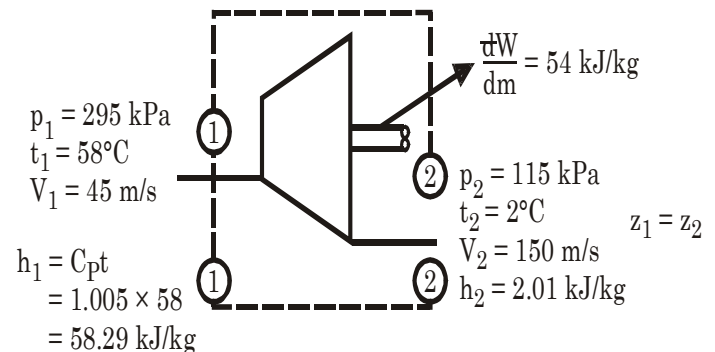
$$h_1 + \frac{V_1^2}{2000} + \frac{dQ}{dm} = h_2 + \frac{V_2^2}{2000} + \frac{dW}{dm}$$

or  $\frac{dQ}{dm} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2000} + \frac{dW}{dm}$

$$= (2.01 - 58.29) + \frac{150^2 - 45^2}{2000} + 54 \text{ kJ/kg}$$

$$= -56.28 + 10.2375 + 54 \text{ kJ/kg}$$

$$= 7.9575 \text{ kJ/kg (have to give to the system)}$$



Q5.14

In a turbo machine handling an incompressible fluid with a density of  $1000 \text{ kg/m}^3$  the conditions of the fluid at the rotor entry and exit are as given below:

	Inlet	Exit
Pressure	1.15 MPa	0.05 MPa
Velocity	30 m/s	15.5 m/s

<b>First Law Applied to Flow Process</b> <b>By: S K Mondal</b>	<b>Chapter 5</b>
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Height above datum      10 m                      2 m

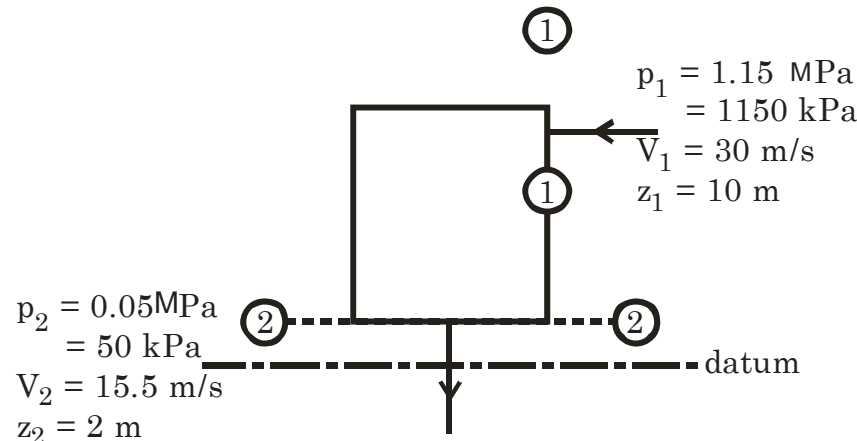
If the volume flow rate of the fluid is 40 m<sup>3</sup>/s, estimate the net energy transfer from the fluid as work.

(Ans. 60.3 MW)

**Solution:**

**By S.F.E.E.**

$$w \left( \frac{p_1}{\rho} + \frac{V_1^2}{2} + g Z_1 \right) + \frac{dQ}{dt} = w \left( \frac{p_2}{\rho} + \frac{V_2^2}{2} + g Z_2 \right) + \frac{dW}{dt}$$



Flow rate = 40 m<sup>3</sup>/s  $\equiv$  40  $\times$  1000 kg/s = w (say)

$$\therefore 40000 \left( \frac{1150}{1000} + \frac{30^2}{2000} + \frac{9.81 \times 10}{1000} \right) + 0$$

$$\begin{aligned} \text{Or } \frac{dW}{dt} &= 40000 \left\{ \frac{p_1 - p_2}{\rho} + \frac{V_1^2 - V_2^2}{2} + g(Z_1 - Z_2) \right\} \\ &= 40000 \left\{ \frac{1150 - 50}{1000} + \frac{30^2 - 15.5^2}{2000} + \frac{9.81 \times (10 - 2)}{1000} \right\} \text{ kW} \\ &= 60.3342 \text{ MW} \end{aligned}$$

**Q5.15**

A room for four persons has two fans, each consuming 0.18 kW power, and three 100 W lamps. Ventilation air at the rate of 80 kg/h enters with an enthalpy of 84 kJ/kg and leaves with an enthalpy of 59 kJ/kg. If each person puts out heat at the rate of 630 kJ/h determine the rate at which heat is to be removed by a room cooler, so that a steady state is maintained in the room.

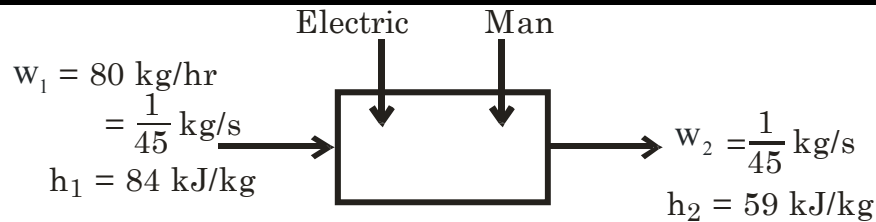
(Ans. 1.92 kW)

**Solution:**

$$\frac{dQ_{\text{person}}}{dt} = + \frac{4 \times 630}{3600} \text{ kJ/s} = 0.7 \text{ kW}$$

$$\frac{dQ_{\text{electric}}}{dt} = + 0.18 \times 2 + \frac{3 \times 100}{1000} \text{ kW} = 0.66 \text{ kW}$$

$$\therefore \frac{dQ}{dt} = 1.36 \text{ kW}$$



For steady state

$$w_1 h_1 + \frac{dQ}{dt} = w_2 h_2 + \frac{dW}{dt}$$

$$\therefore \frac{dW}{dt} = w_1 h_1 - w_2 h_2 + \frac{dQ}{dt} = \frac{1}{45} \times (84 - 59) + 1.36 \text{ kW}$$

$$= 1.9156 \text{ kW}$$

**Q5.16**

Air flows steadily at the rate of 0.4 kg/s through an air compressor, entering at 6 m/s with a pressure of 1 bar and a specific volume of 0.85 m<sup>3</sup>/kg, and leaving at 4.5 m/s with a pressure of 6.9 bar and a specific volume of 0.16 m<sup>3</sup>/kg. The internal energy of the air leaving is 88 kJ/kg greater than that of the air entering. Cooling water in a jacket surrounding the cylinder absorbs heat from the air at the rate of 59 W. Calculate the power required to drive the compressor and the inlet and outlet cross-sectional areas.

(Ans. 45.4 kW, 0.057 m<sup>2</sup>, 0.0142 m<sup>2</sup>)

**Solution:** By S.F.E.E.

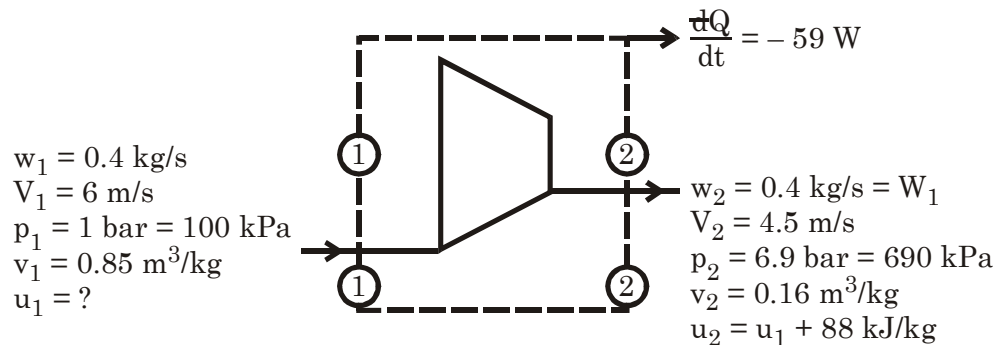
$$w_1 \left( u_1 + p_1 v_1 + \frac{V_1^2}{2000} + 0 \right) + \frac{dQ}{dt} = w_2 \left( u_2 + p_2 v_2 + \frac{V_2^2}{2000} + 0 \right) + \frac{dW}{dt}$$

$$\text{Or } \frac{dW}{dt} = \left[ (u_1 - u_2) + (p_1 v_1 - p_2 v_2) + \frac{V_1^2 - V_2^2}{2000} \right] + \frac{dQ}{dt}$$

$$= 0.4 [-88 + 85 - 110.4 + 0.0076] - 0.059$$

$$= -45.357 - 0.059$$

$$= -45.416 \text{ kW [have to give to compressor]}$$



$$w_1 = \frac{A_1 V_1}{v_1} \quad \therefore A_1 = \frac{w_1 v_1}{V_1} = \frac{0.4 \times 0.85}{6} = 0.0567 \text{ m}^2$$

$$w_2 = \frac{A_2 V_2}{v_2} \quad \therefore A_2 = \frac{w_2 v_2}{V_2} = \frac{0.4 \times 0.16}{4.5} = 0.01422 \text{ m}^2$$



6.

# Second Law of Thermodynamics

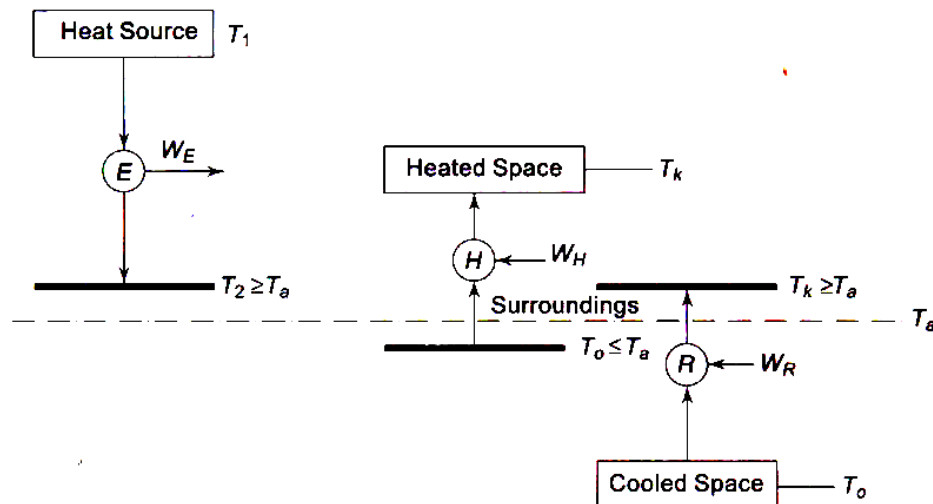
## Some Important Notes

### Regarding Heat Transfer and Work Transfer

- Heat transfer and work transfer are the energy interactions.
- Both heat transfer and work transfer are boundary phenomena.
- It is wrong to say 'total heat' or 'heat content' of a closed system, because heat or work is not a property of the system.
- Both heat and work are path functions and inexact differentials.
- Work is said to be a high grade energy and heat is low grade energy.
- HEAT and WORK are NOT properties because they depend on the path and end states.
- HEAT and WORK are not properties because their net change in a cycle is not zero.
- **Clausius' Theorem:** The cyclic integral of  $\delta Q/T$  for a reversible cycle is equal to zero.

$$\text{or} \quad \oint_R \frac{\delta Q}{T} = 0$$

- The more effective way to increase the cycle efficiency is **to decrease  $T_2$** .
- **Comparison of heat engine, heat pump and refrigerating machine**



$$\frac{Q_C}{Q_H} = \frac{T_C}{T_H}$$

$$\text{hence, } \eta_{\text{Carnot, HE}} = 1 - \frac{Q_C}{Q_H} = 1 - \frac{T_C}{T_H}$$

## Second Law of Thermodynamics

**By: S K Mondal**

**Chapter 6**

$$COP_{Carnot,HP} = \frac{Q_H}{W_{cycle}} = \frac{Q_H}{Q_H - Q_C} = \frac{T_H}{T_H - T_C}$$

$$COP_{Carnot,R} = \frac{Q_C}{W_{cycle}} = \frac{Q_C}{Q_H - Q_C} = \frac{T_C}{T_H - T_C}$$

### Questions with Solution P. K. Nag

**Q6.1** An inventor claims to have developed an engine that takes in 105 MJ at a temperature of 400 K, rejects 42 MJ at a temperature of 200 K, and delivers 15 kWh of mechanical work. Would you advise investing money to put this engine in the market?

(Ans. No)

**Solution:** Maximum thermal efficiency of his engine possible

$$\eta_{\max} = 1 - \frac{200}{400} = 50\%$$

∴ That engine and deliver output =  $\eta \times \text{input}$

$$= 0.5 \times 105 \text{ MJ}$$

$$= 52.5 \text{ MJ} = 14.58 \text{ kWh}$$

As he claims that his engine can deliver more work than ideally possible so I would not advise to investing money.

**Q6.2** If a refrigerator is used for heating purposes in winter so that the atmosphere becomes the cold body and the room to be heated becomes the hot body, how much heat would be available for heating for each kW input to the driving motor? The COP of the refrigerator is 5, and the electromechanical efficiency of the motor is 90%. How does this compare with resistance heating?

(Ans. 5.4 kW)

**Solution:** 
$$COP = \frac{\text{desired effect}}{\text{input}}$$

$$(COP)_{\text{ref.}} = (COP)_{\text{H.P.}} - 1$$

$$\text{or } 6 = \frac{H}{W}$$

$$\therefore (COP)_{\text{H.P.}} = 6$$

$$\text{So input (W)} = \frac{H}{6}$$

But motor efficiency 90% so

$$\text{Electrical energy require (E)} = \frac{W}{0.9} = \frac{H}{0.9 \times 6}$$

$$= 0.1852 H$$

$$= 18.52\% \text{ of Heat (direct heating)}$$

$$H = \frac{100}{18.52} \frac{\text{kW}}{\text{kW of work}} = 5.3995 \text{ kW}$$

**Q6.3** Using an engine of 30% thermal efficiency to drive a refrigerator having a COP of 5, what is the heat input into the engine for each MJ removed from the cold body by the refrigerator?

(Ans. 666.67 kJ)



## Second Law of Thermodynamics

**By: S K Mondal**

## Chapter 6

If this system is used as a heat pump, how many MJ of heat would be available for heating for each MJ of heat input to the engine?

(Ans. 1.8 MJ)

**Solution:** COP of the Ref. is 5

So for each MJ removed from the cold body we need work

$$= \frac{1 \text{ MJ}}{5} = 200 \text{ kJ}$$

For 200 kJ work output of heat engine  $\eta = 30\%$

$$\text{We have to supply heat} = \frac{200 \text{ kJ}}{0.3} = 666.67 \text{ kJ}$$

Now

$$\begin{aligned} \text{COP of H.P.} &= \text{COP of Ref.} + 1 \\ &= 5 + 1 = 6 \end{aligned}$$

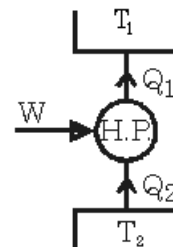
Heat input to the H.E. = 1 MJ

$$\therefore \text{Work output (W)} = 1 \times 0.3 \text{ MJ} = 300 \text{ kJ}$$

That will be the input to H.P.

$$\therefore (\text{COP})_{\text{H.P.}} = \frac{Q_1}{W}$$

$$\therefore Q_1 = (\text{COP})_{\text{H.P.}} \times W = 6 \times 300 \text{ kJ} = 1.8 \text{ MJ}$$



**Q6.4**

An electric storage battery which can exchange heat only with a constant temperature atmosphere goes through a complete cycle of two processes. In process 1–2, 2.8 kWh of electrical work flow into the battery while 732 kJ of heat flow out to the atmosphere. During process 2–1, 2.4 kWh of work flow out of the battery.

(a) Find the heat transfer in process 2–1.

(b) If the process 1–2 has occurred as above, does the first law or the second law limit the maximum possible work of process 2–1? What is the maximum possible work?

(c) If the maximum possible work were obtained in process 2–1, what will be the heat transfer in the process?

(Ans. (a) – 708 kJ (b) Second law,  $W_{2-1} = 9348 \text{ kJ}$  (c)  $Q_{2-1} = 0$ )

**Solution:** From the first Law of thermodynamics

(a) For process 1–2

$$Q_{1-2} = E_2 - E_1 + W_{1-2}$$

$$-732 = (E_2 - E_1) - 10080$$

$$[2.8 \text{ kWh} = 2.8 \times 3600 \text{ kJ}]$$

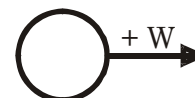
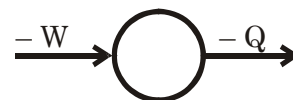
$$\therefore E_2 - E_1 = 9348 \text{ kJ}$$

For process 2–1

$$Q_{21} = E_1 - E_2 + W_{21}$$

$$= -9348 + 8640$$

= –708 kJ i.e. Heat flow out to the atmosphere.



(b) Yes Second Law limits the maximum possible work. As Electric energy stored in a battery is High grade energy so it can be completely converted to the work. Then,

$$W = 9348 \text{ kJ}$$

## Second Law of Thermodynamics

**By: S K Mondal**

## Chapter 6

(c)  $Q_{21} = -9348 + 9348 = 0 \text{ kJ}$

**Q6.5** A household refrigerator is maintained at a temperature of  $2^\circ\text{C}$ . Every time the door is opened, warm material is placed inside, introducing an average of  $420 \text{ kJ}$ , but making only a small change in the temperature of the refrigerator. The door is opened 20 times a day, and the refrigerator operates at 15% of the ideal COP. The cost of work is Rs. 2.50 per kWh. What is the monthly bill for this refrigerator? The atmosphere is at  $30^\circ\text{C}$ .  
(Ans. Rs. 118.80)

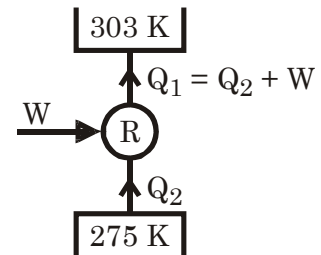
**Solution:** Ideal COP of Ref.  $= \frac{275}{30 - 2} = \frac{275}{28} = 9.82143$

Actual COP  $= 0.15 \times \text{COP}_{\text{ideal}} = 1.4732$

Heat to be removed in a day  
( $Q_2$ )  $= 420 \times 20 \text{ kJ}$   
 $= 8400 \text{ kJ}$

$\therefore$  Work required  $= 5701.873 \text{ kJ/day}$   
 $= 1.58385 \text{ kWh/day}$

Electric bill per month  $= 1.58385 \times 0.32 \times 30 \text{ Rupees}$   
 $= \text{Rs. } 15.20$



**Q6.6** A heat pump working on the Carnot cycle takes in heat from a reservoir at  $5^\circ\text{C}$  and delivers heat to a reservoir at  $60^\circ\text{C}$ . The heat pump is driven by a reversible heat engine which takes in heat from a reservoir at  $840^\circ\text{C}$  and rejects heat to a reservoir at  $60^\circ\text{C}$ . The reversible heat engine also drives a machine that absorbs  $30 \text{ kW}$ . If the heat pump extracts  $17 \text{ kJ/s}$  from the  $5^\circ\text{C}$  reservoir, determine  
(a) The rate of heat supply from the  $840^\circ\text{C}$  source  
(b) The rate of heat rejection to the  $60^\circ\text{C}$  sink.

(Ans. (a)  $47.61 \text{ kW}$ ; (b)  $34.61 \text{ kW}$ )

**Solution:**  
COP of H.P.

$$= \frac{333}{333 - 278} = 6.05454$$

$$Q_3 = W_{\text{H.P.}} + 17$$

$$\therefore \frac{W_{\text{H.P.}} + 17}{W_{\text{H.P.}}} = 6.05454$$

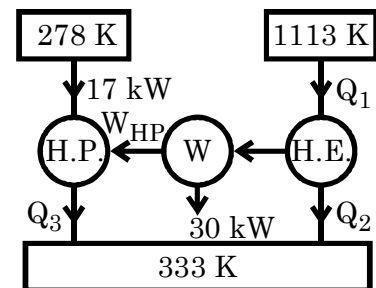
$$\therefore \frac{17}{W_{\text{H.P.}}} = 5.05454$$

$$\therefore W_{\text{H.P.}} = \frac{17}{5.05454} = 3.36 \text{ kW}$$

$$\therefore \text{Work output of the Heat engine}$$

$$W_{\text{H.E.}} = 30 + 3.36 = 33.36 \text{ kW}$$

$$\eta \text{ of the H.E.} = 1 - \frac{333}{1113} = 0.7$$



$$(a) \therefore \frac{W}{Q_1} = 0.7$$

$$\therefore Q_1 = \frac{W}{0.7} = 47.61 \text{ kW}$$

(b) Rate of heat rejection to the 333 K

(i) From H.E. =  $Q_1 - W = 47.61 - 33.36 = 14.25$   
kW

(ii) For H.P. =  $17 + 3.36 = 20.36$  kW

$\therefore$  Total = 34.61 kW

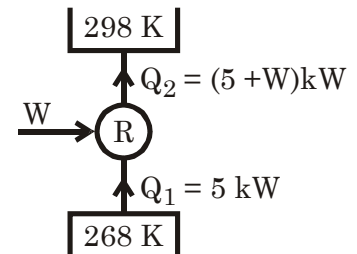
**Q6.7** A refrigeration plant for a food store operates as a reversed Carnot heat engine cycle. The store is to be maintained at a temperature of  $-5^\circ\text{C}$  and the heat transfer from the store to the cycle is at the rate of 5 kW. If heat is transferred from the cycle to the atmosphere at a temperature of  $25^\circ\text{C}$ , calculate the power required to drive the plant.

(Ans. 0.56 kW)

**Solution:**  $(\text{COP})_R = \frac{268}{298 - 268} = 8.933$

$$= \frac{5 \text{ kW}}{W}$$

$$\therefore W = \frac{5}{8.933} \text{ kW} = 0.56 \text{ kW}$$



**Q6.8** A heat engine is used to drive a heat pump. The heat transfers from the heat engine and from the heat pump are used to heat the water circulating through the radiators of a building. The efficiency of the heat engine is 27% and the COP of the heat pump is 4. Evaluate the ratio of the heat transfer to the circulating water to the heat transfer to the heat engine.

(Ans. 1.81)

**Solution:** For H.E.

$$1 - \frac{Q_2}{Q_1} = 0.27$$

## Second Law of Thermodynamics

**By: S K Mondal**

## Chapter 6

$$\frac{Q_2}{Q_1} = 0.73$$

$$Q_2 = 0.73 Q_1$$

$$W = Q_1 - Q_2 = 0.27 Q_1$$

For H.P.

$$\frac{Q_4}{W} = 4$$

$$\therefore Q_4 = 4W = 1.08 Q_1$$

$$\therefore Q_2 + Q_4 = (0.73 + 1.08) Q_1 = 1.81 Q_1$$

$$\therefore \frac{\text{Heat transfer to the circulating water}}{\text{Heat for to the Heat Engine}}$$

$$= \frac{1.81 Q_1}{Q_1} = 1.81$$

**Q6.9**

If 20 kJ are added to a Carnot cycle at a temperature of 100°C and 14.6 kJ are rejected at 0°C, determine the location of absolute zero on the Celsius scale.

(Ans. – 270.37°C)

**Solution:**

$$\frac{Q_1}{Q_2} = \frac{\phi(t_1)}{\phi(t_2)}$$

$$\text{Let } \phi(t) = at + b$$

$$\therefore \frac{Q_1}{Q_2} = \frac{at_1 + b}{at_2 + b}$$

$$\text{or } \frac{20}{14.6} = \frac{a \times 100 + b}{a \times 0 + b} = \frac{a}{b} \times 100 + 1$$

$$\therefore \frac{a}{b} = 3.6986 \times 10^{-3}$$

For absolute zero,  $Q_2 = 0$

$$\therefore \frac{Q_1}{0} = \frac{a \times 100 + b}{a \times t + b}$$

$$\text{or } a \times t + b = 0$$

$$\text{or } t = \frac{-b}{a} = -\frac{1}{3.6986 \times 10^{-3}} = -270.37^\circ \text{C}$$

**Q6.10**

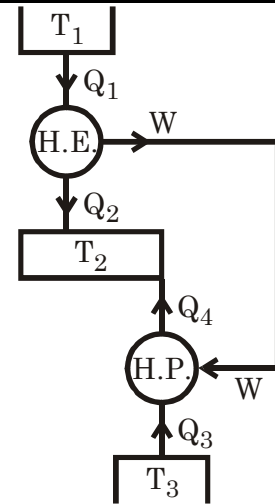
Two reversible heat engines *A* and *B* are arranged in series, *A* rejecting heat directly to *B*. Engine *A* receives 200 kJ at a temperature of 421°C from a hot source, while engine *B* is in communication with a cold sink at a temperature of 4.4°C. If the work output of *A* is twice that of *B*, find

(a) The intermediate temperature between *A* and *B*

(b) The efficiency of each engine

(c) The heat rejected to the cold sink

(Ans. 143.4°C, 40% and 33.5%, 80 kJ)



## Second Law of Thermodynamics

**By: S K Mondal**

## Chapter 6

**Solution:**  $\frac{Q_1}{694} = \frac{Q_2}{T} = \frac{Q_1 - Q_2}{694 - T} = \frac{Q_3}{277.4} = \frac{Q_2 - Q_3}{T - 277.4}$

Hence  $Q_1 - Q_2 = 2 W_2$

$Q_2 - Q_3 = W_2$

$\therefore \frac{2}{694 - T} = \frac{1}{T - 277.4}$

or  $2T - 277.4 \times 2 = 694 - T$

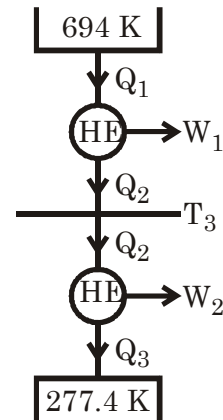
or  $T = 416.27 \text{ K} = 143.27^\circ \text{ C}$

(b)  $\eta_1 = 40\%$

$\eta_2 = 1 - \frac{277.4}{416.27} = 33.36\%$

(c)  $Q_2 = \frac{416.27}{694} \times 200 \text{ kJ} = 119.96 \text{ kJ} ;$

$Q_1 = \frac{277.4}{416.27} \times 119.96 = 79.94 \text{ kJ}$



**Q6.11**

A heat engine operates between the maximum and minimum temperatures of  $671^\circ\text{C}$  and  $60^\circ\text{C}$  respectively, with an efficiency of 50% of the appropriate Carnot efficiency. It drives a heat pump which uses river water at  $4.4^\circ\text{C}$  to heat a block of flats in which the temperature is to be maintained at  $21.1^\circ\text{C}$ . Assuming that a temperature difference of  $11.1^\circ\text{C}$  exists between the working fluid and the river water, on the one hand, and the required room temperature on the other, and assuming the heat pump to operate on the reversed Carnot cycle, but with a COP of 50% of the ideal COP, find the heat input to the engine per unit heat output from the heat pump. Why is direct heating thermodynamically more wasteful?

(Ans. 0.79 kJ/kJ heat input)

**Solution:** Carnot efficiency  $(\eta) = 1 - \frac{273 + 60}{273 + 671} = 1 - \frac{333}{944} = 0.64725$

Actual  $(\eta) = 0.323623 = 1 - \frac{Q'_1}{Q_1}$

$$\therefore \frac{Q'_1}{Q_1} = 0.6764$$

Ideal COP

$$= \frac{305.2}{305.2 - 266.4} = 7.866$$

Actual COP

$$= 3.923 = \frac{Q_3}{W}$$

if  $Q_3 = 1 \text{ kJ}$

$$\therefore W = \frac{Q_3}{3.923} = \frac{1}{3.923}$$

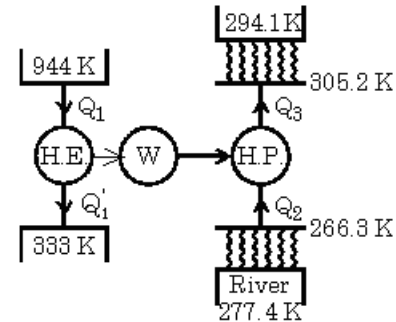
$= 0.2549 \text{ kJ/kJ}$  heat input to block

$$W = Q_1 - Q'_1 = Q_1 - 0.6764 Q_1 = 0.2549$$

$$Q_1 = \frac{0.2549}{(1 - 0.6764)}$$

$= 0.7877 \text{ kJ/kJ}$  heat input to

block.



**Q6.12**

An ice-making plant produces ice at atmospheric pressure and at  $0^\circ\text{C}$  from water. The mean temperature of the cooling water circulating through the condenser of the refrigerating machine is  $18^\circ\text{C}$ . Evaluate the minimum electrical work in kWh required to produce 1 tonne of ice (The enthalpy of fusion of ice at atmospheric pressure is  $333.5 \text{ kJ/kg}$ ).

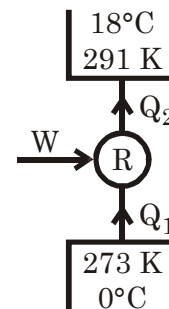
(Ans. 6.11 kWh)

**Solution:** Maximum (COP) =  $\frac{273}{291 - 273} = 15.2$

$$\therefore \frac{Q}{W_{\min}} = 15.2$$

or  $W_{\min} = \frac{Q}{15.2} = \frac{1000 \times 333.5}{15.2} \text{ kJ}$

$$= 21.989 \text{ MJ} = 6.108 \text{ kWh}$$



**Q6.13**

A reversible engine works between three thermal reservoirs, A, B and C. The engine absorbs an equal amount of heat from the thermal reservoirs A and B kept at temperatures  $T_A$  and  $T_B$  respectively, and rejects heat to the thermal reservoir C kept at temperature  $T_C$ . The efficiency of the engine is  $\alpha$  times the efficiency of the reversible engine, which works between the two reservoirs A and C. prove that

$$\frac{T_A}{T_B} = (2\alpha - 1) + 2(1 - \alpha) \frac{T_A}{T_C}$$

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**Solution:**  $\eta$  of H.E. between A and C

$$\eta_A = \left(1 - \frac{T_C}{T_A}\right)$$

$$\eta \text{ of our engine} = \alpha \left(1 - \frac{T_C}{T_A}\right)$$

$$\text{Here } Q_2 = \frac{Q_1}{T_A} \times T_C = Q_3 = \frac{Q_1}{T_B} \times T_C$$

$\therefore$  Total Heat rejection

$$(Q_2 + Q_3) = Q_1 T_C \left( \frac{1}{T_A} + \frac{1}{T_B} \right)$$

Total Heat input =  $2Q_1$

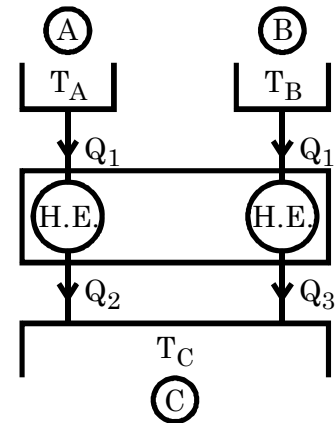
$$\eta \text{ of engine} = \left[ 1 - \frac{Q_1 T_C \left( \frac{1}{T_A} + \frac{1}{T_B} \right)}{2Q_1} \right]$$

$$\therefore \alpha - \frac{\alpha T_C}{T_A} = 1 - \frac{T_C}{2T_A} - \frac{T_C}{2T_B}$$

Multiply both side by  $T_A$  and divide by  $T_C$

$$\text{or } \alpha \frac{T_A}{T_C} - \alpha = \frac{T_A}{T_C} - \frac{1}{2} - \frac{1}{2} \frac{T_A}{T_B}$$

$$\text{or } \frac{T_A}{T_B} = (2\alpha - 1) + 2(1 - \alpha) \frac{T_A}{T_C} \text{ Proved}$$



**Q6.14**

**A reversible engine operates between temperatures  $T_1$  and  $T$  ( $T_1 > T$ ). The energy rejected from this engine is received by a second reversible engine at the same temperature  $T$ . The second engine rejects energy at temperature  $T_2$  ( $T_2 < T$ ).**

**Show that:**

- (a) Temperature  $T$  is the arithmetic mean of temperatures  $T_1$  and  $T_2$  if the engines produce the same amount of work output**
- (b) Temperature  $T$  is the geometric mean of temperatures  $T_1$  and  $T_2$  if the engines have the same cycle efficiencies.**

**Solution:** (a) If they produce same Amount and work

$$\text{Then } W_1 = W_2$$

$$\text{or } \eta_1 Q_1 = \eta_2 Q_2$$

$$\text{or } \left(1 - \frac{T}{T_1}\right) \left(\frac{T_1}{T}\right) Q_2 = \left(1 - \frac{T_2}{T}\right) Q_2$$

$$\text{We know that } \frac{Q_1}{T_1} = \frac{Q_2}{T}$$

$$\text{or } Q_1 = \frac{T_1}{T} Q_2$$

## Second Law of Thermodynamics

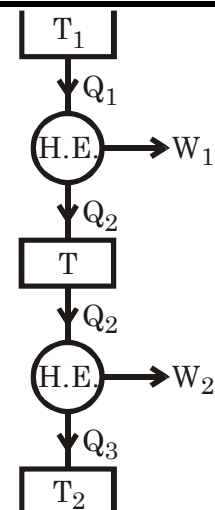
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$$\begin{aligned} \text{or } \frac{T_1}{T} - 1 &= 1 - \frac{T_2}{T} \\ \text{or } \frac{T_1 + T_2}{T} &= 2 \\ \text{or } T &= \frac{T_1 + T_2}{2} \\ \text{i.e., Arithmetic mean of } T_1, T_2 \end{aligned}$$

(b) If their efficiency is same then

$$\begin{aligned} 1 - \frac{T}{T_1} &= 1 - \frac{T}{T_2} \\ \text{or } T &= \sqrt{T_1 T_2} \\ &\quad (\text{as } T \text{ is +ve so -ve sign neglected}) \\ \therefore T &\text{ is Geometric mean of } T_1 \text{ and } T_2. \end{aligned}$$



**Q6.15**

Two Carnot engines A and B are connected in series between two thermal reservoirs maintained at 1000 K and 100 K respectively. Engine A receives 1680 kJ of heat from the high-temperature reservoir and rejects heat to the Carnot engine B. Engine B takes in heat rejected by engine A and rejects heat to the low-temperature reservoir. If engines A and B have equal thermal efficiencies, determine

- The heat rejected by engine B
  - The temperature at which heat is rejected by engine, A
  - The work done during the process by engines, A and B respectively.
- If engines A and B deliver equal work, determine
- The amount of heat taken in by engine B
  - The efficiencies of engines A and B

(Ans. (a) 168 kJ, (b) 316.2 K, (c) 1148.7, 363.3 kJ, (d) 924 kJ, (e) 45%, 81.8%)

**Solution:** As their efficiency is same so

$$\begin{aligned} \eta_A &= \eta_B \\ \text{or } 1 - \frac{T}{1000} &= 1 - \frac{100}{T} \end{aligned}$$

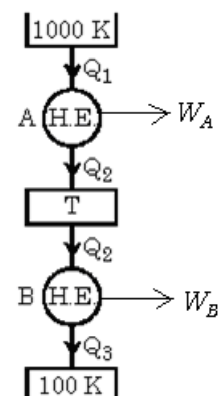
$$(b) T = \sqrt{1000 \times 100} = 316.3 \text{ K}$$

$$\begin{aligned} Q_2 &= \frac{Q_1}{1000} \times T = \frac{1680 \times 316.3}{1000} \\ &= 531.26 \text{ kJ} \end{aligned}$$

$$\begin{aligned} (a) Q_3 &= \frac{Q_2}{316.3} \times 100 = \frac{531.26 \times 100}{316.3} \\ &= 168 \text{ kJ as (a)} \end{aligned}$$

$$\begin{aligned} (c) W_A &= Q_1 - Q_2 = (1680 - 531.26) \text{ kJ} \\ &= 1148.74 \text{ kJ} \end{aligned}$$

$$\begin{aligned} W_B &= (531.26 - 168) \text{ kJ} \\ &= 363.26 \text{ kJ} \end{aligned}$$





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(d) If the equal work then  $T = \frac{100 + 1000}{2} = 550 \text{ K}$

$$\therefore Q_2 = \frac{Q_1}{1000} \times T = \frac{1680 \times 550}{1000} = 924 \text{ kJ}$$

(e)  $\eta_A = 1 - \frac{550}{1000} = 0.45$

$$\eta_B = 1 - \frac{100}{550} = 0.8182$$

**Q6.16**

A heat pump is to be used to heat a house in winter and then reversed to cool the house in summer. The interior temperature is to be maintained at 20°C. Heat transfer through the walls and roof is estimated to be 0.525 kJ/s per degree temperature difference between the inside and outside.

(a) If the outside temperature in winter is 5°C, what is the minimum power required to drive the heat pump?

(b) If the power output is the same as in part (a), what is the maximum outer temperature for which the inside can be maintained at 20°C?

(Ans. (a) 403 W, (b) 35.4°C)

**Solution :**

(a) Estimated Heat rate

$$= 0.525 \times (20 - 5) \text{ kJ/s} = 7.875 \text{ kJ/s}$$

$$\text{COP} = \frac{293}{293 - 278} = 19.53$$

$$W_{\min} = \frac{\dot{Q}}{(\text{COP})_{\max}}$$

$$= \frac{7.875}{19.53} = 0.403 \text{ kW} = 403 \text{ W}$$

(b) Given  $\dot{W} = 403 \text{ W}$

$$\text{Heat rate } (\dot{Q}_1) = 0.525 (T - 293) \text{ kW}$$

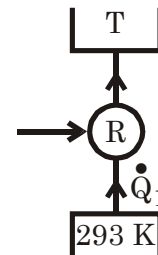
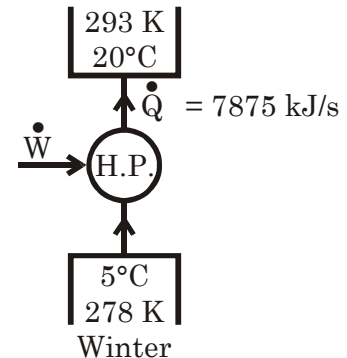
$$= 525(T - 293) \text{ W}$$

$$\therefore \text{COP} = \frac{525(T - 293)}{403} = \frac{293}{(T - 293)}$$

$$\text{or } (T - 293) = \frac{403 \times 293}{525} = 15$$

$$\text{or } T = 308 \text{ K} = 35^\circ \text{ C}$$

$$\therefore \text{Maximum outside Temperature} = 35^\circ \text{C}$$



**Q6.17**

Consider an engine in outer space which operates on the Carnot cycle. The only way in which heat can be transferred from the engine is by radiation. The rate at which heat is radiated is proportional to the fourth power of the absolute temperature and to the area of the radiating surface. Show that for a given power output and a given  $T_1$ , the area of the radiator will be a minimum when

$$\frac{T_2}{T_1} = \frac{3}{4}$$

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**Solution:** Heat have to radiate =  $Q_2$   
 $\therefore Q_2 = \sigma AT_2^4$

From engine side

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \frac{W}{T_1 - T_2}$$

$$\therefore Q_2 = \frac{WT_2}{T_1 - T_2}$$

$$\therefore \frac{WT_2}{T_1 - T_2} = \sigma AT_2^4$$

$$\text{or} \quad A = \frac{W}{\sigma T_2^4} \left\{ \frac{T_2}{T_1 - T_2} \right\} = \frac{W}{\sigma} \left\{ \frac{1}{T_1 T_2^3 - T_2^4} \right\}$$

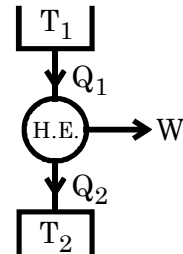
For minimum Area

$$\frac{\partial A}{\partial T_2} = 0 \quad \text{or} \quad \frac{\partial}{\partial T_2} \{T_1 T_2^3 - T_2^4\} = 0$$

$$\text{or} \quad T_1 \times 3 T_2^2 - 4 T_2^3 = 0$$

$$\text{or} \quad 3 T_1 = 4 T_2$$

$$\text{or} \quad \frac{T_2}{T_1} = \frac{3}{4} \text{ proved}$$



**Q6.18**

It takes 10 kW to keep the interior of a certain house at 20°C when the outside temperature is 0°C. This heat flow is usually obtained directly by burning gas or oil. Calculate the power required if the 10 kW heat flow were supplied by operating a reversible engine with the house as the upper reservoir and the outside surroundings as the lower reservoir, so that the power were used only to perform work needed to operate the engine.

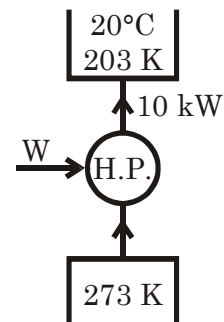
(Ans. 0.683 kW)

**Solution:** COP of the H.P.

$$\frac{10}{W} = \frac{293}{293 - 273}$$

$$\text{or} \quad W = \frac{10 \times 20}{293} \text{ kW}$$

$$= 683 \text{ W only.}$$



**Q6.19**

**Prove that the COP of a reversible refrigerator operating between two given temperatures is the maximum.**

**Solution:** Suppose A is any refrigerator and B is reversible refrigerator and also assume

$$(\text{COP})_A > (\text{COP})_B$$

$$\text{and} \quad Q_{1A} = Q_{1B} = Q$$

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$$\text{or } \frac{Q_{1A}}{W_A} > \frac{Q_{1B}}{W_B}$$

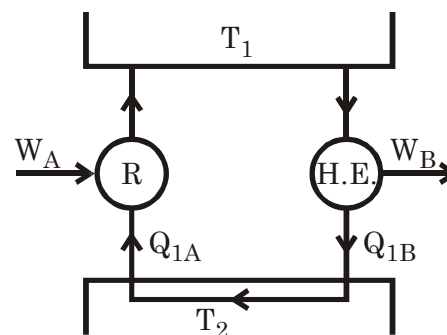
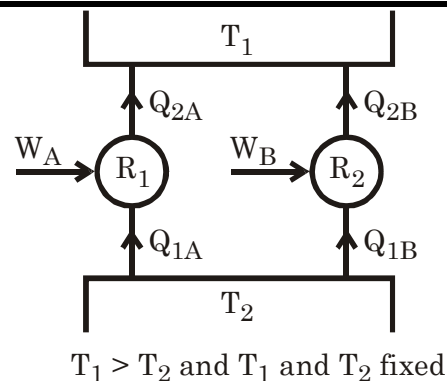
$$\text{or } \frac{Q}{W_A} > \frac{Q}{W_B}$$

$$\text{or } W_A < W_B$$

Then we reversed the reversible refrigerator 'B' and then work output of refrigerator 'B' is  $W_B$  and heat rejection is  $Q_{1B} = Q$  (same)

So we can directly use  $Q$  to feed for refrigerator and Reservoir ' $T_2$ ' is eliminated then also a net work output ( $W_B - W_A$ ) will be available. But it violates the Kelvin-Planck statement i.e. violates Second Law of thermodynamic so our assumption is wrong.

$$\text{So } (\text{COP})_R \geq (\text{COP})_A$$



**Q6.20**

A house is to be maintained at a temperature of  $20^\circ\text{C}$  by means of a heat pump pumping heat from the atmosphere. Heat losses through the walls of the house are estimated at  $0.65 \text{ kW}$  per unit of temperature difference between the inside of the house and the atmosphere.

- If the atmospheric temperature is  $-10^\circ\text{C}$ , what is the minimum power required driving the pump?
- It is proposed to use the same heat pump to cool the house in summer. For the same room temperature, the same heat loss rate, and the same power input to the pump, what is the maximum permissible atmospheric temperature?

(Ans.  $2 \text{ kW}$ ,  $50^\circ\text{C}$ )

**Solution:** Same as 6.16

**Q6.21**

A solar-powered heat pump receives heat from a solar collector at  $T_h$ , rejects heat to the atmosphere at  $T_a$ , and pumps heat from a cold space at  $T_c$ . The three heat transfer rates are  $Q_h$ ,  $Q_a$ , and  $Q_c$  respectively. Derive an expression for the minimum ratio  $Q_h/Q_c$ , in terms of the three temperatures.

If  $T_h = 400 \text{ K}$ ,  $T_a = 300 \text{ K}$ ,  $T_c = 200 \text{ K}$ ,  $Q_c = 12 \text{ kW}$ , what is the minimum  $Q_h$ ? If the collector captures  $0.2 \text{ kW m}^{-2}$ , what is the minimum collector area required?

(Ans.  $26.25 \text{ kW}$ ,  $131.25 \text{ m}^2$ )

**Solution:**

$$W_{\text{output}} = \frac{Q_h}{T_h} (T_h - T_a)$$

## Second Law of Thermodynamics

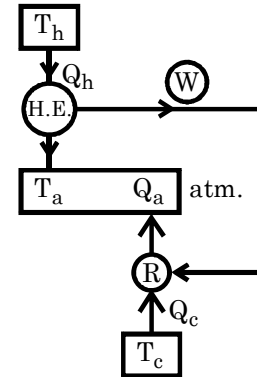
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$$W_{\text{input}} = \frac{Q_c}{T_c} (T_a - T_c)$$

As they same

$$\begin{aligned} \text{So } \frac{Q_h}{Q_c} &= \frac{T_h}{T_c} \times \frac{(T_a - T_c)}{(T_h - T_a)} \\ Q_h &= 12 \times \frac{400}{200} \times \left\{ \frac{300 - 200}{400 - 300} \right\} \text{ kW} \\ &= 24 \text{ kW} \\ \text{Required Area (A)} &= \frac{2.4}{0.2} = 120 \text{ m}^2 \end{aligned}$$



**Q6.22**

A heat engine operating between two reservoirs at 1000 K and 300 K is used to drive a heat pump which extracts heat from the reservoir at 300 K at a rate twice that at which the engine rejects heat to it. If the efficiency of the engine is 40% of the maximum possible and the COP of the heat pump is 50% of the maximum possible, what is the temperature of the reservoir to which the heat pump rejects heat? What is the rate of heat rejection from the heat pump if the rate of heat supply to the engine is 50 kW?

(Ans. 326.5 K, 86 kW)

**Solution:**

$$\eta_{\text{actual}} = 0.4 \left( 1 - \frac{300}{1000} \right) = 0.28$$

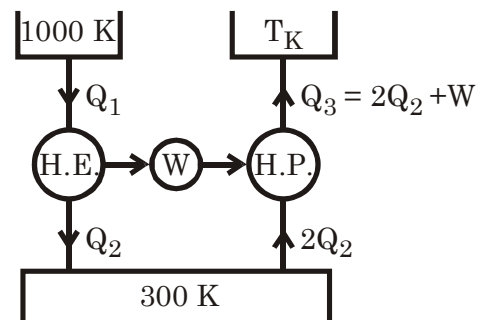
$$\begin{aligned} \therefore W &= 0.28 Q_1 \\ Q_2 &= Q_1 - W = 0.72 Q_1 \\ Q_3 &= 2 Q_2 + W = 1.72 Q_1 \end{aligned}$$

$$\begin{aligned} \therefore (\text{COP})_{\text{actual}} &= \frac{1.72 Q_1}{0.28 Q_1} \\ &= \frac{T}{T - 300} \times (0.5) \end{aligned}$$

$$\text{or } 6.143 T - 300 \times 6.143 = T \times 0.5$$

$$\text{or } T = 326.58 \text{ K}$$

$$Q_3 = 1.72 \times 50 \text{ kW} = 86 \text{ kW}$$



**Q6.23**

A reversible power cycle is used to drive a reversible heat pump cycle. The power cycle takes in  $Q_1$  heat units at  $T_1$  and rejects  $Q_2$  at  $T_2$ . The heat pump abstracts  $Q_4$  from the sink at  $T_4$  and discharges  $Q_3$  at  $T_3$ . Develop an expression for the ratio  $Q_4/Q_1$  in terms of the four temperatures.

$$\left( \text{Ans. } \frac{Q_4}{Q_1} = \frac{T_4(T_1 - T_2)}{T_1(T_3 - T_4)} \right)$$

**Solution:** For H.E.

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$$\text{Work output (W)} = \frac{Q_1}{T_1} (T_1 - T_2)$$

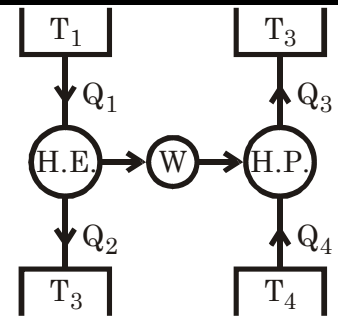
For H.P.

$$\text{Work input (W)} = \frac{Q_4}{T_4} (T_3 - T_4)$$

$$\therefore \frac{Q_1}{T_1} (T_1 - T_2) = \frac{Q_4}{T_4} (T_3 - T_4)$$

$$\text{or} \quad \frac{Q_4}{Q_1} = \frac{T_4}{T_1} \left\{ \frac{T_1 - T_2}{T_3 - T_4} \right\}$$

This is the expression.



**Q6.24**

**Prove that the following propositions are logically equivalent:**

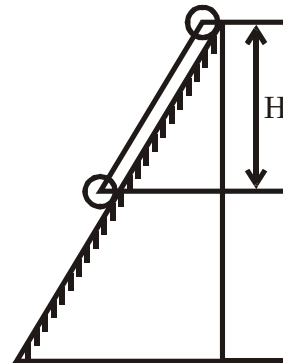
- (a) A PMM2 is Impossible
- (b) A weight sliding at constant velocity down a frictional inclined plane executes an irreversible process.

**Solution:** Applying First Law of Thermodynamics

$$Q_{12} = E_2 - E_1 + W_{1,2}$$

$$\text{or} \quad 0 = E_2 - E_1 - mgh$$

$$\text{or} \quad E_1 - E_2 = mgh$$



## 7.

## Entropy

## Some Important Notes

1. Clausius theorem:  $\oint \left( \frac{\delta Q}{T} \right)_{\text{rev.}} = 0$

2.  $S_f - S_i = \int_i^f \frac{\delta Q_{\text{rev.}}}{T} = (\Delta S)_{\text{irrev. Path}}$

Integration can be performed only on a reversible path.

3. **Clausius Inequality:**  $\oint \frac{\delta Q}{T} \leq 0$

4. At the equilibrium state, the system is at the peak of the entropy hill. (Isolated)

5.  $TdS = dU + pdV$

6.  $TdS = dH - Vdp$

7. Famous relation  $S = K \ln W$

Where  $K$  = Boltzmann constant  
 $W$  = thermodynamic probability.

8. General case of change of entropy of a Gas

$$S_2 - S_1 = m \left\{ c_v \ln \frac{p_2}{p_1} + c_p \ln \frac{V_2}{V_1} \right\}$$

Initial condition of gas  $p_1, V_1, T_1, S_1$  and

Final condition of gas  $p_2, V_2, T_2, S_2$

Questions with Solution P. K. Nag

**Q7.1.** On the basis of the first law fill in the blank spaces in the following table of imaginary heat engine cycles. On the basis of the second law classify each cycle as reversible, irreversible, or impossible.

Cycle	Temperature		Rate of Heat Flow		Rate of work	Efficiency
	Source	Sink	Supply	Rejection	Output	
(a)	327°C	27°C	420 kJ/s	230 kJ/s	...kW	
(b)	1000°C	100°C	...kJ/min	4.2 MJ/min	... kW	65%
(c)	750 K	300 K	...kJ/s	...kJ/s	26 kW	65%
(d)	700 K	300 K	2500 kcal/h	...kcal/h	1 kW	—

(Ans. (a) Irreversible, (b) Irreversible, (c) Reversible, (d) Impossible)

**Solution:**

Cycle	Temperature		Rate of Heat Flow		Rate of work	Efficiency	Remark
	Source	Sink	Supply	Rejection			
(a)	327°C	27°C	420 kJ/s	230 kJ/s	190kW	0.4523	$\eta_{\max} = 50\%$ , irrev.possible
(b)	1000°C	100°C	12000 kJ/km	4.2 kJ/m	7800 kW	65%	$\eta_{\max}=70.7\%$ irrev.possible
(c)	750 K	300 K	43.33 kJ/s	17.33 kJ/s	26 kW	60%	$\eta_{\max}= 60\%$ rev. possible
(d)	700 K	300 K	2500 kcal/h	1640 kcal/h	1 kW	4.4%	$\eta_{\max}=57\%$ irrev.possible

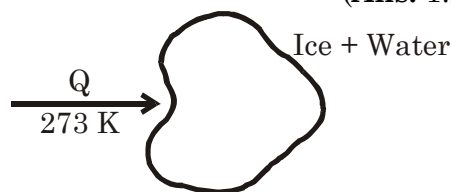
**Q7.2** The latent heat of fusion of water at 0°C is 335 kJ/kg. How much does the entropy of 1 kg of ice change as it melts into water in each of the following ways:

- (a) Heat is supplied reversibly to a mixture of ice and water at 0°C.  
 (b) A mixture of ice and water at 0°C is stirred by a paddle wheel.

(Ans. 1.2271 kJ/K)

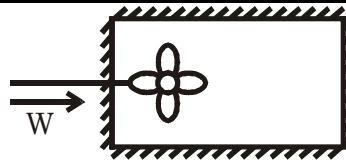
**Solution :** (a)  $(\Delta S)_{\text{system}} = + \frac{1 \times 335}{273} \text{ kJ/ K}$

$$= 1.227 \text{ kJ/K}$$



(b)  $(\Delta S)_{\text{system}}$

$$= \int_{273}^{273} m c_p \frac{dT}{T} = 0$$

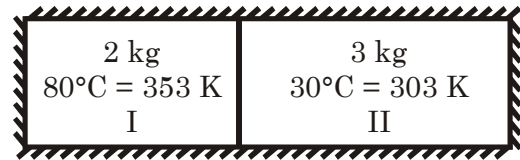


**Q7.3** Two kg of water at 80°C are mixed adiabatically with 3 kg of water at 30°C in a constant pressure process of 1 atmosphere. Find the increase in the entropy of the total mass of water due to the mixing process ( $c_p$  of water = 4.187 kJ/kg K).

(Ans. 0.0576 kJ/K)

**Solution:** If final temperature of mixing is  $T_f$  then  
 $2 \times c_p (353 - T_f)$   
 $= 3 \times c_p (T_f - 303)$

or  $T_f = 323 \text{ K}$



$$(\Delta S)_{\text{system}} = (\Delta S)_I + (\Delta S)_{II}$$

$$\begin{aligned} &= \int_{353}^{323} m_1 c_p \frac{dT}{T} + \int_{303}^{323} m_2 c_p \frac{dT}{T} \\ &= 2 \times 4.187 \ln \left( \frac{323}{353} \right) + 3 \times 4.187 \times \ln \frac{323}{303} \\ &= 0.05915 \text{ kJ/K} \end{aligned}$$

**Q7.4** In a Carnot cycle, heat is supplied at 350°C and rejected at 27°C. The working fluid is water which, while receiving heat, evaporates from liquid at 350°C to steam at 350°C. The associated entropy change is 1.44 kJ/kg K.

- (a) If the cycle operates on a stationary mass of 1 kg of water, how much is the work done per cycle, and how much is the heat supplied?  
 (b) If the cycle operates in steady flow with a power output of 20 kW, what is the steam flow rate?

(Ans. (a) 465.12, 897.12 kJ/kg, (b) 0.043 kg/s)

**Solution:** If heat required for evaporation is  $Q$  kJ/kg then

(a)  $\frac{Q}{(350 + 273)} = 1.44$

or  $Q = 897.12 \text{ kJ/kg}$

It is a Carnot cycle so  $\eta = 1 - \frac{(273 + 27)}{(350 + 273)}$

$\therefore W = \eta \cdot Q = 465.12 \text{ kJ}$

(b)  $P = \dot{m}W$  or  $\dot{m} = \frac{P}{W} = \frac{20}{465.12} \text{ kg/s} = 0.043 \text{ kg/s}$

**Q7.5** A heat engine receives reversibly 420 kJ/cycle of heat from a source at 327°C, and rejects heat reversibly to a sink at 27°C. There are no other heat transfers. For each of the three hypothetical amounts of heat rejected, in (a), (b), and (c) below, compute the cyclic integral of  $dQ/T$ .



from these results show which case is irreversible, which reversible, and which impossible:

- (a) 210 kJ/cycle rejected
- (b) 105 kJ/cycle rejected
- (c) 315 kJ/cycle rejected

(Ans. (a) Reversible, (b) Impossible, (c) Irreversible)

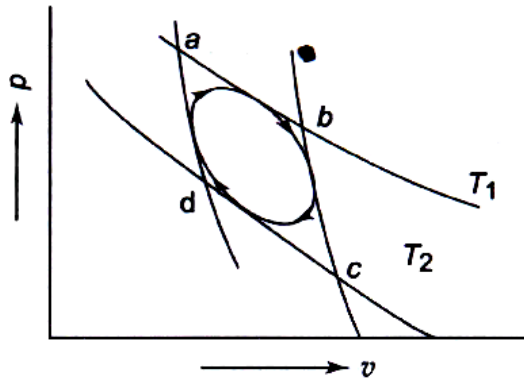
**Solution:** (a)  $\oint \frac{dQ}{T} = \frac{+420}{(327 + 273)} - \frac{210}{(27 + 273)} = 0$   
 $\therefore$  Cycle is Reversible, Possible

(b)  $\oint \frac{dQ}{T} = +\frac{420}{600} - \frac{105}{300} = 0.35$   
 $\therefore$  Cycle is Impossible

(c)  $\oint \frac{dQ}{T} = +\frac{420}{600} - \frac{315}{300} = -0.35$   
 $\therefore$  Cycle is irreversible but possible.

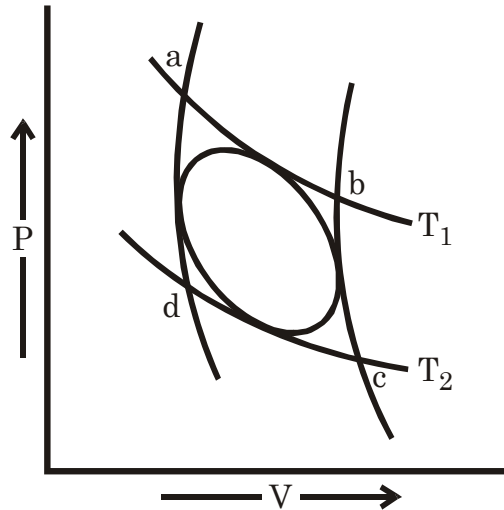
**Q7.6**

In Figure, *abcd* represents a Carnot cycle bounded by two reversible adiabatic and two reversible isotherms at temperatures  $T_1$  and  $T_2$  ( $T_1 > T_2$ ).



The oval figure is a reversible cycle, where heat is absorbed at temperature less than, or equal to,  $T_1$ , and rejected at temperatures greater than, or equal to,  $T_2$ . Prove that the efficiency of the oval cycle is less than that of the Carnot cycle.

Solution:



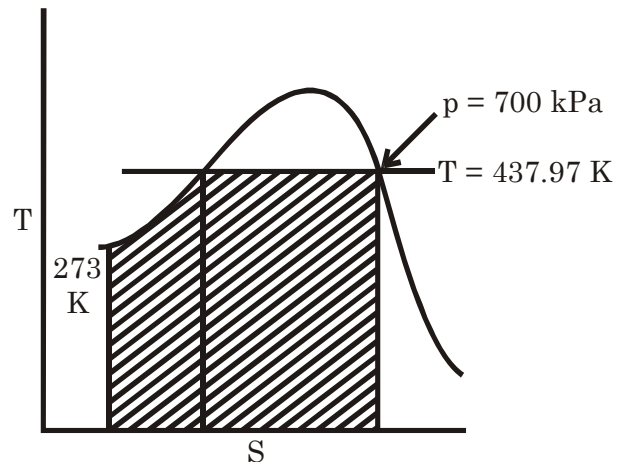
Q7.7

Water is heated at a constant pressure of 0.7 MPa. The boiling point is 164.97°C. The initial temperature of water is 0°C. The latent heat of evaporation is 2066.3 kJ/kg. Find the increase of entropy of water, if the final state is steam

(Ans. 6.6967 kJ/kg K)

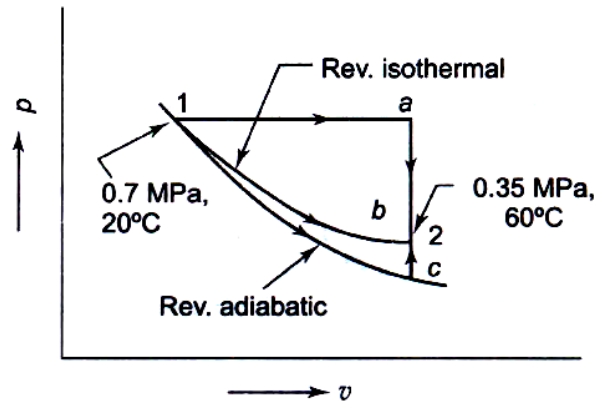
Solution:

$$\begin{aligned}
 (\Delta S)_{\text{Water}} &= \int_{273}^{437.97} 1 \times 4187 \times \frac{dT}{T} \\
 &= 4.187 \ln \left( \frac{437.97}{273} \right) \text{ kJ/K} \\
 &= 1.979 \text{ kJ/K} \\
 (\Delta S)_{\text{Eva pour}} &= \frac{1 \times 2066.3}{437.97} \text{ kJ/K} \\
 &= 4.7179 \text{ kJ/K} \\
 (\Delta S)_{\text{system}} &= 6.697 \text{ kJ/kg - K}
 \end{aligned}$$



Q7.8

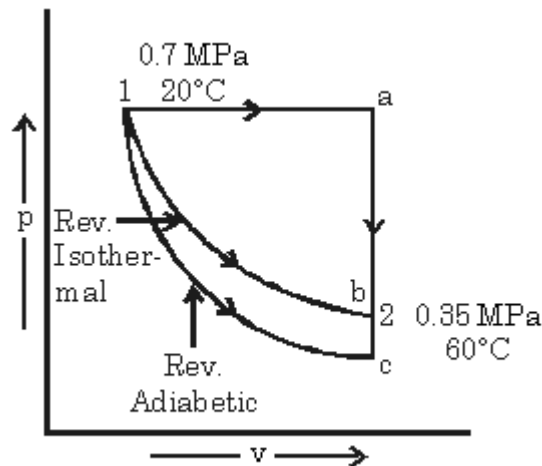
One kg of air initially at 0.7 MPa, 20°C changes to 0.35 MPa, 60°C by the three reversible non-flow processes, as shown in Figure. Process 1: *a-2* consists of a constant pressure expansion followed by a constant volume cooling, process 1: *b-2* an isothermal expansion followed by a constant pressure expansion, and process 1: *c-2* an adiabatic



Expansion followed by a constant volume heating. Determine the change of internal energy, enthalpy, and entropy for each process, and find the work transfer and heat transfer for each process. Take  $c_p = 1.005$  and  $c_v = 0.718$  kJ/kg K and assume the specific heats to be constant. Also assume for air  $pv = 0.287 T$ , where  $p$  is the pressure in kPa,  $v$  the specific volume in  $\text{m}^3/\text{kg}$ , and  $T$  the temperature in K.

**Solution:**

$p_1 = 0.7 \text{ MPa} = 700 \text{ kPa}$	$T_1 = 293 \text{ K}$	
$\therefore v_1 = 0.12013 \text{ m}^3/\text{kg}$	$p_a = 700 \text{ kPa}$	
$\therefore T_a = 666 \text{ K}$	$v_a = 0.27306 \text{ m}^3/\text{kg}$	
$p_2 = 350 \text{ kPa}$	$T_2 = 333 \text{ K}$	
$\therefore v_2 = 0.27306 \text{ m}^3/\text{kg}$		



For process 1-a-2

$$\begin{aligned}
 Q_{1-a} &= u_a - u_1 + \int_{v_1}^{v_a} p \, dv \\
 &= u_a - u_1 + 700(0.27306 - 0.12013) \\
 &= u_a - u_1 + 107
 \end{aligned}$$

$$Q_{a-2} = u_2 - u_a + 0$$

$$\therefore u_a - u_1 = 267.86 \text{ kJ/kg}$$

$$u_2 - u_a = -239 \text{ kJ/kg}$$

$$Q_{1-a} = \int_{T_1}^{T_a} c_p dT$$

$$= 1.005 \times (666 - 293)$$

$$= 374.865 \text{ kJ/kg}$$

$$Q_{a-2} = \int_{T_a}^{T_2} c_v dT$$

$$= 0.718 (333 - 666)$$

$$= -239 \text{ kJ/kg}$$

$$(i) \Delta u = u_2 - u_1 = 28.766 \text{ kJ/kg}$$

$$(ii) \Delta h = h_2 - h_1 = u_2 - u_1 + p_2 v_2 - p_1 v_1$$

$$= 28.766 + 350 \times 0.27306 - 700 \times 0.12013 = 40.246 \text{ kJ/kg}$$

$$(iii) Q = Q_2 + Q_1 = 135.865 \text{ kJ/kg}$$

$$(iv) W = W_1 + W_2 = 107 \text{ kJ/kg}$$

$$(v) \Delta s = s_2 - s_1 = (s_2 - s_a) + (s_a - s_1)$$

$$= C_v \ln\left(\frac{T_2}{T_a}\right) + C_p \ln\left(\frac{T_a}{T_1}\right)$$

$$= 0.3275 \text{ kJ/kg} - \text{K}$$

**Q7.9** Ten grammes of water at 20°C is converted into ice at -10°C at constant atmospheric pressure. Assuming the specific heat of liquid water to remain constant at 4.2 J/gK and that of ice to be half of this value, and taking the latent heat of fusion of ice at 0°C to be 335 J/g, calculate the total entropy change of the system.

(Ans. 16.02 J/K)

**Solution:**

$$S_2 - S_1 = \int_{293}^{273} \frac{m c_p dT}{T}$$

$$= 0.01 \times 4.2 \times \ln \frac{273}{293} \text{ kJ/K}$$

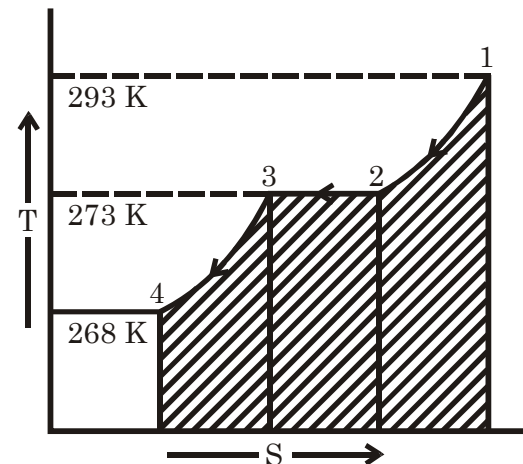
$$= -0.00297 \text{ kJ/K}$$

$$= -2.9694 \text{ J/K}$$

$$S_3 - S_2 = \frac{-mL}{T}$$

$$= \frac{-0.01 \times 335 \times 1000}{273}$$

$$= -12.271 \text{ J/K}$$



$$S_4 - S_3 = \int_{273}^{268} \frac{m c_p dT}{T} = 0.01 \times \left( \frac{4.2}{2} \right) \times \ln \frac{268}{273} \text{ kJ/K}$$

$$= -0.3882 \text{ J/K}$$

$$\therefore S_4 - S_1 = -15.63 \text{ J/K}$$

$$\therefore \text{Net Entropy change} = 15.63 \text{ J/K}$$

**Q7.10** Calculate the entropy change of the universe as a result of the following processes:

(a) A copper block of 600 g mass and with  $C_p$  of 150 J/K at 100°C is placed in a lake at 8°C.

(b) The same block, at 8°C, is dropped from a height of 100 m into the lake.

(c) Two such blocks, at 100 and 0°C, are joined together.

(Ans. (a) 6.69 J/K, (b) 2.095 J/K, (c) 3.64 J/K)

**Solution:**

$$(a) \quad (\Delta S)_{\text{copper}} = \int_{373}^{281} m c_p \frac{dT}{T}$$

$$= 150 \ln \frac{281}{373} \text{ J/K}$$

$$= -42.48 \text{ J/K}$$

As unit of  $C_p$  is J/K there for

$\therefore$  It is heat capacity

$$\text{i.e. } C_p = m c_p$$

$$(\Delta S)_{\text{lake}} = \frac{C_p(100 - 8)}{281} \text{ J/K}$$

$$= \frac{150(100 - 8)}{281} \text{ J/K} = 49.11 \text{ J/K}$$

$$(\Delta S)_{\text{univ}} = (\Delta S)_{\text{COP}} + (\Delta S)_{\text{lake}} = 6.63 \text{ J/K}$$

(b) Work when it touch water =  $0.600 \times 9.81 \times 100 \text{ J} = 588.6 \text{ J}$

As work dissipated from the copper

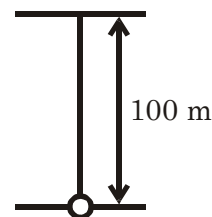
$$(\Delta S)_{\text{copper}} = 0$$

As the work is converted to heat and absorbed by water then

$$(\Delta S)_{\text{lake}} = \frac{W = Q}{281} = \frac{588.6}{281} \text{ J/K} = 2.09466 \text{ J/K}$$

$$\therefore (\Delta S)_{\text{univ}} = 0 + 2.09466 \text{ J/K} = 2.09466 \text{ J/K}$$

$$(c) \text{ Final temperature } (T_f) = \frac{100 + 0}{2} = 50^\circ \text{ C} = 323 \text{ K}$$



$$\begin{aligned}
 (\Delta S)_I &= C_p \int_{T_1}^{T_f} \frac{dT}{T} ; \quad (\Delta S)_{II} = C_p \int_{T_2}^{T_f} \frac{dT}{T} \\
 \therefore (\Delta S)_{\text{system}} &= 150 \ln \left( \frac{T_f}{T_1} \right) + 150 \ln \left( \frac{T_f}{T_2} \right) \\
 &= 150 \left\{ \ln \frac{323}{373} + \ln \frac{323}{273} \right\} \text{ J/K} = 3.638 \text{ J/K}
 \end{aligned}$$

**Q7.11** A system maintained at constant volume is initially at temperature  $T_1$ , and a heat reservoir at the lower temperature  $T_0$  is available. Show that the maximum work recoverable as the system is cooled to  $T_0$  is

$$W = C_v \left[ (T_1 - T_0) - T_0 \ln \frac{T_1}{T_0} \right]$$

**Solution:**

For maximum work obtainable the process should be reversible

$$\begin{aligned}
 (\Delta S)_{\text{body}} &= \int_{T_1}^{T_0} C_v \frac{dT}{T} = C_v \ln \left( \frac{T_0}{T_1} \right) \\
 (\Delta S)_{\text{resoir}} &= \frac{Q - W}{T_0} \\
 (\Delta S)_{\text{cycle}} &= 0 \\
 \therefore (\Delta S)_{\text{univ.}} &= C_v \ln \left( \frac{T_0}{T_1} \right) + \frac{Q - W}{T_0} \geq 0 \\
 \therefore C_v \ln \left( \frac{T_0}{T_1} \right) + \frac{Q - W}{T_0} &\geq 0
 \end{aligned}$$

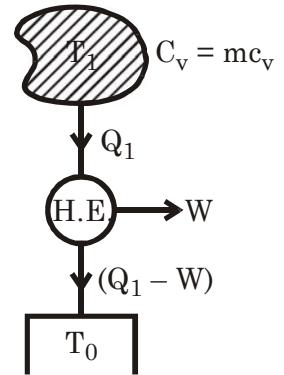
$$\text{or} \quad C_v T_0 \ln \left( \frac{T_0}{T_1} \right) + Q - W \geq 0 \quad \therefore Q = C_v (T_1 - T_0)$$

$$\text{or} \quad W \leq Q + C_v T_0 \ln \left( \frac{T_0}{T_1} \right)$$

$$\text{or} \quad W \leq C_v (T_1 - T_0) + C_v T_0 \ln \left( \frac{T_0}{T_1} \right)$$

$$\text{or} \quad W \leq C_v \left\{ (T_1 - T_0) + T_0 \ln \left( \frac{T_0}{T_1} \right) \right\}$$

$$\therefore \text{Maximum work } W_{\text{max}} = C_v \left\{ (T_1 - T_0) + T_0 \ln \left( \frac{T_0}{T_1} \right) \right\}$$



**Q7.12** If the temperature of the atmosphere is  $5^\circ\text{C}$  on a winter day and if 1 kg of water at  $90^\circ\text{C}$  is available, how much work can be obtained. Take  $c_v$ , of water as  $4.186 \text{ kJ/kg K}$ .

**Solution:** TRY PLEASE

**Q7.13** A body with the equation of state  $U = CT$ , where  $C$  is its heat capacity, is heated from temperature  $T_1$  to  $T_2$  by a series of reservoirs ranging from

$T_1$  to  $T_2$ . The body is then brought back to its initial state by contact with a single reservoir at temperature  $T_1$ . Calculate the changes of entropy of the body and of the reservoirs. What is the total change in entropy of the whole system?

If the initial heating were accomplished merely by bringing the body into contact with a single reservoir at  $T_2$ , what would the various entropy changes be?

**Solution:** TRY PLEASE

**Q7.14** A body of finite mass is originally at temperature  $T_1$ , which is higher than that of a reservoir at temperature  $T_2$ . Suppose an engine operates in a cycle between the body and the reservoir until it lowers the temperature of the body from  $T_1$  to  $T_2$ , thus extracting heat  $Q$  from the body. If the engine does work  $W$ , then it will reject heat  $Q - W$  to the reservoir at  $T_2$ . Applying the entropy principle, prove that the maximum work obtainable from the engine is

$$W (\text{max}) = Q - T_2 (S_1 - S_2)$$

Where  $S_1 - S_2$  is the entropy decrease of the body.

If the body is maintained at constant volume having constant volume heat capacity  $C_v = 8.4 \text{ kJ/K}$  which is independent of temperature, and if  $T_1 = 373 \text{ K}$  and  $T_2 = 303 \text{ K}$ , determine the maximum work obtainable.

(Ans. 58.96 kJ)

**Solution:** Final temperature of the body will be  $T_2$

$$\therefore S_2 - S_1 = \int_{T_1}^{T_2} m c_v \frac{dT}{T} = m c_v \ln \left( \frac{T_2}{T_1} \right)$$

[ $c_v$  = heat energy  $C_v$ ]

$$(\Delta S)_{\text{reservoir}} = \frac{Q - W}{T_2} \quad \therefore (\Delta S)_{\text{H.E.}} = 0$$

$$\therefore (\Delta S)_{\text{univ.}} = (S_2 - S_1) + \frac{Q - W}{T_2} \geq 0$$

$$\text{or} \quad T_2 (S_2 - S_1) + Q - W \geq 0$$

$$\text{or} \quad W \leq Q + T_2 (S_2 - S_1)$$

$$\text{or} \quad W \leq [Q - T_2 (S_1 - S_2)]$$

$$\therefore W_{\text{max}} = [Q - T_2 (S_1 - S_2)]$$

$$\begin{aligned} W_{\text{max}} &= Q - T_2 (S_1 - S_2) \\ &= Q + T_2 C_v \ln \left( \frac{T_2}{T_1} \right) \end{aligned}$$

$$= C_v (T_1 - T_2) + T_2 C_v \ln \left( \frac{T_2}{T_1} \right)$$

$$= 8.4 \left[ 373 - 303 + 303 \ln \left( \frac{303}{373} \right) \right]$$

$$= 58.99 \text{ kJ}$$

**Q7.15** Each of three identical bodies satisfies the equation  $U = CT$ , where  $C$  is the heat capacity of each of the bodies. Their initial temperatures are 200 K, 250 K, and 540 K. If  $C = 8.4 \text{ kJ/K}$ , what is the maximum amount of work that can be extracted in a process in which these bodies are brought to a final common temperature?

(Ans. 756 kJ)

**Solution:**

$$U = CT$$

Therefore heat capacity of the body is  $C = 8.4 \text{ kJ/K}$

Let final temperature will be ( $T_f$ )

$$\therefore W = W_1 + W_2$$

$$Q = Q_1 + Q_2$$

$$(\Delta S)_{540\text{K body}} = C \ln \frac{T_f}{540} \text{ kJ/K}$$

$$(\Delta S)_{250\text{ K}} = C \ln \left( \frac{T_f}{250} \right)$$

$$(\Delta S)_{200\text{ K}} = C \ln \left( \frac{T_f}{200} \right)$$

$$(\Delta S)_{\text{surrounds}} = 0$$

$$(\Delta S)_{\text{H.E.}} = 0$$

$$\therefore (\Delta S)_{\text{univ.}} = C \ln \left( \frac{T_f^3}{540 \times 250 \times 200} \right) \geq 0$$

For minimum  $T_f$

$$T_f^3 = 540 \times 250 \times 200$$

$$\therefore T_f = 300 \text{ K}$$

$$\therefore Q = 8.4(540 - 300) = 2016 \text{ kJ}$$

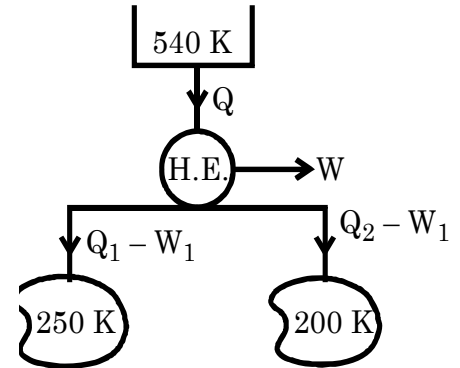
$$Q_1 - W_1 = 8.4(300 - 250) = 420 \text{ kJ}$$

$$Q_2 - W_2 = 8.4(300 - 200) = 840 \text{ kJ}$$

$$\therefore Q_1 + Q_2 - (W_1 + W_2) = 1260$$

$$\text{or } (W_1 + W_2) = 2016 - 1260 \text{ kJ} = 756 \text{ kJ}$$

$$\therefore W_{\text{max}} = 756 \text{ kJ}$$



**Q7.16** In the temperature range between  $0^\circ\text{C}$  and  $100^\circ\text{C}$  a particular system maintained at constant volume has a heat capacity.

$$C_v = A + 2BT$$

With  $A = 0.014 \text{ J/K}$  and  $B = 4.2 \times 10^{-4} \text{ J/K}^2$

A heat reservoir at  $0^\circ\text{C}$  and a reversible work source are available. What is the maximum amount of work that can be transferred to the reversible work source as the system is cooled from  $100^\circ\text{C}$  to the temperature of the reservoir?

(Ans. 4.508 J)

**Solution:**



Find temperature of body is 273 K

$$\begin{aligned}\therefore Q &= \int_{373}^{273} C_v dT = AT + BT^2 \Big|_{373}^{273} \\ &= -A(100) + B(273^2 - 373^2) \text{ J} \\ &= -28.532 \text{ J (flow from the system)}\end{aligned}$$

$$\begin{aligned}(\Delta S)_{\text{body}} &= \int_{373}^{273} C_v \frac{dT}{T} \\ &= \int_{373}^{273} \left( \frac{A + 2BT}{T} \right) dT \\ &= A \ln \frac{273}{373} + 2B(273 - 373) \text{ J/K} \\ &= -0.08837 \text{ J/K}\end{aligned}$$

$$(\Delta S)_{\text{res.}} = \frac{Q - W}{273}; (\Delta S)_{\text{H.E.}} = 0 \quad (\Delta S)_{\text{surrounds}} = 0$$

$$\therefore (\Delta S)_{\text{univ}} = -0.08837 + \frac{Q - W}{273} \geq 0$$

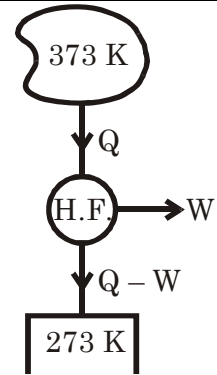
$$\text{or } -24.125 + Q - W \geq 0$$

$$\text{or } W \leq Q - 24.125$$

$$\text{or } W \leq (28.532 - 24.125) \text{ J}$$

$$\text{or } W \leq 4.407 \text{ J}$$

$$W_{\text{max}} = 4.407 \text{ J}$$



**Q7.17** Each of the two bodies has a heat capacity at constant volume

$$C_v = A + 2BT$$

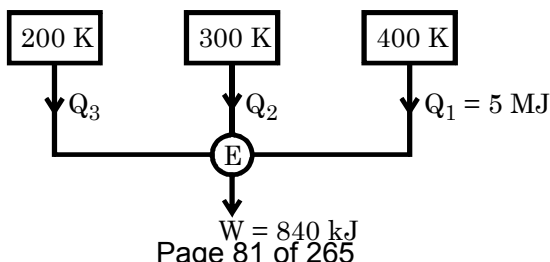
Where  $A = 8.4 \text{ J/K}$  and  $B = 2.1 \times 10^{-2} \text{ J/K}^2$

If the bodies are initially at temperatures 200 K and 400 K and if a reversible work source is available, what are the maximum and minimum final common temperatures to which the two bodies can be brought? What is the maximum amount of work that can be transferred to the reversible work source?

(Ans.  $T_{\text{min}} = 292 \text{ K}$ )

**Solution:** TRY PLEASE

**Q7.18** A reversible engine, as shown in Figure during a cycle of operations draws 5 MJ from the 400 K reservoir and does 840 kJ of work. Find the amount and direction of heat interaction with other reservoirs.

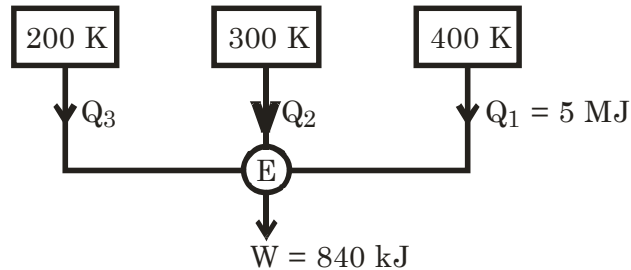


(Ans.  $Q_2 = + 4.98 \text{ MJ}$  and  $Q_3 = - 0.82 \text{ MJ}$ )

**Solution:** Let  $Q_2$  and  $Q_3$  both incoming i.e. out from the system

$\therefore Q_2 \rightarrow +ve, \quad Q_3 \rightarrow +ve$

$$(\Delta S)_{\text{univ}} = \frac{Q_3}{200} + \frac{Q_2}{300} + \frac{5000}{400} + (\Delta S)_{\text{H.E.}} + (\Delta S)_{\text{surrounds}} = 0$$



Or 
$$\frac{Q_3}{2} + \frac{Q_2}{3} + \frac{5000}{4} + 0 + 0 = 0$$

or 
$$6 Q_3 + 4 Q_2 + 3 \times 5000 = 0 \quad \dots (i)$$

$$Q_3 + Q_2 + 5000 - 840 = 0 \quad \dots (ii)$$

Heat balance

or 
$$4 Q_3 + 4 Q_2 + 16640 = 0 \quad \dots (iii)$$

$\therefore$  (i) – (iii) gives

$$2 Q_3 = +1640$$

$\therefore Q_3 = +820 \text{ kJ}$

(Here –ve sign means heat flow opposite to our assumption)

$\therefore Q_2 = -4980 \text{ kJ}$

**Q7.19**

For a fluid for which  $pv/T$  is a constant quantity equal to  $R$ , show that the change in specific entropy between two states  $A$  and  $B$  is given by

$$s_B - s_A = \int_{T_A}^{T_B} \left( \frac{C_p}{T} \right) dT - R \ln \frac{p_B}{p_A}$$

A fluid for which  $R$  is a constant and equal to  $0.287 \text{ kJ/kg K}$ , flows steadily through an adiabatic machine, entering and leaving through two adiabatic pipes. In one of these pipes the pressure and temperature are  $5 \text{ bar}$  and  $450 \text{ K}$  and in the other pipe the pressure and temperature are  $1 \text{ bar}$  and  $300 \text{ K}$  respectively. Determine which pressure and temperature refer to the inlet pipe.

(Ans.  $A$  is the inlet pipe)

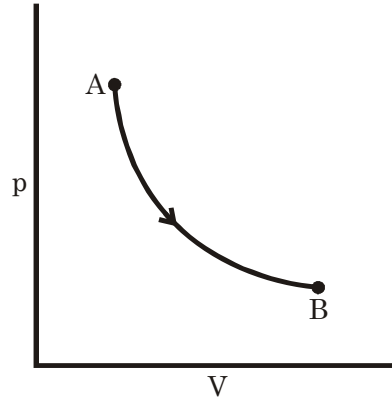
For the given temperature range,  $c_p$  is given by

$$C_p = a \ln T + b$$

Where  $T$  is the numerical value of the absolute temperature and  $a = 0.026 \text{ kJ/kg K}$ ,  $b = 0.86 \text{ kJ/kg K}$ .

(Ans.  $s_B - s_A = 0.0509 \text{ kJ/kg K}$ .  $A$  is the inlet pipe.)

Solution:



$$dS = \frac{C_v dT}{T} + \frac{R}{V} dV$$

$$\frac{pV}{T} = R$$

$$\therefore \frac{V}{T} = \frac{R}{p}$$

$$dQ = dH - Vdp$$

$$\text{or } TdS = dH - Vdp$$

$$\text{or } ds = \frac{C_p dT}{T} - \frac{Vdp}{T}$$

$$\text{or } ds = \frac{C_p dT}{T} - \frac{R}{p} dp$$

Integration both side with respect A to B

$$\int_{s_A}^{s_B} ds = \int_{T_A}^{T_B} \left( \frac{C_p}{T} \right) dT - R \int_{p_A}^{p_B} \frac{dp}{p}$$

$$\text{or } s_B - s_A = \left[ \int_{T_A}^{T_B} \left( \frac{C_p}{T} \right) dT - R \ln \left( \frac{p_B}{p_A} \right) \right] \text{ proved}$$

$$\begin{aligned} s_B - s_A &= \int_{450}^{300} \left( \frac{a \ln T + b}{T} \right) dT - 0.287 \times \ln \left( \frac{1}{5} \right) \\ &= \left[ a \frac{(\ln T)^2}{2} + b \ln T \right]_{450}^{300} - 0.287 \times \ln \left( \frac{1}{5} \right) \end{aligned}$$

$$s_B - s_A = \frac{a}{2} \{ (\ln 300)^2 - (\ln 450)^2 \} + b \ln \frac{300}{450} - 0.287 \ln \left( \frac{1}{5} \right)$$

$$\text{or } s_B - s_A = 0.05094 \text{ kJ/kg} - \text{K}$$

$\therefore$  A is the inlet of the pipe

**Q7.20**

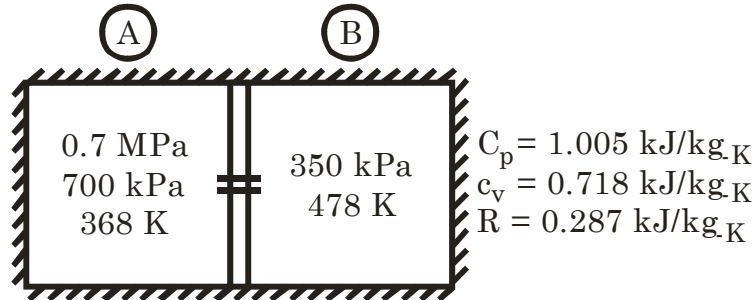
Two vessels, A and B, each of volume 3 m<sup>3</sup> may be connected by a tube of negligible volume. Vessel A contains air at 0.7 MPa, 95 °C, while vessel B contains air at 0.35 MPa, 205°C. Find the change of entropy when A is connected to B by working from the first principles and assuming the

mixing to be complete and adiabatic. For air take the relations as given in Example 7.8.

(Ans. 0.959 kJ/K)

**Solution:** Let the final temperature be ( $T_f$ )

$$\begin{aligned}\text{Mass of } (m_A) &= \frac{p_A V_A}{RT_A} \\ &= \frac{700 \times 3}{0.287 \times 368} \text{ kg} \\ &= 19.88335 \text{ kg}\end{aligned}$$



$$\text{Mass of gas } (m_B) = \frac{p_B V_B}{R T_B} = \frac{350 \times 3}{0.287 \times 478} = 7.653842 \text{ kg}$$

For adiabatic mixing of gas Internal Energy must be same

$$\begin{aligned}\therefore u_A &= m_A c_v T_A \\ &= 19.88335 \times 0.718 \times 368 \text{ kJ} = 5253.66 \text{ kJ} \\ u_B &= m_B c_v T_B \\ &= 7.653842 \times 0.718 \times 478 \text{ kJ} = 2626.83 \text{ kJ}\end{aligned}$$

$$U_{\text{mixture}} = (m_A c_v + m_B c_v) T_f$$

$$\text{Or } T_f = 398.6 \text{ K}$$

If final pressure ( $p_f$ )

$$\begin{aligned}\therefore p_f \times V_f &= m_f R T_f \\ \therefore p_f &= \frac{27.5372 \times 0.287 \times 398.6}{6} \text{ kPa} = 525 \text{ kPa}\end{aligned}$$

$$(\Delta S)_A = m_A \left[ c_p \ln \frac{T_f}{T_A} - R \ln \left( \frac{p_f}{p_A} \right) \right] = 3.3277$$

$$(\Delta S)_B = m_B \left[ c_p \ln \frac{T_f}{T_B} - R \ln \left( \frac{p_f}{p_B} \right) \right] = -2.28795 \text{ kJ/K}$$

$$\therefore (\Delta S)_{\text{univ}} = (\Delta S)_A + (\Delta S)_B + 0 = 0.9498 \text{ kJ/K}$$

**Q7.21**

(a) An aluminium block ( $c_p = 400 \text{ J/kg K}$ ) with a mass of 5 kg is initially at  $40^\circ\text{C}$  in room air at  $20^\circ\text{C}$ . It is cooled reversibly by transferring heat to a completely reversible cyclic heat engine until the block reaches  $20^\circ\text{C}$ . The  $20^\circ\text{C}$  room air serves as a constant temperature sink for the engine. Compute (i) the change in entropy for the block,

(ii) the change in entropy for the room air, (iii) the work done by the engine.

- (b) If the aluminium block is allowed to cool by natural convection to room air, compute (i) the change in entropy for the block, (ii) the change in entropy for the room air (iii) the net the change in entropy for the universe.

(Ans. (a)  $-134 \text{ J/K}$ ,  $+134 \text{ J/K}$ ,  $740 \text{ J}$ ;  
(b)  $-134 \text{ J/K}$ ,  $+136.5 \text{ J/K}$ ,  $2.5 \text{ J/K}$ )

**Solution:**

(a) 
$$(\Delta S)_{Al} = \int_{313}^{293} \frac{m c_p dT}{T}$$

$$5 \times 400 \times \ln \frac{293}{313} \text{ J/K} = -132.06 \text{ J/K}$$

$$(\Delta S)_{air} = \frac{Q - W}{293}$$

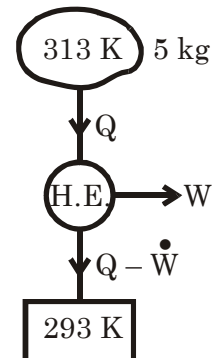
$$\text{And } Q = m c_p (313 - 293) = 40000 \text{ J}$$

As heat is reversibly flow then

$$(\Delta S)_{Al} + (\Delta S)_{air} = 0$$

$$\text{or } -132.06 + 136.52 - \frac{W}{293} = 0$$

$$\text{or } W = 1.306 \text{ kJ}$$



(b)  $(\Delta S)_{\Delta f} = \text{Same for reversible or irreversible} = -132.06 \text{ J/K}$

$$(\Delta S)_{air} = \frac{4000}{293} = 136.52 \text{ J/K}$$

$$(\Delta S)_{air} = +4.4587 \text{ J/K}$$

**Q7.22**

Two bodies of equal heat capacities  $C$  and temperatures  $T_1$  and  $T_2$  form an adiabatically closed system. What will the final temperature be if one lets this system come to equilibrium (a) freely? (b) Reversibly? (c) What is the maximum work which can be obtained from this system?

**Solution:**

(a) Freely  $T_f = \frac{T_1 + T_2}{2}$

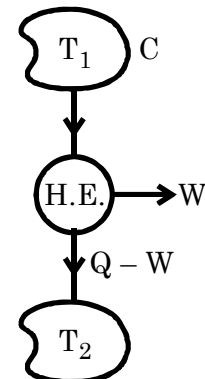
(b) Reversible

Let find temperature be  $T_f$

$$\begin{aligned} \text{the } (\Delta S)_{hot} &= \int_{T_1}^{T_f} C \frac{dT}{T} \\ &= C \ln \frac{T_f}{T_1} \end{aligned}$$

$$(\Delta S)_{cold} = \int_{T_2}^{T_f} C \frac{dT}{T} = C \ln \left( \frac{T_f}{T_2} \right)$$

$$\begin{aligned} \therefore (\Delta S)_{univ.} &= (\Delta S)_{hot} + (\Delta S)_{cold} = (\Delta S)_{surroundings} \\ &= C \ln \frac{T_f}{T_1} + C \ln \frac{T_f}{T_2} = 0 \end{aligned}$$



$$\begin{aligned}
 \text{or} \quad T_f &= \sqrt{T_1 T_2} \\
 \therefore Q &= C(T_1 - T_f) \\
 Q - W &= C(T_f - T_2) \\
 \hline
 W &= C(T_1 - T_f - T_f + T_2) \\
 &= C \{T_1 + T_2 - 2 T_f\} \\
 &= C[T_1 + T_2 - 2\sqrt{T_1 T_2}]
 \end{aligned}$$

**Q7.23** A resistor of 30 ohms is maintained at a constant temperature of 27°C while a current of 10 amperes is allowed to flow for 1 sec. Determine the entropy change of the resistor and the universe.

(Ans.  $(\Delta S)_{\text{resistor}} = 0$ ,  $(\Delta S)_{\text{univ}} = 10 \text{ J/K}$ )

If the resistor initially at 27°C is now insulated and the same current is passed for the same time, determine the entropy change of the resistor and the universe. The specific heat of the resistor is 0.9 kJ/kg K and the mass of the resistor is 10 g.

(Ans.  $(\Delta S)_{\text{univ}} = 6.72 \text{ J/K}$ )

**Solution:** As resistor is in steady state therefore no change in entropy. But the work = heat is dissipated to the atmosphere.

$$\begin{aligned}
 \text{So } (\Delta S)_{\text{atm}} &= \frac{i^2 R t}{T_{\text{atm}}} \\
 &= \frac{10^2 \times 30 \times 1}{300} = 10 \text{ kJ/kg}
 \end{aligned}$$

If the resistor is insulated then no heat flow to surroundings

$$\text{So } (\Delta S)_{\text{surroundings}} = 0$$

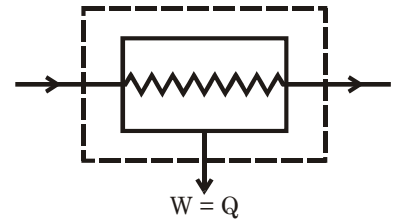
And, Temperature of resistance ( $\Delta t$ )

$$= \frac{10^2 \times 30 \times 1}{900 \times 0.01} = 333.33^\circ \text{C}$$

$$\therefore \text{Final temperature } (T_f) = 633.33 \text{ K}$$

Initial temperature ( $T_o$ ) = 300 K

$$\begin{aligned}
 \therefore (\Delta S) &= \int_{300}^{633.33} m c \frac{dT}{T} \\
 &= 0.01 \times 0.9 \times \ln \left( \frac{633.33}{300} \right) = 6.725 \text{ J/K} \\
 (\Delta S)_{\text{univ}} &= (\Delta S)_{\text{rev.}} = 6.725 \text{ J/K}
 \end{aligned}$$



**Q7.24** An adiabatic vessel contains 2 kg of water at 25°C. By paddle-wheel work transfer, the temperature of water is increased to 30°C. If the specific heat of water is assumed constant at 4.187 kJ/kg K, find the entropy change of the universe.

(Ans. 0.139 kJ/K)

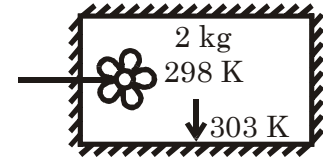
**Solution:**

$$(\Delta S)_{\text{surr.}} = 0$$

$$(\Delta S)_{\text{sys}} = \int_{298}^{303} m c \frac{dT}{T}$$

$$= 2 \times 4.187 \times \ln \frac{303}{298} = 0.13934 \text{ kJ/K}$$

$$\therefore (\Delta S)_{\text{univ}} = (\Delta S)_{\text{sys}} + (\Delta S)_{\text{surr}} = 0.13934 + 0 = 0.13934 \text{ kJ/K}$$

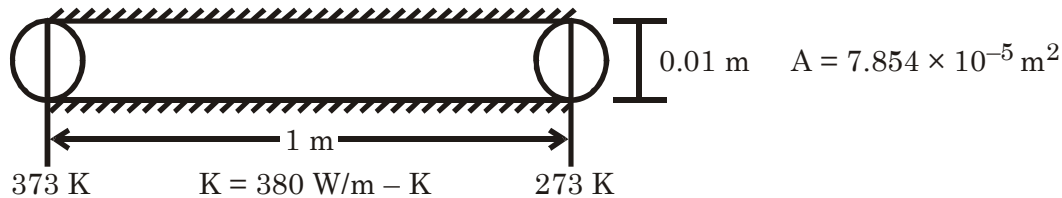


**Q7.25**

A copper rod is of length 1 m and diameter 0.01 m. One end of the rod is at 100°C, and the other at 0°C. The rod is perfectly insulated along its length and the thermal conductivity of copper is 380 W/mK. Calculate the rate of heat transfer along the rod and the rate of entropy production due to irreversibility of this heat transfer.

(Ans. 2.985 W, 0.00293 W/K)

**Solution:**



$$\begin{aligned} \dot{Q} &= kA \frac{\Delta T}{\Delta x} \\ &= 380 \times 7.854 \times 10^{-5} \times \frac{100}{1} \text{ W} = 2.9845 \text{ W} \end{aligned}$$

At the 373 K end from surrounding  $\dot{Q}$  amount heat is go to the system. So at this end

$$(\Delta \dot{S})_{\text{charge}} = -\frac{\dot{Q}}{373}$$

And at the 273 K and from system  $\dot{Q}$  amount of heat is rejected to the surroundings.

$$\therefore (\Delta \dot{S})_{\text{charge}} = \frac{\dot{Q}}{273}$$

$$\therefore (\Delta \dot{S})_{\text{univ.}} = \frac{\dot{Q}}{273} - \frac{\dot{Q}}{373} = 0.00293 \text{ W/K}$$

**Q7.26**

A body of constant heat capacity  $C_p$  and at a temperature  $T_i$  is put in contact with a reservoir at a higher temperature  $T_f$ . The pressure remains constant while the body comes to equilibrium with the reservoir. Show that the entropy change of the universe is equal to

$$C_p \left[ \frac{T_i - T_f}{T_f} - \ln \left( 1 + \frac{T_i - T_f}{T_f} \right) \right]$$

Prove that entropy change is positive.

Given  $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$  {where  $x < 1$ }

**Solution:**

Final temperature of the body will be  $T_f$

$$\therefore (\Delta S)_{\text{body}} = C_p \int_{T_i}^{T_f} \frac{dT}{T} = C_p \ln \left( \frac{T_f}{T_i} \right)$$

$$(\Delta S)_{\text{reservoir}} = \frac{C_p(T_f - T_i)}{T_f}$$

$\therefore$  Total entropy change

$$\begin{aligned} (\Delta S)_{\text{univ}} &= C_p \left[ \frac{T_f - T_i}{T_f} + \ln \frac{T_f}{T_i} \right] \\ &= C_p \left[ \frac{T_f - T_i}{T_f} - \ln \frac{T_i}{T_f} \right] \\ &= C_p \left[ \frac{T_f - T_i}{T_f} - \ln \left( 1 + \frac{T_i - T_f}{T_f} \right) \right] \end{aligned}$$

Let  $\frac{T_f - T_i}{T_f} = x$  as  $T_f > T_i$

$$\therefore \frac{T_f - T_i}{T_f} < 1$$

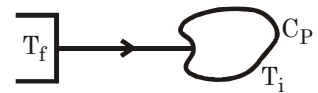
$$\therefore (\Delta S)_{\text{in}} = C_p \{x - \ln(1+x)\}$$

$$= C_p \left[ x - x + \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4} + \dots \alpha \right]$$

$$= C_p \left[ \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4} - \frac{x^5}{5} + \dots \alpha \right]$$

$$= C_p \left[ \frac{x^2(3-2x)}{6} + \frac{x^4(5-4x)}{20} + \dots \alpha \right]$$

$$\therefore (\Delta S)_{\text{univ}} \text{ is +ve}$$



**Q7.27**

An insulated 0.75 kg copper calorimeter can containing 0.2 kg water is in equilibrium at a temperature of 20°C. An experimenter now places 0.05 kg of ice at 0°C in the calorimeter and encloses the latter with a heat insulating shield.

- (a) When all the ice has melted and equilibrium has been reached, what will be the temperature of water and the can? The specific heat of copper is 0.418 kJ/kg K and the latent heat of fusion of ice is 333 kJ/kg.



- (b) Compute the entropy increase of the universe resulting from the process.  
 (c) What will be the minimum work needed by a stirrer to bring back the temperature of water to 20°C?

(Ans. (a) 4.68°C, (b) 0.00276 kJ/K, (c) 20.84 kJ)

**Solution:**

Mass of ice = 0.05 kg

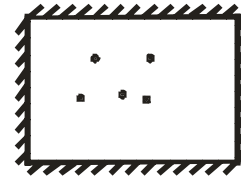
(a) Let final temperature be ( $T_f$ )

$$\begin{aligned} \therefore & 0.75 \times 0.418 \times (293 - T_f) \\ & + 0.2 \times 4.187 \times (293 - T_f) \\ = & 333 \times 0.05 + 0.05 \times 4.187 \\ & \times (T_f - 273) \end{aligned}$$

$$\begin{aligned} \text{or } & 1.1509(293 - T_f) \\ & = 16.65 - 57.15255 + 0.20935 T_f \end{aligned}$$

$$\text{or } 337.2137 - 1.1509 T_f$$

$$\text{or } T_f = 277.68 \text{ K} = 4.68^\circ \text{C}$$



$$\begin{aligned} W_{ab} &= 0.2 \text{ kg} \\ c_v &= 0.75 \text{ kJ/kg-K} \\ T_1 &= 293 \text{ K} \end{aligned}$$

(b)  $(\Delta S)_{\text{system}}$

$$\begin{aligned} = & 0.75 \times 0.418 \times \ln\left(\frac{T_f}{293}\right) + 0.2 \times 4.187 \times \ln\left(\frac{T_f}{293}\right) \\ & + \frac{333 \times 0.05}{273} + 0.05 \times 4.187 \ln\left(\frac{T_f}{273}\right) \\ = & 0.00275 \text{ kJ/K} = 2.75 \text{ J/K} \end{aligned}$$

(c) Work fully converted to heat so no Rejection.

$$\therefore W = C \times (20 - 4.68) = 20.84 \text{ kJ}$$

$$\therefore C = (\text{Heat capacity}) = 1.36025$$

**Q7.28**

Show that if two bodies of thermal capacities  $C_1$  and  $C_2$  at temperatures  $T_1$  and  $T_2$  are brought to the same temperature  $T$  by means of a reversible heat engine, then

$$\ln T = \frac{C_1 \ln T_1 + C_2 \ln T_2}{C_1 + C_2}$$

**Solution:**

$$(\Delta S)_1 = \int_{T_1}^T C_1 \frac{dT}{T} = C_1 \ln\left(\frac{T}{T_1}\right)$$

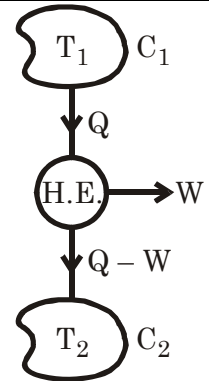
$$(\Delta S)_2 = \int_{T_2}^T C_2 \frac{dT}{T} = C_2 \ln\left(\frac{T}{T_2}\right)$$

$$(\Delta S)_{\text{univ}} = (\Delta S)_1 + (\Delta S)_2$$

For reversible process for an isolated system  $(\Delta S)$  since.

$$0 = C_1 \ln\left(\frac{T}{T_1}\right) + C_2 \ln\left(\frac{T}{T_2}\right)$$

$$\begin{aligned} \text{or} \quad & \left(\frac{T}{T_1}\right)^{C_1} \left(\frac{T}{T_2}\right)^{C_2} = 1 \\ \text{or} \quad & T^{C_1 + C_2} = T_1^{C_1} T_2^{C_2} \\ \text{or} \quad & (C_1 + C_2) \ln T = C_1 \ln T_1 + C_2 \ln T_2 \\ \text{or} \quad & \ln T = \frac{C_1 \ln T_1 + C_2 \ln T_2}{C_1 + C_2} \quad \text{Proved} \end{aligned}$$



**Q7.29** Two blocks of metal, each having a mass of 10 kg and a specific heat of 0.4 kJ/kg K, are at a temperature of 40°C. A reversible refrigerator receives heat from one block and rejects heat to the other. Calculate the work required to cause a temperature difference of 100°C between the two blocks.

**Solution:** Mass = 10 kg

$$C = 0.4 \text{ kJ/kg} - \text{K}$$

$$T = 40^\circ \text{C} = 313 \text{ K}$$

$$\therefore (\Delta S)_{\text{hot}} = mc \ln \left( \frac{T_f}{313} \right)$$

$$(\Delta S)_{\text{cold}} = mc \ln \left( \frac{T_f - 100}{313} \right)$$

For minimum work requirement process must be reversible  
so  $(\Delta S)_{\text{univ}} = 0$

$$\therefore \ln \frac{T_f(T_f - 100)}{(313)^2} = 0 = \ln 1$$

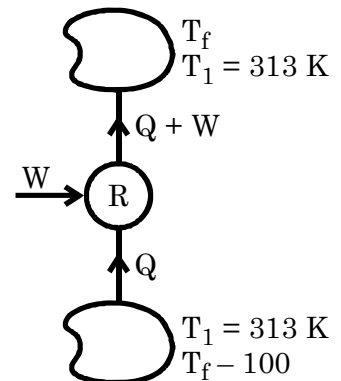
$$\text{or} \quad T_f^2 - 100 T_f - 313^2 = 0$$

$$\begin{aligned} \text{or} \quad T_f &= \frac{100 \pm \sqrt{100^2 + 4 \times 313^2}}{2} \\ &= 367 \text{ K or } (-267) \end{aligned}$$

$$\therefore Q + W = 10 \times 10.4 \times (367 - 313) = 215.87 \text{ kJ}$$

$$Q = 10 \times 0.4 \times (313 - 267) = 184 \text{ kJ}$$

$$\therefore W_{\text{min}} = 31.87 \text{ kJ}$$



**Q7.30** A body of finite mass is originally at a temperature  $T_1$ , which is higher than that of a heat reservoir at a temperature  $T_2$ . An engine operates in infinitesimal cycles between the body and the reservoir until it lowers the temperature of the body from  $T_1$  to  $T_2$ . In this process there is a heat flow  $Q$  out of the body. Prove that the maximum work obtainable from the engine is  $Q + T_2 (S_1 - S_2)$ , where  $S_1 - S_2$  is the decrease in entropy of the body.

**Solution:** Try please.

**Q7.31** A block of iron weighing 100 kg and having a temperature of 100°C is immersed in 50 kg of water at a temperature of 20°C. What will be the change of entropy of the combined system of iron and water? Specific heats of iron and water are 0.45 and 4.18 kJ/kg K respectively.

(Ans. 1.1328 kJ/K)

**Solution:** Let final temperature is  $t_f$  °C

$$\therefore 100 \times 0.45 \times (100 - t_f) = 50 \times 4.18 \times (t_f - 20)$$

$$100 - t_f = 4.644 t_f - 20 \times 4.699$$

$$\text{or } 5.644 t_f = 192.88$$

$$\text{or } t_f = 34.1732^\circ \text{ C}$$

$$\therefore t_f = 307.1732 \text{ K}$$

$$\text{ENTROPY} = (\Delta S)_{\text{iron}} + (\Delta S)_{\text{water}}$$

$$= 100 \times 0.45 \ln \left( \frac{307.1732}{373} \right) + 50 \times 4.180 \times \ln \left( \frac{307.1732}{293} \right)$$

$$= 1.1355 \text{ kJ/K}$$

**Q7.32** 36 g of water at 30°C are converted into steam at 250°C at constant atmospheric pressure. The specific heat of water is assumed constant at 4.2 J/g K and the latent heat of vaporization at 100°C is 2260 J/g. For water vapour, assume  $pV = mRT$  where  $R = 0.4619 \text{ kJ/kg K}$ , and

$$\frac{C_p}{R} = a + bT + cT^2, \text{ where } a = 3.634,$$

$$b = 1.195 \times 10^{-3} \text{ K}^{-1} \text{ and } c = 0.135 \times 10^{-6} \text{ K}^{-2}$$

Calculate the entropy change of the system.

(Ans. 277.8 J/K)

**Solution:**

$$m = 36 \text{ g} = 0.036 \text{ kg}$$

$$T_1 = 30^\circ\text{C} = 303 \text{ K}$$

$$T_2 = 373 \text{ K}$$

$$T_3 = 523 \text{ K}$$

$$(\Delta S)_{\text{Water}}$$

$$= m c_p \ln \left( \frac{373}{303} \right) \text{ kJ/K}$$

$$= 0.03143 \text{ kJ/K}$$

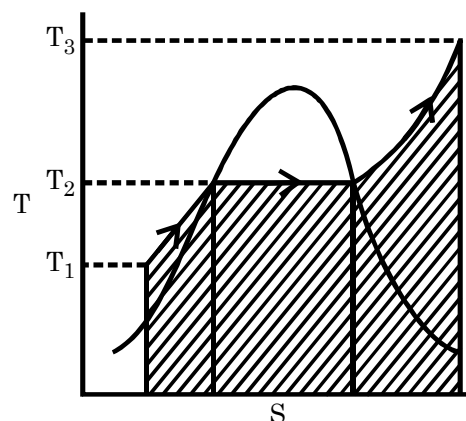
$$(\Delta S)_{\text{Vaporization}} = \frac{mL}{T_2}$$

$$= \frac{0.036 \times 2260}{373}$$

$$= 0.21812 \text{ kJ/K}$$

$$(\Delta S)_{\text{Vapor}} = \int_{373}^{523} m c_p \frac{dT}{T}$$

$$= mR \int_{373}^{523} \left( \frac{a}{T} + b + cT \right) dT$$



$$= mR \left[ a \ln T + bT + \frac{CT^2}{2} \right]_{373}^{523}$$

$$= mR \left[ a \ln \frac{523}{373} + b \times (523 - 373) + \frac{C}{2} (523^2 - 373^2) \right]$$

$$= 0.023556 \text{ kJ/kg}$$

$$(\Delta S)_{\text{System}} = (\Delta S)_{\text{water}} + (\Delta S)_{\text{vaporization}} + (\Delta S)_{\text{vapor}} = 273.1 \text{ J/K}$$

- Q7.33**      A 50 ohm resistor carrying a constant current of 1 A is kept at a constant temperature of 27°C by a stream of cooling water. In a time interval of 1s  
 (a) What is the change in entropy of the resistor?  
 (b) What is the change in entropy of the universe?

(Ans. (a) 0, (b) 0.167 J/K)

**Solution:**      Try please.

- Q7.34**      A lump of ice with a mass of 1.5 kg at an initial temperature of 260 K melts at the pressure of 1 bar as a result of heat transfer from the environment. After some time has elapsed the resulting water attains the temperature of the environment, 293 K. Calculate the entropy production associated with this process. The latent heat of fusion of ice is 333.4 kJ/kg, the specific heat of ice and water are 2.07 and 4.2 kJ/kg K respectively, and ice melts at 273.15 K.

(Ans. 0.1514 kJ/K)

**Solution:**      Try please.

- Q7.35**      An ideal gas is compressed reversibly and adiabatically from state a to state b. It is then heated reversibly at constant volume to state c. After expanding reversibly and adiabatically to state d such that  $T_b = T_d$ , the gas is again reversibly heated at constant pressure to state e such that  $T_e = T_c$ . Heat is then rejected reversibly from the gas at constant volume till it returns to state a. Express  $T_a$  in terms of  $T_b$  and  $T_c$ . If  $T_b = 555 \text{ K}$  and  $T_c = 835 \text{ K}$ , estimate  $T_a$ . Take  $\gamma = 1.4$ .

$$\left( \text{Ans. } T_a = \frac{T_b^{\gamma+1}}{T_c^{\gamma}}, 313.29 \text{ K} \right)$$

**Solution:**

$$(\Delta S)_{bc} = C_v \ln \left( \frac{T_c}{T_b} \right)$$

$$(\Delta S)_{de} = C_p \ln \left( \frac{T_e}{T_b} \right)$$

$$(\Delta S)_{ea} = C_v \ln \left( \frac{T_a}{T_c} \right)$$

$$(\Delta S)_{\text{Cycles}} = 0$$

$$\therefore (C_p + C_v) \ln \left( \frac{T_c}{T_b} \right) + C_v \ln \left( \frac{T_a}{T_c} \right) = 0$$

$$\text{or } (\gamma + 1) \ln \left( \frac{T_c}{T_b} \right) + \ln \left( \frac{T_a}{T_c} \right) = \ln 1$$

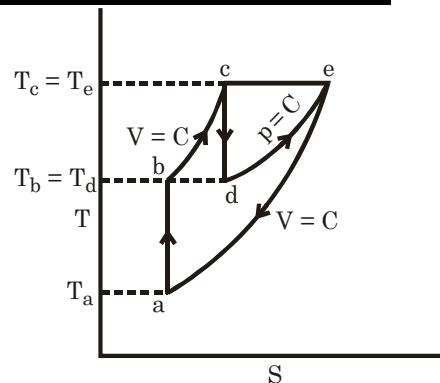
$$\text{or } \left( \frac{T_c}{T_b} \right)^{\gamma+1} \left( \frac{T_a}{T_c} \right) = 1$$

$$\text{Or } (T_c)^\gamma \cdot T_a = T_b^{\gamma+1}$$

$$\therefore T_a = \frac{T_b^{\gamma+1}}{T_c^\gamma}$$

Given  $T_b = 555 \text{ K}$ ,  $T_c = 835 \text{ K}$ ,  $\gamma = 1.4$  + Gas

$$T_a = \frac{(555)^{1.4+1}}{835^{1.4}} = 313.286 \text{ K}$$



**Q7.36** Liquid water of mass 10 kg and temperature 20°C is mixed with 2 kg of ice at - 5°C till equilibrium is reached at 1 atm pressure. Find the entropy change of the system. Given:  $c_p$  of water = 4.18 kJ/kg K,  $c_p$  of ice = 2.09 kJ/kg K and latent heat of fusion of ice = 334 kJ/kg.

(Ans. 190 J/K)

**Solution:** Try please.

**Q7.37** A thermally insulated 50-ohm resistor carries a current of 1 A for 1 s. The initial temperature of the resistor is 10°C. Its mass is 5 g and its specific heat is 0.85 J g K.

(a) What is the change in entropy of the resistor?

(b) What is the change in entropy of the universe?

(Ans. (a) 0.173 J/K (b) 0.173 J/K)

**Solution:** Try please.

**Q7.38** The value of  $c_p$  for a certain substance can be represented by  $c_p = a + bT$ .

(a) Determine the heat absorbed and the increase in entropy of a mass  $m$  of the substance when its temperature is increased at constant pressure from  $T_1$  to  $T_2$ .

(b) Find the increase in the molal specific entropy of copper, when the temperature is increased at constant pressure from 500 to 1200 K. Given for copper: when  $T = 500 \text{ K}$ ,  $c_p = 25.2 \times 10^3$  and when  $T = 1200 \text{ K}$ ,  $c_p = 30.1 \times 10^3 \text{ J/k mol K}$ .

$$\text{Ans. } \left( \text{(a) } m \left[ a(T_2 - T_1) + \frac{b}{2}(T_2^2 - T_1^2) \right], m \left[ a \ln \frac{T_2}{T_1} + b(T_2 - T_1) \right] \right);$$

(b) 24.7 kJ/k mol K

**Solution:**  $dQ = C_p dT$

$$\therefore Q = m \int_{T_1}^{T_2} c_p dT$$

$$= m \int_{T_1}^{T_2} (a + bT) dT = m \left[ aT + \frac{bT^2}{2} \right]_{T_1}^{T_2}$$

$$= m \left[ a(T_2 - T_1) + \frac{b}{2}(T_2^2 - T_1^2) \right]$$

$$TdS = C_p dT$$

$$\text{or} \quad dS = m c_p \frac{dT}{T}$$

$$\text{or} \quad \int_{S_1}^{S_2} dS = m \int_1^2 c_p \frac{dT}{T} = m \int_{T_1}^{T_2} \frac{(a + bT)}{T} dT$$

$$(S_2 - S_1) = [a \ln T + bT]_{T_1}^{T_2} = m \left[ a \ln \frac{T_2}{T_1} + b(T_2 - T_1) \right]$$

$\Rightarrow$  For a and b find

$$25.2 = a + b \times 500$$

$$30.1 = a + b \times 1200$$

$$\therefore b \times 700 = 4.9 \quad \therefore b = 0.007 \text{ kJ/kg K} \quad \therefore a = 21.7 \text{ kJ/kg - K}$$

$$\therefore S_2 - S_1 = \left[ 21.7 \ln \left( \frac{1200}{500} \right) + 0.007 (1200 - 500) \right] \text{ kJ/K} = 23.9 \text{ kJ/K}$$

8.

# Availability & Irreversibility

## Some Important Notes

### 1. Available Energy (A.E.)

$$W_{\max} = Q_1 \left( 1 - \frac{T_0}{T_1} \right) = m c_p \int_{T_0}^T \left( 1 - \frac{T_0}{T} \right) dT$$

$$= (T_1 - T_0) \Delta S$$

$$= u_1 - u_2 - T_0 (s_1 - s_2)$$

(For closed system), it is not  $(\phi_1 - \phi_2)$  because change of volume is present there.

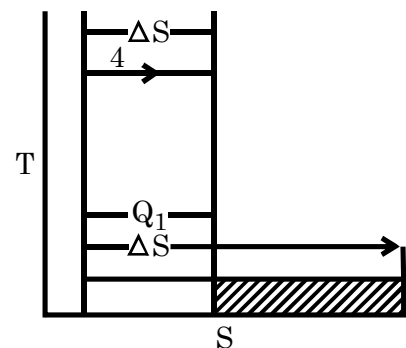
$$= h_1 - h_2 - T_0 (s_1 - s_2)$$

(For steady flow system), it is  $(A_1 - A_2)$  as in steady state no change in volume is CONSTANT VOLUME (i.e. change in availability in steady flow)

### 2. Decrease in Available Energy

$$= T_0 [\Delta S' - \Delta S]$$

Take  $\Delta S'$  &  $\Delta S$  both +Ve Quantity



### 3. Availability function:

$$A = h - T_0 s + \frac{V^2}{2} + gZ$$

Availability = maximum useful work

**For steady flow**

$$\text{Availability} = A_1 - A_0 = (h_1 - h_0) - T_0 (s_1 - s_0) + \frac{V_1^2}{2} + gZ \quad (\because V_0 = 0, Z_0 = 0)$$

$$\phi = u - T_0 s + p_0 v$$

**For closed system**

$$\text{Availability} = \phi_1 - \phi_0 = u_1 - u_0 - T_0 (s_1 - s_0) + p_0 (V_1 - V_0)$$

Available energy is maximum work obtainable not USEFULWORK.

### 4. Unavailable Energy (U.E.)

$$= T_0 (S_1 - S_2)$$

### 5. Increase in unavailable Energy = Loss in availability

$$= T_0 (\Delta S)_{\text{univ.}}$$

### 6. Irreversibility

$$I = W_{\max} - W_{\text{actual}} \\ = T_0(\Delta S)_{\text{univ.}}$$

### 7. Irreversibility rate = $\dot{I}$ rate of energy degradation

$$S_{\text{gen}} = \int_1^2 \dot{m} dS \\ = \text{rate of energy loss } (\dot{W}_{\text{lost}}) \\ = T_0 \times \dot{S}_{\text{gen}} \quad \text{for all processes}$$

### 8. $W_{\text{actual}} \Rightarrow dQ = du + dW_{\text{act}}$ this for closed system

$$h_1 + \frac{V_1^2}{2} + gZ_1 + \frac{dQ}{dm} = h_2 + \frac{V_2^2}{2} + gZ_2 + \frac{dW_{\text{act}}}{dm} \quad \text{this for steady flow}$$

### 9. Helmholtz function, $F = U - TS$

### 10. Gibb's function, $G = H - TS$

### 11. Entropy Generation number ( $N_s$ ) = $\frac{\dot{S}_{\text{gen}}}{\dot{m} c_p}$

### 12. Second law efficiency

$$\eta_{II} = \frac{\text{Minimum exergy intake to perform the given task } (X_{\min})}{\text{Actual exergy intake to perform the given task } (X)} = \eta_1 / \eta_{\text{Carnot}}$$

$X_{\min} = W$ , if work is involved

$$= Q \left( 1 - \frac{T_0}{T} \right) \quad \text{if Heat is involved.}$$

### 13. To Calculate $dS$

$$\text{i) Use } S_2 - S_1 = m \left[ c_v \ln \frac{p_2}{p_1} + c_p \ln \frac{V_2}{V_1} \right]$$

For closed system

$$TdS = dU + pdV$$

$$\text{or} \quad dS = m c_v \frac{dT}{T} + \frac{p}{T} dV$$

$$= m c_v \frac{dT}{T} + mR \frac{dV}{V}$$

$$\int_1^2 dS = m c_v \int_1^2 \frac{dT}{T} + mR \int_1^2 \frac{dV}{V}$$

For steady flow system

$$TdS = dH - Vdp$$

$$\text{or} \quad dS = m c_p \frac{dT}{T} - \frac{V}{T} dp$$

$$pV = mRT$$

$$\int_1^2 dS = m c_p \int_1^2 \frac{dT}{T} - mR \int_1^2 \frac{dp}{p}$$

$$\frac{V}{T} = \frac{mR}{p}$$

But Note that



$$\begin{aligned} \text{And } TdS &= dU + pdV \\ TdS &= dH - Vdp \end{aligned}$$

Both valid for closed system only

**14. In Pipe Flow Entropy generation rate**



Due to lack of insulation it may be  
 $T_1 > T_2$  for hot fluid  $T_1 < T_2$  for cold fluid

$$\begin{aligned} \dot{S}_{\text{gen}} &= \dot{S}_{\text{sys}} - \frac{\dot{Q}}{T_0} \\ &= \dot{m}(S_2 - S_1) - \frac{\dot{m} c_p (T_2 - T_1)}{T_0} \end{aligned}$$

$$\therefore \text{Rate of Irreversibility } (\dot{I}) = T_0 \dot{S}_{\text{gen}}$$

**15. Flow with friction**

$$\text{Decrease in availability} = \dot{m} R T_0 \times \frac{\Delta p}{p_1}$$

Questions with Solution P. K. Nag

- Q8.1** What is the maximum useful work which can be obtained when 100 kJ are abstracted from a heat reservoir at 675 K in an environment at 288 K? What is the loss of useful work if
- A temperature drop of 50°C is introduced between the heat source and the heat engine, on the one hand, and the heat engine and the heat sink, on the other
  - The source temperature drops by 50°C and the sink temperature rises by 50°C during the heat transfer process according to the linear law  $\frac{dQ}{dT} = \pm \text{constant}$ ?

(Ans. (a) 11.2 kJ, (b) 5.25 kJ)

**Solution:**

Entropy change for this process

$$\Delta S = \frac{-100}{675} \text{ kJ/K}$$

$$= 0.14815 \text{ kJ/K}$$

$$W_{\max} = (T - T_0) \Delta S$$

$$= (675 - 288) \Delta S = 57.333 \text{ kJ}$$

(a) Now maximum work obtainable

$$W'_{\max} = 100 \left( 1 - \frac{338}{625} \right)$$

$$= 45.92 \text{ kJ}$$

$$\therefore \text{Loss of available work} = 57.333 - 45.92$$

$$= 11.413 \text{ kJ}$$

(b) Given  $\frac{dQ}{dT} = \pm \text{constant}$

$$\text{Let } dQ = \pm m c_p dT$$

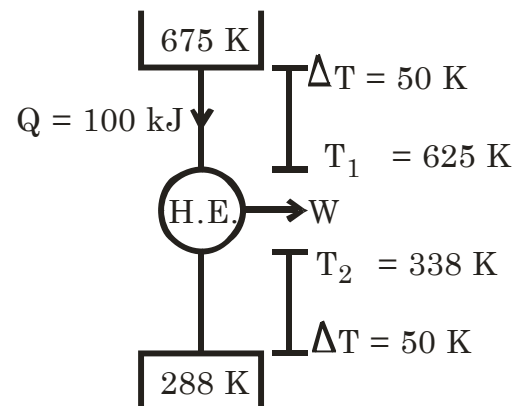
$\therefore$  When source temperature is  $(675 - T)$  and since temperature  $(288 + T)$  at that time if  $dQ$  heat is flow then maximum. Available work from that  $dQ$  is  $dW$ .

$$\therefore dW_{\max} = dQ \left( 1 - \frac{288 + T}{675 - T} \right)$$

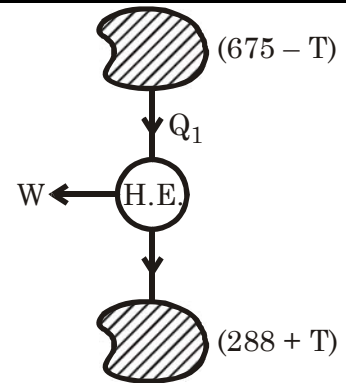
$$= \left( 1 - \frac{288 + T}{675 - T} \right) m c_p dT$$

$$\therefore W_{\max} = m c_p \int_0^{50} \left( 1 - \frac{288 + T}{675 - T} \right) dT$$

$$\left\{ \frac{-288 - T}{675 - T} = \frac{-963 + 675 - T}{675 - T} \right\}$$



$$\begin{aligned}
 &= m c_p \int_0^{50} \left\{ 1 + 1 - \frac{963}{675 - T} \right\} dT \\
 &= m c_p \left\{ 2(50 - 0) + 963 \ln \left( \frac{675 - 50}{675 - 0} \right) \right\} \\
 &= 25.887 m c_p \text{ kJ} \\
 m c_p \times 50 &= 100 \text{ kJ} \\
 &= 51.773 \text{ kJ} \\
 \therefore m c_p &= 2 \text{ kJ/K} \\
 \therefore \text{Loss of availability} &= (57.333 - 51.773) \text{ kJ} \\
 &= 5.5603 \text{ kJ}
 \end{aligned}$$



**Q 8.2** In a steam generator, water is evaporated at 260°C, while the combustion gas ( $c_p = 1.08 \text{ kJ/kg K}$ ) is cooled from 1300°C to 320°C. The surroundings are at 30°C. Determine the loss in available energy due to the above heat transfer per kg of water evaporated (Latent heat of vaporization of water at 260°C = 1662.5 kJ/kg).

(Ans. 443.6 kJ)

**Solution:** Availability decrease of gas

$$\begin{aligned}
 A_{\text{gas}} &= h_1 - h_2 - T_0 (s_1 - s_2) \\
 &= m c_p (T_1 - T_2) - T_0 m c_p \ln \left( \frac{T_1}{T_2} \right) \\
 &= m c_p \left[ (T_1 - T_2) - T_0 \ln \frac{T_1}{T_2} \right] \\
 \therefore T_1 &= 1573 \text{ K}; T_2 = 593 \text{ K}; T_0 = 303 \text{ K} \\
 &= m \times 739.16 \text{ kJ}
 \end{aligned}$$

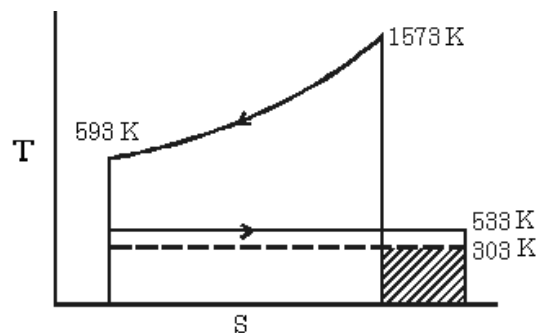
Availability increase of water

$$\begin{aligned}
 A_w &= (T_1 - T_0) \Delta S \\
 &= (T_1 - T_0) \times \frac{mL}{T_1} \\
 &= 1 \times 1662.5 \left\{ 1 - \frac{303}{533} \right\} \\
 &= 717.4 \text{ kJ}
 \end{aligned}$$

For mass flow rate of gas (m)

$$\begin{aligned}
 m_g c_p (T_2 - T_1) &= m_w \times L \\
 \therefore m_g \times 1.08 \times (1300 - 320) &= 1 \times 1662.5 \\
 \dot{m}_g &= 1.5708 \text{ kg/ of water of evaporator} \\
 A_{\text{gas}} &= 1161.1 \text{ kJ}
 \end{aligned}$$

$$\begin{aligned}
 \text{Loss of availability} &= \dot{A}_{\text{gas}} - A_w \\
 &= (1161.1 - 717.4) \text{ kJ} \\
 &= 443.7 \text{ kJ}
 \end{aligned}$$



**Q 8.3** Exhaust gases leave an internal combustion engine at 800°C and 1 atm, after having done 1050 kJ of work per kg of gas in the engine ( $c_p$  of gas = 1.1 kJ/kg K). The temperature of the surroundings is 30°C.

(a) How much available energy per kg of gas is lost by throwing away the exhaust gases?

(b) What is the ratio of the lost available energy to the engine work?

(Ans. (a) 425.58 kJ, (b) 0.405)

**Solution:** Loss of availability

$$\begin{aligned} (a) &= \int_{303}^{1073} m c_p dT \left( 1 - \frac{T_0}{T} \right) \\ &= 1 \times 1.1 \left\{ (1073 - 303) - 303 \ln \left( \frac{1073}{303} \right) \right\} \\ &= 425.55 \text{ kJ} \\ (b) \quad r &= \frac{425.55}{1050} = 0.40528 \end{aligned}$$

**Q 8.4** A hot spring produces water at a temperature of 56°C. The water flows into a large lake, with a mean temperature of 14°C, at a rate of 0.1 m<sup>3</sup> of water per min. What is the rate of working of an ideal heat engine which uses all the available energy?

(Ans. 19.5 kW)

**Solution:** Maximum work obtainable

$$\begin{aligned} W_{\max} &= \int_{287}^{329} \dot{m} c_p \left( 1 - \frac{287}{T} \right) dT \\ &= \dot{V} \rho c_p \left\{ (329 - 287) - 287 \ln \frac{329}{287} \right\} \\ &= \frac{0.1}{60} \times 1000 \times 4.187 \left\{ (329 - 287) - 287 \ln \frac{329}{287} \right\} \text{ kW} \\ &= 19.559 \text{ kW} \end{aligned}$$

**Q8.5** 0.2 kg of air at 300°C is heated reversibly at constant pressure to 2066 K. Find the available and unavailable energies of the heat added. Take  $T_0 = 30^\circ\text{C}$  and  $c_p = 1.0047 \text{ kJ/kg K}$ .

(Ans. 211.9 and 78.1 kJ)

**Solution:** Entropy increase

$$\Delta S = S_2 - S_1 = \int_{573}^{2066} m c_p \frac{dT}{T} = 0.2 \times 1.0047 \times \ln \frac{2066}{573} = 0.2577 \text{ kJ/K}$$

**Availability increases**

$$\begin{aligned} A_{\text{increase}} &= h_2 - h_1 - T_0 (s_2 - s_1) \\ &= m c_p (T_2 - T_1) - T_0 \times 0.2577 \\ &= 1250.24 - 78.084 \\ &= 1172.2 \text{ kJ} \end{aligned}$$

$$\text{Heat input} = m c_p (T_2 - T_1) = 1250.24 \text{ kJ}$$

$$\text{Unavailable entropy} = 78.086 \text{ kJ}$$

**Q8.6** Eighty kg of water at 100°C are mixed with 50 kg of water at 60°C, while the temperature of the surroundings is 15°C. Determine the decrease in available energy due to mixing.

**Solution:**

$m_1 = 80 \text{ kg}$	$m_2 = 50 \text{ kg}$
$T_1 = 100^\circ = 373 \text{ K}$	$T_2 = 60^\circ \text{C} = 333 \text{ K}$
$T_0 = 288 \text{ K}$	

Availability decrease of 80 kg

$$\begin{aligned} A_{\text{dec}} &= \int_{357.62}^{373} m c_p dT \left( 1 - \frac{T_0}{T} \right) \\ &= m c_p \left[ (373 - 357.62) - 288 \ln \left( \frac{373}{357.62} \right) \right] \\ &= 1088.4 \text{ kJ} \end{aligned}$$

$$\begin{aligned} A_{\text{in}} &= \int_{333}^{357.62} m c_p \left( 1 - \frac{T_0}{T} \right) dT \\ &= m c_p \left[ (357.62 - 333) - 288 \ln \left( \frac{357.62}{333} \right) \right] \\ &= 853.6 \text{ kJ} \end{aligned}$$

$$\begin{aligned} \therefore \text{Availability loss due to mixing} &= (1088.4 - 853.6) \text{ kJ} \\ &= 234.8 \text{ kJ} \end{aligned}$$

**Q8.7** A lead storage battery used in an automobile is able to deliver 5.2 MJ of electrical energy. This energy is available for starting the car.

**Solution:** Electrical Energy is high Grade Energy so full energy is available

$$\begin{aligned} \therefore A_{\text{electric}} &= 5.2 \text{ MJ} = 5200 \text{ kJ} \\ \text{Availability of compressed air} \\ A_{\text{air}} &= u_1 - u_0 - T_0(s_1 - s_0) \\ &= m c_v (T_1 - T_0) - T_0 (s_1 - s_0) \\ (s_1 - s_0) &= c_v \ln \frac{p_1}{p_0} + c_p \ln \frac{v_1}{v_0} = c_p \ln \frac{T_1}{T_0} - R \ln \frac{p_1}{p_0} \end{aligned}$$

Here  $T_1 = T_0 = 25^\circ \text{ C} = 298 \text{ K}$

Let atm  $p_r = 1 \text{ bar} = 100 \text{ kPa}$

Given  $p_1 = 7 \text{ MPa} = 7000 \text{ kPa}$

$$\therefore \text{ Required mass of air} = \frac{5200}{363.36} \text{ kg} = 14.311 \text{ kg}$$

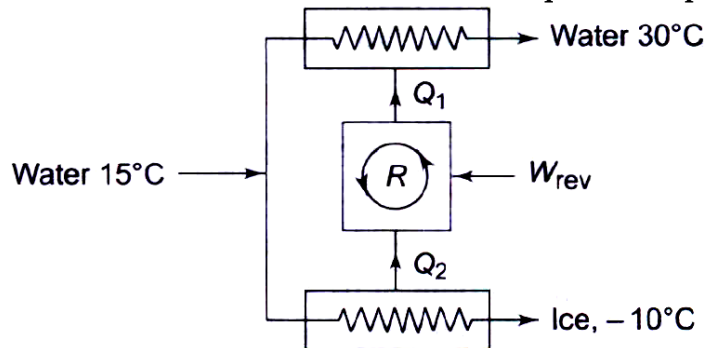
Specific volume of air at 7 MPa, 25°C then

$$v = \frac{RT}{p} = \frac{0.287 \times 298}{7000} \text{ m}^3/\text{kg} = 0.012218 \text{ m}^3/\text{kg}$$

$$\therefore \text{ Required storage volume (V)} = 0.17485 \text{ m}^3$$

Q8.8

Ice is to be made from water supplied at 15°C by the process shown in Figure. The final temperature of the ice is -10°C, and the final temperature of the water that is used as cooling water in the condenser is 30°C. Determine the minimum work required to produce 1000 kg of ice.



Take  $c_p$  for water = 4.187 kJ/kg K,  $c_p$  for ice = 2.093 kJ/kg K, and latent heat of fusion of ice = 334 kJ/kg.

(Ans. 33.37 MJ)

**Solution:**

Let us assume that heat rejection temperature is ( $T_0$ )

(i) Then for 15°C water to 0° C water if we need  $W_R$  work minimum.

$$\text{Then (COP)} = \frac{Q_2}{W_R} = \frac{T_2}{T_0 - T_2}$$

$$\text{or } W_R = Q_2 \frac{(T_0 - T_2)}{T_2}$$

$$= Q_2 \left( \frac{T_0}{T_2} - 1 \right)$$

When temperature of water is

T if change is dT

$$\text{Then } dQ_2 = -mc_p dT$$

(heat rejection so -ve)

$$\therefore dW_R = -mc_p dT \left( \frac{T_0}{T} - 1 \right)$$

$$\therefore W_{R1} = -mc_p \int_{288}^{273} \left( \frac{T_0}{T} - 1 \right) dT$$

$$= mc_p \left[ T_0 \ln \frac{288}{273} - (288 - 273) \right]$$

$$= 4187 \left[ T_0 \ln \frac{288}{273} - 15 \right] \text{ kJ}$$

(ii)  $W_R$  required for 0° C water to 0 ° C ice

$$\begin{aligned}
 W_{R_{II}} &= Q_2 \left( \frac{T_0}{T_2} - 1 \right) \\
 &= mL \left( \frac{T_0}{T_2} - 1 \right) \\
 &= 1000 \times 335 \left( \frac{T_0}{273} - 1 \right) \\
 &= 335000 \left( \frac{T_0}{273} - 1 \right) \text{ kJ}
 \end{aligned}$$

(iii)  $W_R$  required for  $0^\circ \text{C}$  ice to  $-10^\circ \text{C}$  ice.

When temperature is  $T$  if  $dT$  temperature decreases

$$\therefore dQ_2 = -m c_{p, \text{ice}} dT$$

$$\therefore dW_R = -m c_{p, \text{ice}} dT \left( \frac{T_0}{T} - 1 \right)$$

$$\therefore W_{R_{II}} = m c_{p, \text{ice}} \int_{263}^{273} \left( \frac{T_0}{T} - 1 \right) dT = m c_{p, \text{ice}} \left[ T_0 \ln \frac{273}{263} - (273 - 263) \right]$$

$$\begin{aligned}
 \text{Let } c_{p, \text{ice}} &= \frac{1}{2} c_{p, \text{water}} = \frac{4.187}{2} \text{ kJ/kg} \\
 &= 1000 \times \frac{4.187}{2} \left[ T_0 \ln \frac{273}{263} - 10 \right] \\
 &= 2093.5 \left[ T_0 \ln \frac{273}{263} - 10 \right] \text{ kJ}
 \end{aligned}$$

$\therefore$  Total work required

$$\begin{aligned}
 W_R &= (i) + (ii) + (iii) \\
 &= [1529.2 T_0 - 418740] \text{ kJ}
 \end{aligned}$$

$\therefore W_R$  and  $T_0$  has linear relationship

$$\therefore T_0 = \frac{15 + 30}{2} ^\circ\text{C} = 22.5^\circ\text{C} = 295.5 \text{ K}$$

$$\therefore W_R = 33138.6 \text{ kJ} = 33.139 \text{ MJ}$$

**Q8.9**

**A pressure vessel has a volume of  $1 \text{ m}^3$  and contains air at  $1.4 \text{ MPa}$ ,  $175^\circ\text{C}$ . The air is cooled to  $25^\circ\text{C}$  by heat transfer to the surroundings at  $25^\circ\text{C}$ . Calculate the availability in the initial and final states and the irreversibility of this process. Take  $p_0 = 100 \text{ kPa}$ .**

(Ans.  $135 \text{ kJ/kg}$ ,  $114.6 \text{ kJ/kg}$ ,  $222 \text{ kJ}$ )

**Solution:**

Given  $T_i = 175^\circ\text{C} = 448 \text{ K}$

$V_i = 1 \text{ m}^3$

$p_i = 1.4 \text{ MPa} = 1400 \text{ kPa}$

$T_f = 25^\circ\text{C} = 298 \text{ K}$

$V_f = 1 \text{ m}^3$

$p_f = 931.25 \text{ kPa}$

**Calculated Data:**

$p_0 = 101.325 \text{ kPa}$ ,  $T_0 = 298 \text{ K}$

$c_p = 1.005 \text{ kJ/kg} - \text{K}$ ,  $c_v = 0.718 \text{ kJ/kg} - \text{K}$ ;  $R = 0.287 \text{ kJ/kg} - \text{K}$

$$\therefore \text{Mass of air (m)} = \frac{p_i V_i}{RT_i} = \frac{1400 \times 1}{0.287 \times 448} = 10.8885 \text{ kg}$$

$$\therefore \text{Final volume } (V_0) = \frac{mRT_0}{p_0} = \frac{10.8885 \times 0.287 \times 298}{101.325} = 9.1907 \text{ m}^3$$

$\therefore$  Initial availability

$$\begin{aligned} A_i &= \phi_1 - \phi_0 \\ &= u_1 - u_0 - T_0 (s_1 - s_0) + p_0 (V_1 - V_0) \\ &= mc_v(T_1 - T_0) - T_0 \left\{ mc_p \ln \frac{V_1}{V_0} + mc_v \ln \frac{p_1}{p_0} \right\} + p_0(V_1 - V_0) \\ &= m \left[ 0.718(448 - 298) - 298 \left\{ 1.005 \ln \frac{1}{9.1907} \right. \right. \\ &\quad \left. \left. + 0.718 \ln \frac{1400}{101.325} \right\} + 101.325 (1 - 9.1907) \right] \text{ kJ} \\ &= 1458.58 \text{ kJ} = 133.96 \text{ kJ/kg} \end{aligned}$$

Final Availability

$$\begin{aligned} A_f &= \phi_f - \phi_0 \\ &= mc_v(T_f - T_0) - T_0 \left\{ mc_p \ln \frac{V_f}{V_0} + mc_v \ln \frac{p_f}{p_0} \right\} + p_0(V_f - V_0) \\ &\quad \left[ p_f = \frac{mRT_f}{V_f} = 931.25 \text{ kPa and } T_f = T_0 \right] \\ &= 0 - T_0 m \left\{ c_p \ln \frac{V_f}{V_0} + c_v \ln \frac{p_f}{p_0} \right\} + p_0(V_f - V_0) \\ &= (2065.7 - 829.92) \text{ kJ} \\ &= 1235.8 \text{ kJ} = 113.5 \text{ kJ/kg} \\ \therefore \text{Irreversibility} &= \text{Loss of availability} \\ &= (1458.5 - 1235.8) \text{ kJ} = 222.7 \text{ kJ} \end{aligned}$$

**Q8.10** Air flows through an adiabatic compressor at 2 kg/s. The inlet conditions are 1 bar and 310 K and the exit conditions are 7 bar and 560 K. Compute the net rate of availability transfer and the irreversibility. Take  $T_0 = 298 \text{ K}$ .

(Ans. 481.1 kW and 21.2 kW)

**Solution:** Mass flow rate ( $\dot{m}$ ) = 2 kg/s

$$p_i = 1 \text{ bar} = 100 \text{ kPa}$$

$$p_f = 7 \text{ bar} = 700 \text{ kPa}$$

$$T_0 = 298 \text{ K}$$

$$T_i = 310 \text{ K}$$

$$T_f = 560 \text{ K}$$

**Calculated data:**

$$\dot{V}_i = \frac{\dot{m} RT_i}{p_i} = 1.7794 \text{ m}^3/\text{s} \quad \dot{V}_f = \frac{\dot{m} RT_f}{p_f} = 0.4592 \text{ m}^3/\text{s}$$

Availability increase rate of air =  $B_2 - B_1$

$$\begin{aligned} &= h_2 - h_1 - T_0 (s_2 - s_1) \\ &= \dot{m} c_p(T_2 - T_1) - T_0 \left\{ \dot{m} c_p \ln \frac{V_2}{V_1} + \dot{m} c_v \ln \frac{p_2}{p_1} \right\} \end{aligned}$$



$$\begin{aligned}
 &= \dot{m} \left[ c_p (T_2 - T_1) - T_0 \left\{ c_p \ln \frac{v_2}{v_1} + c_v \ln \frac{p_2}{p_1} \right\} \right] \\
 &= 2[251.25 - 10.682] \text{ kW} \\
 &= 481.14 \text{ kW}
 \end{aligned}$$

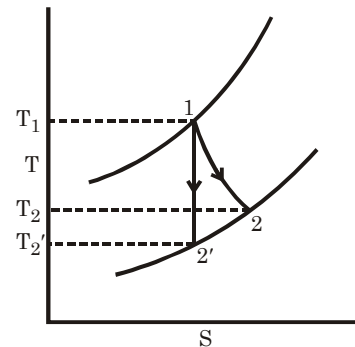
$$\begin{aligned}
 \text{Actual work required} &= \dot{m}(h_2 - h_1) \\
 W &= 2 \times 251.25 \text{ kW} = 502.5 \text{ kW} \\
 \therefore \text{Irreversibility} &= W_{\text{act.}} - W_{\text{min.}} \\
 &= (502.5 - 481.14) \text{ kW} \\
 &= 21.36 \text{ kW}
 \end{aligned}$$

**Q8.11** An adiabatic turbine receives a gas ( $c_p = 1.09$  and  $c_v = 0.838$  kJ/kg K) at 7 bar and 1000°C and discharges at 1.5 bar and 665°C. Determine the second law and isentropic efficiencies of the turbine. Take  $T_0 = 298$  K.  
(Ans. 0.956, 0.879)

**Solution:**

$$\begin{aligned}
 T_1 &= 1273 \text{ K} \\
 R &= c_p - c_v = 0.252 \\
 p_1 &= 7 \text{ bar} = 700 \text{ kPa}
 \end{aligned}$$

$$\begin{aligned}
 \therefore v_1 &= \frac{RT_1}{p_1} = \frac{(c_p - c_v) T_1}{p_1} \\
 &= \frac{0.252 \times 1273}{700} \text{ m}^3/\text{kg} \\
 &= 0.45828 \text{ m}^3/\text{kg}
 \end{aligned}$$



$$\begin{aligned}
 T_2 &= 938 \text{ K} & T_0 &= 298 \text{ K}
 \end{aligned}$$

$$p_2 = 1.5 \text{ bar} = 150 \text{ kPa}$$

$$v_2 = \frac{RT_2}{p_2} = 1.57584 \text{ m}^3/\text{kg}$$

$$\begin{aligned}
 W_{\text{actual}} &= h_1 - h_2 = mc_p (T_1 - T_2) \\
 &= 1 \times 1.09 \times (1273 - 938) \text{ kW} = 365.15 \text{ kW}
 \end{aligned}$$

$$\begin{aligned}
 S_2 - S_1 &= m \left[ c_v \ln \frac{p_2}{p_1} + c_p \ln \frac{V_2}{V_1} \right] \\
 &= 1 \times \left\{ 0.838 \ln \frac{150}{700} + 1.09 \times \ln \frac{1.57584}{0.45828} \right\} \text{ kW/K} \\
 &= 0.055326 \text{ kW/K}
 \end{aligned}$$

$$S_2 - S_2' = mc_p \ln \frac{T_2}{T_2'} = S_2 - S_1 = 0.055326$$

$$\therefore 1 \times 1.09 \ln \frac{T_2}{T_2'} = 0.055326$$

$$\therefore \frac{T_2}{T_2'} = 1.05207$$

$$\therefore T_2' = \frac{T_2}{1.05207} = \frac{938}{(1.05207)} = 891.6 \text{ K}$$

$$\begin{aligned}\text{Isentropic work} &= h_1 - h'_2 = m c_p (T_1 - T'_2) \\ &= 3 \times 1.09(1273 - 891.6) \text{ kW} = 415.75 \text{ kW}\end{aligned}$$

$$\therefore \text{Isentropic efficiency} = \frac{365.15}{415.75} = 87.83\%$$

**Change of availability**

$$\begin{aligned}\Delta A &= A_1 - A_2 \\ &= h_1 - h_2 - T_0(S_1 - S_2) \\ &= m c_p (T_1 - T_2) + T_0 (S_2 - S_1) \\ &= 1 \times 1.09 (1273 - 938) + 298(0.055326) \text{ kW} = 381.64 \text{ kW}\end{aligned}$$

$$\begin{aligned}\therefore \eta_{II} &= \frac{\text{Minimum exergy required to perform the task}}{\text{Actual availability loss}} \\ &= \frac{365.15}{381.64} = 95.7\%\end{aligned}$$

**Q8.12** Air enters an adiabatic compressor at atmospheric conditions of 1 bar, 15°C and leaves at 5.5 bar. The mass flow rate is 0.01 kg/s and the efficiency of the compressor is 75%. After leaving the compressor, the air is cooled to 40°C in an after-cooler. Calculate

- (a) The power required to drive the compressor  
(b) The rate of irreversibility for the overall process (compressor and cooler).

(Ans. (a) 2.42 kW, (b) 1 kW)

**Solution:**

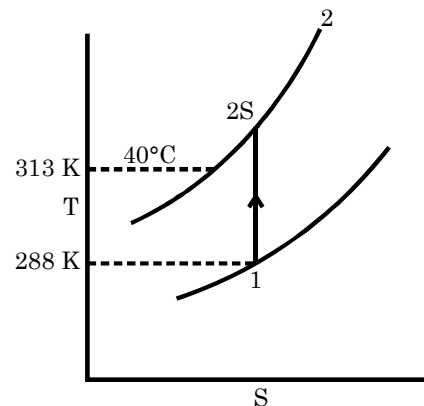
$$p_1 = 1 \text{ bar} = 100 \text{ kPa}$$

$$T_1 = 288 \text{ K}$$

$$\dot{m} = 0.01 \text{ kg/s}$$

$$v_1 = \frac{RT_1}{p_1} = 0.82656 \text{ m}^3/\text{kg}$$

$$p_2 = 5.5 \text{ bar} = 550 \text{ kPa}$$



For minimum work required to compressor is isentropic

$$\begin{aligned}W_{\text{isentropic}} &= \frac{\gamma(p_2 V_2 - p_1 V_1)}{\gamma - 1} \\ &= \frac{\gamma}{\gamma - 1} RT_1 \left[ \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \\ &= \frac{1.4}{0.4} \times 0.287 \times 288 \left[ \left( \frac{550}{100} \right)^{\frac{0.4}{1.4}} - 1 \right] \text{ kJ/kg} = 181.55 \text{ kW/kg}\end{aligned}$$

$\therefore$  **Actual work required**

$$W_{\text{act}} = \frac{181.55}{0.75} = 242 \text{ kJ/kg}$$

(a)  $\therefore$  Power required driving the compressor

$$= \dot{m} W_{\text{act}} = 2.42 \text{ kW}$$

Extra work added in 2' to 2 is  $(242 - 181.55) = 60.85 \text{ kJ/kg}$

$$\therefore \text{If } C_p(T_2 - T_2') = 60.85$$

$$\text{or } T_2 = T_2' + \frac{60.85}{1.005} = 529.25 \text{ K}$$

$\therefore$  Availability loss due to cooling

$$= \int_{313}^{529.25} 1 \times 1.005 \left(1 - \frac{288}{T}\right) dT$$

$$= 1.005 \left\{ (529.21 - 313) - 288 \ln \left( \frac{529.25}{313} \right) \right\} \text{ kJ/kg}$$

$$= 65.302 \text{ kJ/kg}$$

$\therefore$  Total available energy loss

$$= (60.85 + 65.302) \text{ kJ/kg} = 126.15 \text{ kJ/kg}$$

$\therefore$  Power loss due to irreversibility = 1.2615 kW

**Q8.13**

In a rotary compressor, air enters at 1.1 bar, 21 ° C where it is compressed adiabatically to 6.6 bar, 250°C. Calculate the irreversibility and the entropy production for unit mass flow rate. The atmosphere is at 1.03 bar, 20°C. Neglect the K.E. changes.

(Ans. 19 kJ/kg, 0.064 kJ/kg K)

**Solution:**

$$p_1 = 1.1 \text{ bar} = 110 \text{ kPa}$$

$$T_1 = 294 \text{ K}$$

$$p_2 = 6.6 \text{ bar} = 660 \text{ kPa}$$

$$T_2 = 523 \text{ K}$$

$$p_0 = 103 \text{ kPa}$$

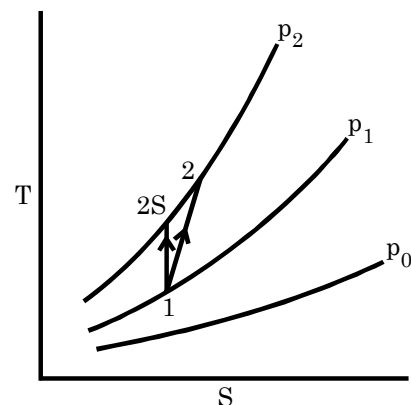
$$T_0 = 293 \text{ K}$$

$$\Delta s = s_2 - s_1 = \int_1^2 \left( \frac{dh}{T} - v \frac{dp}{T} \right)$$

$$= \left[ C_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \right]$$

$$= \left[ 1.005 \ln \frac{523}{294} - 0.287 \ln \left( \frac{660}{110} \right) \right]$$

$$= 0.064647 \text{ kJ/kg} - \text{K} = 64.647 \text{ J/kg} - \text{K}$$



**Minimum work required**

$$W_{\text{min}} = \text{Availability increase}$$

$$= h_2 - h_1 - T_0 (s_2 - s_1)$$

$$= mc_p (T_2 - T_1) - T_0 \Delta s$$

$$= 1 \times 1.005 (523 - 294) - 293 \times 0.064647 = 211.2 \text{ kJ/kg}$$

Actual work required ( $W_{act}$ ) = 230.145 kJ/kg

∴ Irreversibility =  $T_0 \Delta s$

$$= 293 \times 0.064647 = 18.942 \text{ kJ/kg}$$

**Q8.14** In a steam boiler, the hot gases from a fire transfer heat to water which vaporizes at a constant temperature of 242.6°C (3.5 MPa). The gases are cooled from 1100 to 430°C and have an average specific heat,  $c_p = 1.046$  kJ/kg K over this temperature range. The latent heat of vaporization of steam at 3.5 MPa is 1753.7 kJ/kg. If the steam generation rate is 12.6 kg/s and there is negligible heat loss from the boiler, calculate:

- (a) The rate of heat transfer
- (b) The rate of loss of exergy of the gas
- (c) The rate of gain of exergy of the steam
- (d) The rate of entropy generation. Take  $T_0 = 21^\circ\text{C}$ .

(Ans. (a) 22096 kW, (b) 15605.4 kW

(c) 9501.0 kW, (d) 20.76 kW/K)

**Solution:** (a) Rate of heat transfer =  $12.6 \times 1752.7 \text{ kW} = 22.097 \text{ MW}$

If mass flow rate at gas is  $\dot{m}_g$

$$\text{Then } \dot{m}_g c_{p_g} (1100 - 430) = 22097$$

$$\text{or } \dot{m}_g = 31.53 \text{ kg/s}$$

$$\begin{aligned} \text{Loss of exergy of the gas} &= \int_{703}^{1373} \dot{m}_g c_{p_g} \left(1 - \frac{294}{T}\right) dT \\ &= \dot{m}_g c_{p_g} \left[ (1373 - 703) - 294 \ln \left( \frac{1373}{703} \right) \right] \\ &= 15606 \text{ kJ/s} = 15.606 \text{ MW} \end{aligned}$$

$$\text{Gain of exergy of steam} = \dot{m}_w L_w \left(1 - \frac{294}{515.4}\right) = 9.497 \text{ MW}$$

$$\text{Rate of entropy gas} = \frac{\text{Irreversibility}}{T_0}$$

$$= 20.779 \text{ kW/K}$$

**Q8.15** An economizer, a gas-to-water finned tube heat exchanger, receives 67.5 kg/s of gas,  $c_p = 1.0046$  kJ/kg K, and 51.1 kg/s of water,  $c_p = 4.186$  kJ/kg K. The water rises in temperature from 402 to 469 K, where the gas falls in temperature from 682 K to 470 K. There are no changes of kinetic energy and  $p_0 = 1.03$  bar and  $T_0 = 289$  K. Determine:

- (a) Rate of change of availability of the water
- (b) The rate of change of availability of the gas
- (c) The rate of entropy generation

(Ans. (a) 4802.2 kW, (b) 7079.8 kW, (c) 7.73 kW/K)

**Solution:** (a) Rate of change of availability of water =  $Q \left(1 - \frac{T_0}{T}\right)$

$$\begin{aligned} &= \int_{402}^{469} \dot{m}_w c_{p_w} dT \left(1 - \frac{289}{T}\right) \\ &= 51.1 \times 4.186 \times \left[ (469 - 402) - 289 \ln \frac{469}{402} \right] \text{ kW} \end{aligned}$$

$$= 4.823 \text{ MW (gain)}$$

(b) Rate of availability loss of gas

$$\begin{aligned} &= \int_{470}^{682} \dot{m}_g c_{Pg} \left(1 - \frac{289}{T}\right) dT \\ &= 67.5 \times 1.0046 \left[ (682 - 470) - 289 \ln \frac{682}{470} \right] \\ &= 7.0798 \text{ MW} \end{aligned}$$

$$\therefore \text{(c) Rate of irreversibility } (\dot{I}) = 2.27754 \text{ MW}$$

$$\therefore \text{Entropy generation rate } \dot{S}_{\text{gas}} = \frac{\dot{I}}{T_0} = 7.8808 \text{ kW/K}$$

**Q8.16** The exhaust gases from a gas turbine are used to heat water in an adiabatic counter flow heat exchanger. The gases are cooled from 260 to 120°C, while water enters at 65°C. The flow rates of the gas and water are 0.38 kg/s and 0.50 kg/s respectively. The constant pressure specific heats for the gas and water are 1.09 and 4.186 kJ/kg K respectively. Calculate the rate of exergy loss due to heat transfer. Take  $T_0 = 35^\circ\text{C}$ .

(Ans. 12.5 kW)

<b>Solution:</b> $T_{gi} = 260^\circ\text{C} = 533 \text{ K}$ $T_{go} = 120^\circ\text{C} = 393 \text{ K}$ $\dot{m}_g = 0.38 \text{ kg/s}$ $c_{pg} = 1.09 \text{ kJ/kg} - \text{K}$ $T_0 = 35^\circ\text{C} = 308 \text{ K}$	$T_{wi} = 65^\circ\text{C} = 338 \text{ K}$ $T_{wo} = 365.7 \text{ K (Calculated)}$ $\dot{m}_w = 0.5 \text{ kg/s}$ $c_{pw} = 4.186 \text{ kJ/kg} - \text{K}$
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To calculate  $T_{wo}$  from heat balance

$$\dot{m}_g c_{Pg} (T_{gi} - T_{go}) = \dot{m}_w c_{Pw} (T_{wo} - T_{wi})$$

$$\therefore T_{wo} = 365.7 \text{ K}$$

Loss rate of availability of gas

$$= \dot{m}_g c_{pg} \left[ (533 - 393) - 308 \ln \left( \frac{533}{393} \right) \right] = 19.115 \text{ kW}$$

Rate of gain of availability of water

$$= \dot{m}_w c_{pw} \left[ (365.7 - 338) - 308 \ln \left( \frac{365.7}{338} \right) \right] = 7.199 \text{ kW}$$

$$\therefore \text{Rate of exergy loss} = 11.916 \text{ kW}$$

**Q8.17** The exhaust from a gas turbine at 1.12 bar, 800 K flows steadily into a heat exchanger which cools the gas to 700 K without significant pressure drop. The heat transfer from the gas heats an air flow at constant pressure, which enters the heat exchanger at 470 K. The mass flow rate of air is twice that of the gas and the surroundings are at 1.03 bar, 20°C. Determine:

- (a) The decrease in availability of the exhaust gases.
- (b) The total entropy production per kg of gas.
- (c) What arrangement would be necessary to make the heat transfer reversible and how much would this increase the power output of

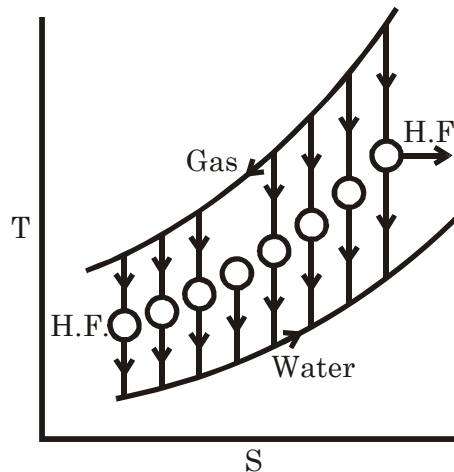
the plant per kg of turbine gas? Take  $c_p$  for exhaust gas as 1.08 and for air as 1.05 kJ/kg K. Neglect heat transfer to the surroundings and the changes in kinetic and potential energy.

(Ans. (a) 66 kJ/kg, (b) 0.0731 kJ/kg K, (c) 38.7 kJ/kg)

**Solution:** (a) Availability decrease of extra gases =  $Q \left( 1 - \frac{T_0}{T} \right)$

$$= \int_{700}^{800} m c_p \left( 1 - \frac{293}{T} \right) dT = 1 \times 1.08 \left[ (800 - 700) - 293 \ln \left( \frac{800}{700} \right) \right]$$

$$= 65.745 \text{ kJ/kg}$$



(b) Exit air temperature  $T_{\text{exit}}$

$$2m c_{pa} (T_e - 470) = m \times c_{pg} (800 - 700)$$

or  $T_e = 521.5 \text{ K}$

$\therefore$  Availability increases

$$= 2 \times 1.05 \times \left[ (521.5 - 470) - 293 \ln \frac{521.5}{470} \right] = 44.257 \text{ kJ/kg}$$

$\therefore \dot{S}_{\text{gas}} = 73.336 \text{ J/K}$  of per kg gas flow

For reversible heat transfer

$$(\Delta S)_{\text{univ}} = 0$$

$$(\Delta S)_{\text{Gas}} = -(\Delta S)_{\text{water}}$$

$$m \times 1.08 \ln \frac{800}{700}$$

$$= -2m \times 1.05 \times \ln \left( \frac{470}{T_0} \right)$$

or  $\ln \frac{T_0}{470} = 0.068673$

$\therefore T_0 = 503.4 \text{ K}$

$\therefore Q_1 = m \times 1.08(800 - 700) = 108 \text{ kJ/kg}$

$Q_2 = 2m \times 1.05 (503.4 - 470) = 70.162 \text{ kJ/kg}$  of gas

$W = Q_1 - Q_2 = 37.84 \text{ kJ/kg}$  of gas flow [i.e. extra output]

**Q8.18**

An air preheater is used to heat up the air used for combustion by cooling the outgoing products of combustion from a furnace. The rate of flow of the products is 10 kg/s, and the products are cooled from 300°C to

200°C, and for the products at this temperature  $c_p = 1.09 \text{ kJ/kg K}$ . The rate of air flow is 9 kg/s, the initial air temperature is 40°C, and for the air  $c_p = 1.005 \text{ kJ/kg K}$ .

- (a) What is the initial and final availability of the products?
- (b) What is the irreversibility for this process?
- (c) If the heat transfer from the products were to take place reversibly through heat engines, what would be the final temperature of the air?

What power would be developed by the heat engines? Take  $T_o = 300 \text{ K}$ .

(Ans. (a) 85.97, 39.68 kJ/kg, (b) 256.5 kW,  
(c) 394.41 K, 353.65 kW)

**Solution:** To calculate final air temperature ( $T_f$ )

$$\dot{m}_g c_{pg} (573 - 473) = \dot{m}_a c_{pa} (T_f - 313)$$

$$10 \times 1.09 (573 - 473) = 9 \times 1.005 (T_f - 313)$$

Or  $T_f = 433.5 \text{ K}$

- (a) Initial availability of the product

$$= c_{pg} \left[ (573 - 300) - 300 \ln \frac{573}{300} \right]$$

$$= 85.97 \text{ kJ/kg of product}$$

**Final availability**

$$= c_{pg} \left[ (473 - 300) - 300 \ln \frac{473}{300} \right] = 39.68 \text{ kJ/kg of product}$$

$\therefore$  Loss of availability = 46.287 kJ/kg of product

Availability gain by air

$$= c_{pg} \left[ (433.5 - 313) - 300 \ln \left( \frac{433.5}{313} \right) \right] = 22.907 \text{ kJ/kg of air}$$

- (b)  $\therefore$  Rate of irreversibility

$$\dot{I} = (10 \times 46.287 - 22.907 \times 9) \text{ kW} = 256.7 \text{ kW}$$

- (c) For reversible heat transfer

$$(\Delta S)_{\text{Univ}} = 0$$

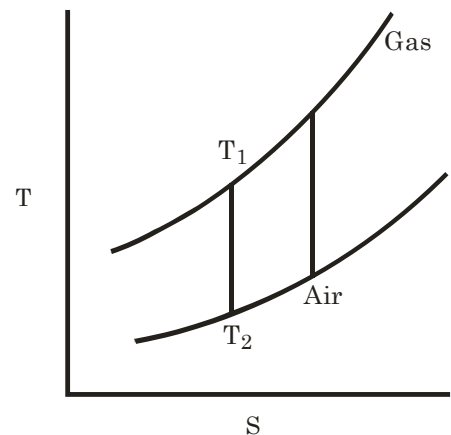
$$\therefore (\Delta S)_{\text{gas}} + (\Delta S)_{\text{air}} = 0$$

or  $(\Delta S)_{\text{gas}} = -(\Delta S)_{\text{air}}$

$$\text{or } \dot{m}_g c_{pg} \ln \left( \frac{T_f}{T_i} \right)$$

$$= \dot{m}_a c_{pa} \ln \left( \frac{T_f}{T_i} \right)$$

$$\text{or } 10 \times 1.09 \ln \left( \frac{473}{10 \times 1.09 \ln (473) 573} \right)$$



$$= -9 \times 1.005 \times \ln \frac{T_f}{313}$$

$$\text{or } T_f = 394.4 = 399.4 \text{ K}$$

$$\therefore \dot{Q}_1 = \dot{m}_g c_{pg}(T_i - T_f) = 1090 \text{ kJ}$$

$$Q_2 = \dot{m}_a c_{pa}(394.4 - 313) = 736.263 \text{ kJ}$$

$$\therefore \dot{W} = \dot{Q}_1 - Q_2 = 353.74 \text{ kW output of engine.}$$

**Q8.19** A mass of 2 kg of air in a vessel expands from 3 bar, 70°C to 1 bar, 40°C, while receiving 1.2 kJ of heat from a reservoir at 120°C. The environment is at 0.98 bar, 27°C. Calculate the maximum work and the work done on the atmosphere.

(Ans. 177 kJ, 112.5 kJ)

**Solution:**

Maximum work from gas

$$= u_1 - u_2 - T_0 (s_1 - s_2)$$

$$= m c_v (T_1 - T_2)$$

$$- T_0 \left[ m c_p \ln \frac{T_1}{T_2} - m R \ln \frac{p_1}{p_2} \right]$$

$$= 2 \left[ 0.718(343 - 313) - 300 \left[ 1.005 \ln \frac{343}{313} - 0.287 \ln \left( \frac{3}{1} \right) \right] \right]$$

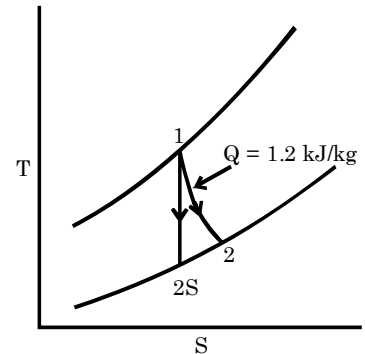
$$= 177.07 \text{ kJ}$$

$$\text{Work done on the atmosphere} = p_0 (V_2 - V_1)$$

$$= 98 \left[ m R \frac{T_0}{p_2} - m R \frac{T_1}{p_1} \right]$$

$$= 98 m R \left[ \frac{T_2}{p_2} - \frac{T_1}{p_1} \right]$$

$$= 111.75 \text{ kJ}$$



**Q8.20** Air enters the compressor of a gas turbine at 1 bar, 30°C and leaves the compressor at 4 bar. The compressor has an efficiency of 82%. Calculate per kg of air

(a) The work of compression

(b) The reversible work of compression

(c) The irreversibility. For air, use  $\frac{T_{2s}}{T_1} = \left( \frac{p_2}{p_1} \right)^{\gamma-1/\gamma}$

Where  $T_{2s}$  is the temperature of air after isentropic compression and  $\gamma = 1.4$ . The compressor efficiency is defined as  $(T_{2s} - T_1) / (T_2 - T_1)$ , where  $T_2$  is the actual temperature of air after compression.

(Ans. (a) 180.5 kJ/kg, (b) 159.5 kJ/kg (c) 21 kJ/kg)

**Solution:**

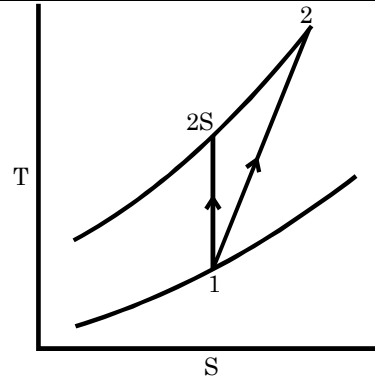


$$p_1 = 1 \text{ bar} = 100 \text{ kPa}$$

$$T_1 = 30^\circ \text{ C} = 303 \text{ K}$$

$$p_2 = 4 \text{ bar} = 400 \text{ kPa}$$

$$T_2 = ? \quad \eta_{\text{com}} = 6.82$$



(b) Minimum work required for compression is isentropic work

$$\begin{aligned} \therefore W_R &= \frac{\gamma}{\gamma-1} mRT \left\{ \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\} \\ &= \frac{1.4 \times 1 \times 0.287 \times 303}{(1.4-1)} \left\{ \left( \frac{400}{100} \right)^{\frac{0.4}{1.4}} - 1 \right\} = 147.92 \text{ kJ/kg} \end{aligned}$$

$$(a) \quad \text{Actual work} = \frac{147.92}{0.82} = 180.4 \text{ kJ/kg}$$

$\therefore$  Extra work 32.47 kJ will heat the gas from  $T_2'$  to  $T_2$

$$\frac{T_2'}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \quad \therefore T_2' = 450.3 \text{ K}$$

$$\therefore 32.47 = mc_p (T_2 - T_2')$$

$$\therefore T_2 = 482.6 \text{ K}$$

$$(c) \quad \text{Irreversibility (I)} = (180.4 - 147.92) \text{ kJ/kg} = 32.48 \text{ kJ/kg}$$

**Q8.21**

**A mass of 6.98 kg of air is in a vessel at 200 kPa, 27°C. Heat is transferred to the air from a reservoir at 727°C. Until the temperature of air rises to 327°C. The environment is at 100 kPa, 17°C. Determine**

**(a) The initial and final availability of air**

**(b) The maximum useful work associated with the process.**

(Ans. (a) 103.5, 621.9 kJ (b) 582 kJ)

**Solution:**

$$p_1 = 200 \text{ kPa}$$

$$p_2 = \frac{mRT_2}{V_2} = 400 \text{ kPa}$$

$$p_o = 100 \text{ kPa}$$

$$T_1 = 300 \text{ K}$$

$$T_2 = 600 \text{ K}$$

$$T_o = 290 \text{ K}$$

$$V_1 = \frac{mRT_1}{P_1} = 3.005 \text{ m}^3$$

$$V_2 = V_1 = 3.005 \text{ m}^3$$

$$V_o = 5.8095 \text{ m}^3$$

$$m = 6.98 \text{ kg}$$

**(a) Initial availability**

$$\begin{aligned} A_i &= u_1 - u_o - T_o (s_1 - s_o) + p_o (V_1 - V_o) \\ &= m c_v (T_1 - T_o) - m T_o \left[ m c_p \ln \frac{T_1}{T_o} - R \ln \frac{p_1}{p_o} \right] + p_o (V_1 - V_o) \\ &= 6.98 \times 0.718 (300 - 290) - 6.98 \times 290 \end{aligned}$$

$$-6.98 \times 290 \left[ 1.005 \ln \frac{300}{290} - 0.287 \ln \left( \frac{200}{100} \right) \right] + 100(3.005 - 5.8095) \\ = 103.4 \text{ kJ}$$

**Final availability**

$$A_f = m c_v (T_2 - T_0) - m T_0 \left[ c_p \ln \frac{T_2}{T_0} - R \ln \frac{p_2}{p_0} \right] + p_0 (V_2 - V_0) \\ = 6.98 \times 0.718(600 - 290) - 6.98 \times 290 \\ \times \left[ 1.005 \ln \frac{600}{290} - 0.287 \ln \left( \frac{400}{100} \right) \right] + 100 (3.005 - 5.8095) \\ = 599.5 \text{ kJ}$$

**(b) Maximum useful work**

$$= u_2 - u_1 - T_0 (s_2 - s_1) + p_0 (V_2 - V_1) \\ = m c_v (T_2 - T_1) - T_0 m \left[ c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \right] + p_0 (V_2 - V_1) \\ = 6.98 \times 0.718(600 - 300) - 300 \\ \times 6.98 \left[ 1.005 \ln \frac{600}{300} - 0.287 \ln \left( \frac{400}{200} \right) \right] + p_0 \times 0 = 461.35 \text{ kJ}$$

$$\therefore V_2 = V_1$$

Heat transfer to the vessel

$$Q = \int m c_v dT \quad p = \frac{mRT}{V} \\ = m c_v (T_2 - T_1) = 6.98 \times 0.718 \times (600 - 300) \text{ kJ} \\ = 1503.402 \text{ kJ}$$

$$\therefore \text{Useful work loss of reservoir} = Q \left( 1 - \frac{T_0}{T} \right) \\ = 1503.402 \left( 1 - \frac{290}{1000} \right) \\ = 1067.47 \text{ kJ}$$

**Q8.22** Air enters a compressor in steady flow at 140 kPa, 17°C and 70 m/s and leaves it at 350 kPa, 127°C and 110 m/s. The environment is at 100 kPa, 7°C. Calculate per kg of air

**(a) The actual amount of work required**

**(b) The minimum work required**

**(c) The irreversibility of the process**

(Ans. (a) 114.4 kJ, (b) 97.3 kJ, (c) 17.1 kJ)

**Solution:** Minimum work required

$$T_2 = 127^\circ\text{C} = 400 \text{ K}$$

$$T_1 = 290 \text{ K}$$

$$T_0 = 280 \text{ K}$$

$$w = h_2 - h_1 - T_0 (s_2 - s_1) + \frac{V_2^2 - V_1^2}{2000} \\ = m c_p (T_2 - T_1) - m T_0 \left[ c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \right] + \frac{V_2^2 - V_1^2}{2000}$$

$$\begin{aligned}
 &= 1 \times 1.005(400 - 290) - 1 \times 280 \left[ 1.005 \ln \frac{400}{290} - 0.287 \ln \frac{350}{140} \right] \\
 &\quad + \frac{1.110^2 - 70^2}{2000} \text{ kJ} \\
 &= 110.55 - 16.86 + 3.6 = 97.29 \text{ kJ/kg}
 \end{aligned}$$

**Actual work required**

$$= h_2 - h_1 + \frac{V_2^2 - V_1^2}{2000} = (110.55 + 3.6) \text{ kJ} = 114.15 \text{ kJ}$$

$\therefore$  Irreversibility of the process

$$= T_0(s_2 - s_1) = T_0(\Delta S)_{\text{univ}} = 16.86 \text{ kJ/kg}$$

**Q8.23** Air expands in a turbine adiabatically from 500 kPa, 400 K and 150 m/s to 100 kPa, 300 K and 70 m/s. The environment is at 100 kPa, 17°C. Calculate per kg of air

(a) The maximum work output

(b) The actual work output

(c) The irreversibility

(Ans. (a) 159 kJ, (b) 109 kJ, (c) 50 kJ)

**Solution:** Maximum work output

$$\begin{aligned}
 w &= h_1 - h_2 - T_0(s_1 - s_2) + \frac{V_1^2 - V_2^2}{2000} \\
 &= C_p(T_1 - T_2) - T_0 \left\{ C_p \ln \frac{T_1}{T_2} - R \ln \frac{p_1}{p_2} \right\} + \frac{V_1^2 - V_2^2}{2000} \\
 &= 1.005(400 - 300) - 290 \left\{ 1.005 \ln \frac{400}{300} - 0.287 \ln \frac{500}{100} \right\} + \frac{150^2 - 70^2}{2000} \\
 &= 159.41 \text{ kJ/kg}
 \end{aligned}$$

$$\text{Actual output} = h_1 - h_2 + \frac{V_1^2 - V_2^2}{2000} = 100.5 + 8.8 = 109.3 \text{ kJ/kg}$$

The irreversibility (I) =  $T_0(\Delta S)_{\text{univ}} = 50.109 \text{ kJ/kg}$

**Q8.24** Calculate the specific exergy of air for a state at 2 bar, 393.15 K when the surroundings are at 1 bar, 293.15 K. Take  $c_p = 1$  and  $R = 0.287 \text{ kJ/kg K}$ .

(Ans. 72.31 kJ/kg)

**Solution:** Exergy = Available energy

$$= h_1 - h_2 - T_0(s_1 - s_2)$$

$$= C_p(T_1 - T_0) - T_0 \left[ C_p \ln \frac{T_1}{T_0} - R \ln \frac{p_1}{p_0} \right]$$

$$= 1 \times (393.15 - 293.15) - 293.15 \left[ 1 \times \ln \frac{393.15}{293.15} - 0.287 \ln \left( \frac{2}{1} \right) \right] \text{ kJ/kg}$$

$$= 72.28 \text{ kJ/kg}$$

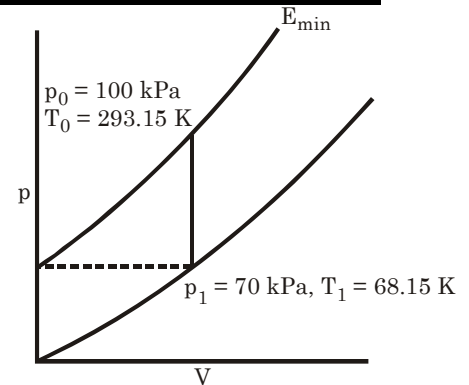
**Q8.25** Calculate the specific exergy of  $\text{CO}_2$  ( $c_p = 0.8659$  and  $R = 0.1889 \text{ kJ/kg K}$ ) for a state at 0.7 bar, 268.15 K and for the environment at 1.0 bar and 293.15 K.

(Ans. – 18.77 kJ/kg)

**Solution:**

Exergy = Available energy

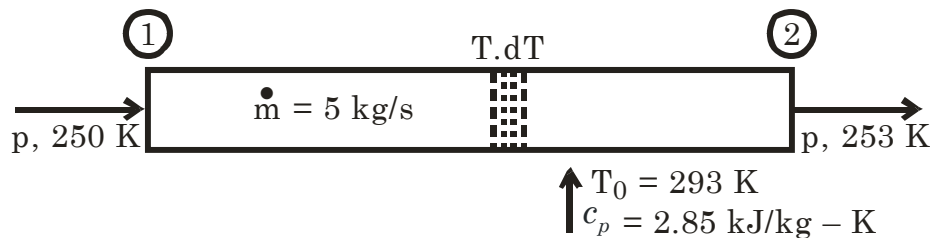
$$\begin{aligned}
 h_1 - h_0 - T_0 (s_1 - s_2) \\
 &= C_p (T_1 - T_0) - T_0 \left[ C_p \ln \frac{T_1}{T_0} - R \ln \frac{p_1}{p_0} \right] \\
 &= 0.8659 (268.15 - 293.15) - 293.15 \\
 &\quad \times \left\{ 0.8659 \ln \frac{268.15}{293.15} - 0.1889 \ln \frac{70}{100} \right\} \text{ kJ/kg} \\
 &= -18.772 \text{ kJ/kg}
 \end{aligned}$$



**Q8.26** A pipe carries a stream of brine with a mass flow rate of 5 kg/s. Because of poor thermal insulation the brine temperature increases from 250 K at the pipe inlet to 253 K at the exit. Neglecting pressure losses, calculate the irreversibility rate (or rate of energy degradation) associated with the heat leakage. Take  $T_0 = 293$  K and  $c_p = 2.85$  kJ/kg K.

(Ans. 7.05 kW)

**Solution:**



Entropy generation rate

$$\begin{aligned}
 \dot{S}_{\text{gas}} &= \dot{S}_{\text{sys}} - \frac{\dot{Q}}{T_0} \\
 &= \dot{m} (S_2 - S_1) - \frac{\dot{m} c_p (253 - 250)}{T_0} \\
 &= \dot{m} c_p \left[ \ln \frac{T_2}{T_1} - \frac{3}{T_0} \right] \text{ kW/K} \\
 &= 0.0240777 \text{ kW/K}
 \end{aligned}$$

Where,  $\dot{Q} = -\dot{m} c_p (253 - 250)$

–ve because  $\dot{Q}$  flux from surroundings.

$$S_2 - S_1 = c_p \ln \frac{T_2}{T_1}$$

$\therefore \dot{I}$  = rate of energy degradation  
= rate of exergy loss

$$T_0 \dot{S}_{\text{gen}} = 293 \times 0.0240777 \text{ kW} = 7.0548 \text{ kW}$$

**Q8.27** In an adiabatic throttling process, energy per unit mass of enthalpy remains the same. However, there is a loss of exergy. An ideal gas flowing at the rate  $m$  is throttled from pressure  $p_1$  to pressure  $p_2$  when the environment is at temperature  $T_0$ . What is the rate of exergy loss due to throttling?

$$\left( \text{Ans. } \dot{I} = \dot{m} R T_0 \ln \frac{p_1}{p_2} \right)$$

**Solution:**

Adiabatic throttling process  $h_1 = h_2$

∴ Rate of entropy generation ( $\dot{S}_{\text{gen}}$ )

$$\dot{S}_{\text{gen}} = (\dot{\Delta S})_{\text{sys}} + (\dot{\Delta S})_{\text{surr.}}$$

$$= (\dot{\Delta S})_{\text{sys}} + 0$$

$$= \dot{m}(S_2 - S_1)$$

$$= \dot{m} R \ln \left( \frac{p_1}{p_2} \right)$$

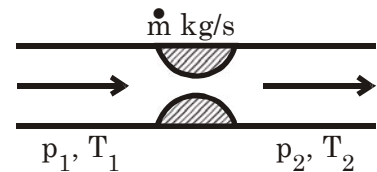
(as no heat interaction with surroundings)

$$T dS = dh - V dp$$

$$\text{or } dS = \frac{dh}{T} - V \frac{dp}{T} \quad \frac{V}{T} = \frac{mR}{p}$$

$$dS = 0 - mR \frac{dp}{p}$$

$$\text{or } S_2 - S_1 = - \int_1^2 mR \frac{dp}{p} = -mR \ln \frac{p_2}{p_1} = mR \ln \frac{p_1}{p_2}$$



∴ Irreversibility rate ( $\dot{I}$ )

$$= T_0 \times \dot{S}_{\text{gen}}$$

$$= T_0 \times mR \ln \left( \frac{p_1}{p_2} \right)$$

$$= mR T_0 \ln \left( \frac{p_1}{p_2} \right)$$

**Q8.28.**

Air at 5 bar and 20°C flows into an evacuated tank until the pressure in the tank is 5 bar. Assume that the process is adiabatic and the temperature of the surroundings is 20°C.

(a) What is the final temperature of the air?

(b) What is the reversible work produced between the initial and final states of the air?

(c) What is the net entropy change of the air entering the tank?

(d) Calculate the irreversibility of the process.

(Ans. (a) 410.2 K, (b) 98.9 kJ/kg,

(c) 0.3376 kJ/kg K, (d) 98.9 kJ/kg)

**Solution:**

If  $m$  kg of air is entered to the tank then the enthalpy of entering fluid is equal to internal energy of tank fluid.

(a)  $h = u$

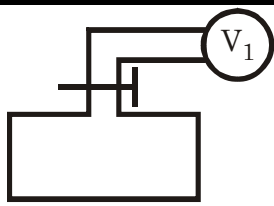
$$\therefore C_p T_1 = C_v T_2$$

$$\text{or } T_2 = \left( \frac{C_p}{C_v} \right) T_1 = \gamma T_1$$

$$= 1.4 \times 293 \text{ K} = 410.2 \text{ K}$$

(b) Reversible work

$$\begin{aligned} W &= p dV \text{ work} \\ &= p (V_2 - V_1) \\ &= p (0 - V_1) \\ &= -pV_1 \end{aligned}$$

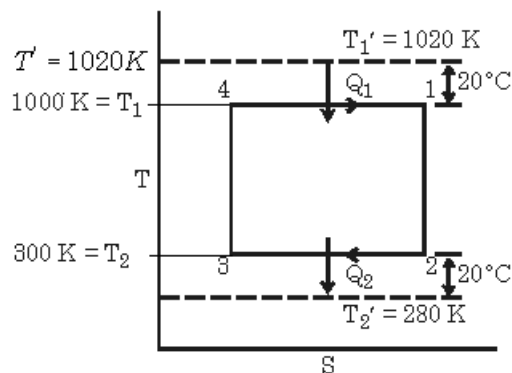
$$v_1 = \frac{RT_1}{p_1} = 0.168182 \text{ m}^3/\text{kg}$$


Q8.29

A Carnot cycle engine receives and rejects heat with a 20°C temperature differential between itself and the thermal energy reservoirs. The expansion and compression processes have a pressure ratio of 50. For 1 kg of air as the working substance, cycle temperature limits of 1000 K and 300 K and  $T_0 = 280 \text{ K}$ , determine the second law efficiency.

(Ans. 0.965)

Solution:



Let  $Q_1$  amount of heat is in input. Then actual Carnot cycle produces work

$$W = Q_1 \left( 1 - \frac{360}{1000} \right) = 0.7 Q_1$$

If there is no temperature differential between inlet and outlet then from  $Q_1$  heat input Carnot cycle produce work.

$$W_{\max} = Q_1 \left( 1 - \frac{280}{1020} \right) = 0.72549 Q_1$$

$$\therefore \text{Second law efficiency } (\eta_{II}) = \frac{W}{W_{\max}} = \frac{0.7}{0.72549} = 0.965$$

Q8.30

Energy is received by a solar collector at the rate of 300 kW from a source temperature of 2400 K. If 60 kW of this energy is lost to the surroundings at steady state and if the user temperature remains constant at 600 K, what are the first law and the second law efficiencies? Take  $T_0 = 300 \text{ K}$ .

(Ans. 0.80, 0.457)

Solution:

First law efficiency

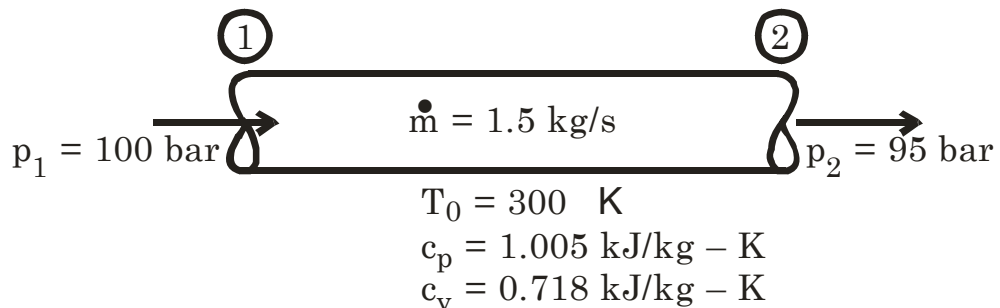
$$= \frac{300 - 60}{300} = 0.8$$

$$\text{Second law efficiency} = \frac{(300 - 60) \left(1 - \frac{300}{600}\right)}{300 \left(1 - \frac{300}{2400}\right)} = 0.457$$

**Q8.31** For flow of an ideal gas through an insulated pipeline, the pressure drops from 100 bar to 95 bar. If the gas flows at the rate of 1.5 kg/s and has  $c_p = 1.005$  and  $c_v = 0.718$  kJ/kg-K and if  $T_0 = 300$  K, find the rate of entropy generation, and rate of loss of exergy.

(Ans. 0.0215 kW/K, 6.46 kW)

**Solution:**



Rate of entropy generation

$$\dot{S}_{\text{gen}} = (\Delta \dot{S})_{\text{sys}} - \frac{\dot{Q}}{T_0}$$

As it is insulated pipe so  $\dot{Q} = 0$

$$= (\Delta \dot{S})_{\text{sys}}$$

$$TdS = dh - Vdp$$

$$= \dot{m}(S_2 - S_1)$$

Here  $h_1 = h_2$  so  $dh = 0$

$$= \dot{m} R \ln \left( \frac{p_1}{p_2} \right)$$

$$\therefore TdS = -Vdp$$

$$= 1.5 \times 0.287 \times \ln \left( \frac{100}{95} \right) \text{ kW/K}$$

$$dS = -\frac{V}{T} dp$$

$$= 0.022082 \text{ kW/K}$$

$$\int_1^2 dS = -mR \int_1^2 \frac{dp}{p} = mR \ln \frac{p_2}{p_1}$$

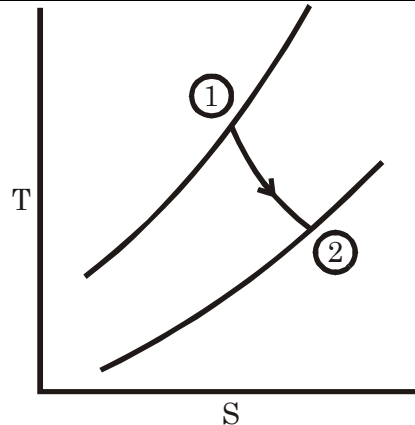
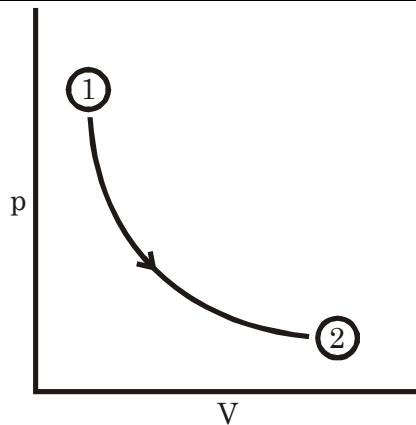
Rate of loss of exergy = Irreversibility rate ( $\dot{I}$ )

$$T_0 \dot{S}_{\text{gen}} = 300 \times 0.022082 = 6.6245 \text{ kW}$$

**Q8.32** The cylinder of an internal combustion engine contains gases at 2500°C, 58 bar. Expansion takes place through a volume ratio of 9 according to  $pv^{1.38} = \text{const}$ . The surroundings are at 20°C, 1.1 bar. Determine the loss of availability, the work transfer and the heat transfer per unit mass. Treat the gases as ideal having  $R = 0.26$  kJ/kg-K and  $c_v = 0.82$  kJ/kg-K.

(Ans. 1144 kJ/kg, 1074 kJ/kg, -213 kJ/kg)

**Solution:**



$$p_1 = 58 \text{ bar} = 5800 \text{ kPa}$$

$$v_1 = 0.1243 \text{ m}^3/\text{kg} \text{ (calculating)}$$

$$T_1 = 2500^\circ\text{C} = 2773 \text{ K}$$

$$p_2 = 279.62 \text{ kPa (calculated)}$$

$$v_2 = 9 v_1 = 1.11876 \text{ m}^3/\text{kg}$$

$$T_2 = 1203.2 \text{ K (calculated)}$$

$$\therefore v_1 = m \frac{RT_1}{p_1} = 0.1243 \text{ m}^3/\text{kg}$$

$$p_0 = 1.1 \text{ bar} = 110 \text{ kPa}$$

$$R = 0.26 \text{ kJ/kg} \cdot \text{K}$$

$$T_0 = 20^\circ\text{C} = 293 \text{ K}$$

$$W = 0.82 \text{ kJ/kg} \cdot \text{K}$$

$$\therefore c_p = c_v + R = 1.08 \text{ kJ/kg}$$

$$\therefore \frac{p_2}{p_1} = \left( \frac{v_1}{v_2} \right)^n \text{ or } p_2 = p_1 \left( \frac{v_1}{v_2} \right)^{1.38} = \frac{p_1}{9^{1.38}}$$

$$\frac{T_2}{T_1} = \left( \frac{v_1}{v_2} \right)^{n-1} = \frac{1}{9^{0.38}}$$

$$\therefore T_2 = \frac{T_1}{9^{0.38}} = 1203.2 \text{ K}$$

$\Rightarrow$  Loss of availability

$$\therefore \phi_1 - \phi_2$$

$$= (u_1 - u_2) - T_0 (s_1 - s_2) + p_0 (v_1 - v_2)$$

$$= C_v(T_1 - T_2) - T_0 \left[ C_p \ln \frac{T_1}{T_2} - R \ln \frac{p_1}{p_2} \right] + p_0(v_1 - v_2)$$

$$= 0.82(2773 - 1203.2) - 293 \left[ 1.08 \ln \frac{2773}{1203.2} - 0.26 \ln \left( \frac{5800}{279.62} \right) \right]$$

$$+ 110(0.1243 - 1.11876) \text{ kJ/kg}$$

$$= 1211 \text{ kJ/kg}$$

$$\text{Work transfer (W)} = \frac{p_1 v_1 - p_2 v_2}{n-1} = 1074 \text{ kJ/kg}$$

$$\delta Q = du + \delta W$$

$$\therefore Q_{1-2} = C_v (T_2 - T_1) + W_{1-2}$$

$$= -1287.2 + 1074 = -213.2 \text{ kJ/kg}$$

**Q8.33**

In a counterflow heat exchanger, oil ( $c_p = 2.1 \text{ kJ/kg} \cdot \text{K}$ ) is cooled from 440 to 320 K, while water ( $c_p = 4.2 \text{ kJ/kg} \cdot \text{K}$ ) is heated from 290 K to temperature  $T$ . The respective mass flow rates of oil and water are 800



and 3200 kg/h. Neglecting pressure drop, KE and PE effects and heat loss, determine

- (a) The temperature  $T$
- (b) The rate of exergy destruction
- (c) The second law efficiency

Take  $T_0 = 17^\circ\text{C}$  and  $p_0 = 1 \text{ atm}$ .

(Ans. (a) 305 K, (b) 41.4 MJ/h, (c) 10.9%)

**Solution:**

From energy balance

$$\begin{aligned} \text{(a)} \quad \dot{m}_P c_p (440 - 320) \\ = \dot{m}_W c_{pW} (T - 290) \\ \therefore T = 290 + 15 = 305 \text{ K} \end{aligned}$$

$$\text{(b)} \quad \dot{S}_{\text{gen}} = (\dot{\Delta S})_0 + (\dot{\Delta S})$$

$$\begin{aligned} &= \dot{m}_O c_{pO} \ln \frac{T_{fo}}{T_{io}} + \dot{m}_W c_{pW} \ln \frac{T_{fw}}{T_{iw}} \\ &= \left[ \frac{800}{3600} \times 2.1 \times \ln \frac{320}{440} + \frac{3200}{3600} \times 4.2 \times \ln \frac{305}{290} \right] \\ &= 0.039663 \text{ kW/K} = 39.6634 \text{ W/K} \end{aligned}$$

$$\begin{aligned} \therefore \text{Rate of energy destruction} &= T_0 \times \dot{S}_{\text{gen}} = 290 \times 0.039663 \text{ kW} \\ &= 11.5024 \text{ kW} = 41.4 \text{ MJ/K} \end{aligned}$$

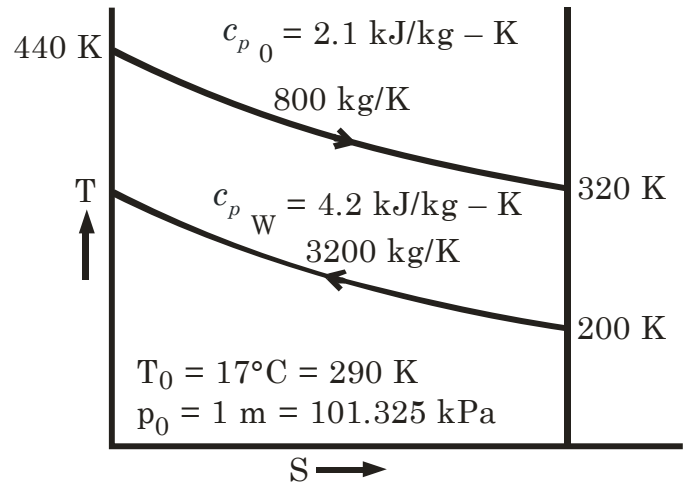
$$\begin{aligned} \text{(c)} \quad \text{Availability decrease of oil} \\ = A_1 - A_2 = h_1 - h_2 - T_0 (s_1 - s_2) \end{aligned}$$

$$\begin{aligned} &= \dot{m}_O c_{pO} \left[ (T_1 - T_2) - T_0 \ln \frac{T_1}{T_2} \right] \\ &= \frac{800}{3600} \times 2.1 \times \left[ (440 - 320) - 290 \ln \frac{440}{320} \right] \\ &= 12.903 \text{ kW} \end{aligned}$$

Availability decrease of water

$$\begin{aligned} A_1 - A_2 &= h_1 - h_2 - T_0 (s_1 - s_2) \\ &= \dot{m}_W c_{pW} \left[ (T_1 - T_2) - T_0 \ln \frac{T_1}{T_2} \right] \\ &= \frac{3200}{3600} \times 4.2 \times \left[ (305 - 290) - 290 \ln \frac{305}{290} \right] \text{ kW} = 1.4 \text{ kW} \end{aligned}$$

$$\therefore \text{Second law efficiency } (\eta_{II}) = \frac{\text{Gain of availability}}{\text{Loss for that}} = \frac{1.4}{12.903} = 10.85\%$$





# 9.

## Properties of Pure Substances

### Some Important Notes

#### 1. Triple point

On p-T diagram	It is a Point.
On p-V diagram	It is a Line
On T-s diagram	It is a Line
On U-V diagram	It is a Triangle

#### 2. Triple point of water

T = 273.16 K = 0.01°C	p = 0.00612 bar = 4.587 mm of Hg	Entropy (S) = 0 Internal Energy (u) = 0 Enthalpy (h) = u + pV = Slightly positive
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#### 3. Triple point of CO<sub>2</sub>

p ≈ 5 atm	And T = 216.55 K = - 56.45° C that so why sublimation occurred.
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#### 3. Critical Point

For water     $p_c = 221.2 \text{ bar} \approx 225.5 \text{ kgf/cm}^2$   
 $T_c = 374.15^\circ\text{C} \approx 647.15 \text{ K}$   
 $v_c = 0.00317 \text{ m}^3/\text{kg}$

#### At critical point

$$h_{fg} = 0; \quad v_{fg} = 0; \quad S_{fg} = 0$$

#### 4. Mollier Diagram

Basis of the h-S diagram is  $\left(\frac{\partial h}{\partial S}\right)_p = T$        $\left[ \begin{array}{l} \because TdS = dh - vdp \\ \therefore \left(\frac{\partial h}{\partial S}\right)_p = T \end{array} \right]$

∴ The slope of an isobar on the h-s co-ordinates is equal to the absolute saturation temperature at that pressure. And for that isobars on Mollier diagram diverges from one another.

5. Dryness fraction

$$x = \frac{m_v}{m_v + m_l}$$

6.	$v = (1 - x) v_f + x v_g$	$v = v_f + x v_{fg}$
	$u = (1 - x) u_f + x u_g$	$u = u_f + x u_{fg}$
	$h = (1 - x) h_f + x h_g$	$h = h_f + x h_{fg}$
	$s = (1 - x) s_f + x s_g$	$s = s_f + x s_{fg}$

7. **Super heated vapour:** When the temperature of the vapour is greater than the saturation temperature corresponding to the given pressure.

8. **Compressed liquid:** When the temperature of the liquid is less than the saturation temperature at the given pressure, the liquid is called compressed liquid.

9. In combined calorimeter

$x = x_1 \times x_2$	$x_1 = \text{from throttle calorimeter}$
	$x_2 = \text{from separation calorimeter}$

Questions with Solution P. K. Nag

Q9.1 Complete the following table of properties for 1 kg of water (liquid, vapour or mixture)

	$p$ (bar)	$t$ (°C)	$v$ (m <sup>3</sup> /kg)	$x$ (%)	Super- heat (°C)	$h$ (kJ/kg)	$s$ (kJ/kg K)
(a)	—	35	25.22	—	—	—	—
(b)	—	—	0.001044	—	—	419.04	—
(c)	—	212.42	—	90	—	—	—
(d)	1	—	—	—	—	—	6.104
(e)	10	320	—	—	—	—	—
(f)	5	—	0.4646	—	—	—	—
(g)	4	—	0.4400	—	—	—	—
(h)	—	500	—	—	—	3445.3	—
(i)	20	—	—	—	50	—	—
(j)	15	—	—	—	—	—	7.2690

Solution:

	p bar	t°C	v m <sup>3</sup> /kg	x/%	Superheat 0°C	h kJ/kg	s kJ/ kg – K
a	0.0563	35	25.22	100	0	2565.3	8.353
b	1.0135	100°	0.001044	0	0	419.04	1.307
c	20	212.42	0.089668	90	0	2608.3	5.94772
d	1	99.6	1.343	79.27	0	2207.3	6.104
e	10	320	0.2676	100	140	3093.8	7.1978
f	5	238.8°C	0.4646	100	87.024	2937.1	7.2235
g	4	143.6	0.4400	95.23	0	2635.9	6.6502
h	40	500	0.0864	100	249.6	3445.3	7.090
i	20	212.4°C	0.1145	100	50	2932.5	6.600
j	15	400	0.203	100	201.70	3255.8	7.2690

Calculations: For (a) ..... For (b)

$$\text{For (c) } v = v_f + x(v_g - v_f) \quad h = h_f + x h_{fg} \quad \Rightarrow s = s_f + x s_{fg}$$

$$\text{For (d) } s = s_f + x s_{fg} \quad \therefore x = \frac{s - s_f}{s_{fg}} = 0.7927 \quad \therefore h = h_f + x h_{fg}$$

$$v = v_f + x(v_{fg} - v_f)$$

$$\text{For (e) } t_{\text{sat}} = 180^\circ\text{C} \quad v = 0.258 + \frac{20}{50}(0.282 - 0.258),$$

$$h = 3051.2 + \frac{20}{50}(3157.8 - 3051.2) = 3093.8$$

$$s = 7.123 + \frac{20}{50}(7.310 - 7.123) = 7.1978$$

$$\text{For (f) } t = 200 + \frac{0.4646 - 0.425}{0.476 - 0.425} \times 50 = 238.8^\circ\text{C}$$

$$h = 2855.4 + \frac{38.8}{50}(2960.7 - 2855.4) = 2937.1$$

$$s = 7.059 + \frac{38.8}{50}(7.271 - 7.059) = 7$$

$$(g) \quad 0.4400 = 0.001084 + x(0.462 - 0.001084) \quad \therefore x = 0.9523$$

$$h = 604.7 + x \times 2133, \quad s = 1.7764 + x \times 5.1179 = 6.6502$$

$$(i) \quad t = 262.4^\circ\text{C} \quad v = 0.111 + \frac{12.4}{50}(0.121 - 0.111),$$

$$h = 2902.5 + \frac{12.4}{50}(3023.5 - 2902.5) = 2932.5$$

$$s = 6.545 + \frac{12.9}{50}(6.766 - 6.545) = 6.600$$

**Q9.2**

**(a) A rigid vessel of volume 0.86 m<sup>3</sup> contains 1 kg of steam at a pressure of 2 bar. Evaluate the specific volume, temperature, dryness fraction, internal energy, enthalpy, and entropy of steam.**

**(b) The steam is heated to raise its temperature to 150°C. Show the process on a sketch of the  $p$ - $v$  diagram, and evaluate the pressure, increase in enthalpy, increase in internal energy, increase in entropy of steam, and the heat transfer. Evaluate also the pressure at which the steam becomes dry saturated.**

**(Ans. (a) 0.86 m<sup>3</sup>/kg, 120.23°C, 0.97, 2468.54 kJ/kg, 2640.54 kJ/kg, 6.9592 kJ/kg K**

**(b) 2.3 bar, 126 kJ/kg, 106.6 kJ/kg, 0.2598 kJ/kg K, 106.6 kJ/K)**

**Solution:** (a)  $\rightarrow$  Specific volume = Volume/mass =  $\frac{0.86 \text{ m}^3}{1 \text{ kg}} = 0.86 \text{ m}^3/\text{kg}$

$\rightarrow$  at 2 bar pressure saturated steam sp. Volume = 0.885 m<sup>3</sup>/kg

So it is wet steam and temperature is saturation temperature  
= 120.2° C

$$\rightarrow v = v_f + x(v_g - v_f) \quad \therefore x = \frac{v - v_f}{v_g - v_f}$$

$$= \frac{0.86 - 0.001061}{0.885 - 0.001061} = 0.97172$$

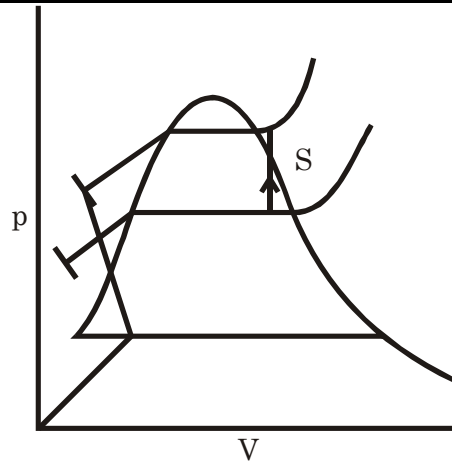
$\rightarrow$  Internal energy (u) = h - pv = 2644 - 200 × 0.86 = 2472 kJ/kg

$\rightarrow$  Here h = h<sub>f</sub> + x h<sub>fg</sub> = 504.7 + 0.97172 × 2201.6 = 2644 kJ/kg

$\rightarrow s = s_f + x s_{fg} = 1.5301 + 0.97172 \times 5.5967 = 6.9685 \text{ kJ/kg} - \text{K}$

**(b)  $T_2 = 150^\circ\text{C} = 423 \text{ K}$**

**$v_2 = 0.86 \text{ m}^3/\text{kg}$**



$$p_s = 2 + \frac{0.885 - 0.86}{0.885 - 0.846} + (2 + 0 - 2) = 2.0641 \text{ bar}$$

$$v_s = 0.86 \text{ m}^3/\text{kg}$$

$$T_s = \frac{0.0691}{0.1} (121.8 - 120.2) + 120.2 = 121.23^\circ \text{C} = 394.23 \text{ K}$$

Path 2 – 5 are in superheated zone so gas law (obey)

$$\therefore \frac{p_s v_1}{T_s} = \frac{p_2 v_2}{T_2} \quad [\because v_2 = v_1]$$

$$\therefore p_2 = \frac{T_2}{T_s} \times p_s = \frac{423}{394.23} \times 2.0641 = 2.215 \text{ bar}$$

From Molier diagram  $p_s = 2.3 \text{ bar}$ ,  $h_2 = 2770 \text{ kJ/kg}$ ,  $s_2 = 7.095$

$$\therefore \Delta h = 127 \text{ kJ/kg}, \Delta s = 0.1265 \text{ kJ/kg} - \text{K},$$

$$u_2 = h_2 - p_2 v_2 = 2580$$

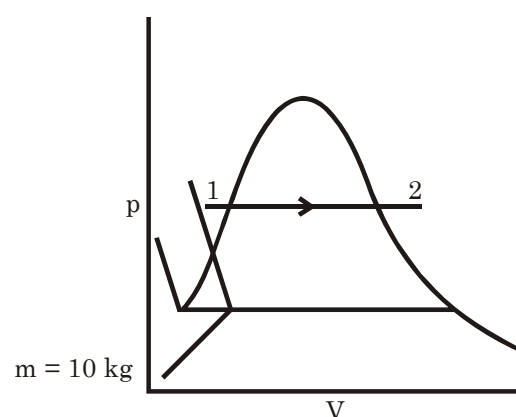
$$\therefore \Delta q = u_2 - u_1 = 107.5 \text{ kJ/kg}$$

**Q9.3**

**Ten kg of water at 45°C is heated at a constant pressure of 10 bar until it becomes superheated vapour at 300°C. Find the change in volume, enthalpy, internal energy and entropy.**

(Ans. 2.569 m<sup>3</sup>, 28627.5 kJ, 26047.6 kJ, 64.842 kJ/K)

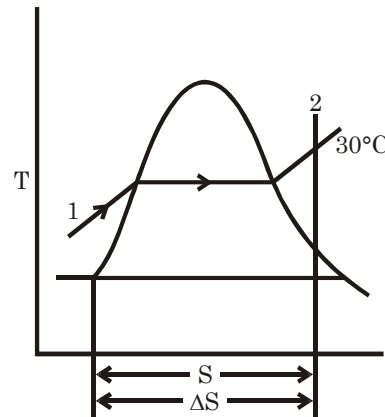
**Solution:**



At state (1)

$$p_1 = 10 \text{ bar} = 1000 \text{ kPa}$$

$$T_1 = 45^\circ \text{C} = 318 \text{ K}$$



At state (2)

$$p_2 = p_1 = 10 \text{ bar}$$

$$T_2 = 300^\circ \text{C}$$

For Steam Table

$$v_1 = 0.001010 \text{ m}^3/\text{kg}$$

$$h_1 = 188.4 \text{ kJ/kg}$$

$$u_1 = h_1 - p_1 v_1 = 187.39 \text{ kJ/kg}$$

$$s_1 = 0.693 \text{ kJ/kg} - \text{K}$$

$$v_2 = 0.258 \text{ m}^3/\text{kg}$$

$$u_2 = 2793.2 \text{ kJ/kg}$$

$$h_2 = 3051.2 \text{ kJ/kg}$$

$$s_2 = 7.123 \text{ kJ/kg} - \text{K}$$

$$\therefore \text{Change in volume} = m (v_2 - v_1) = 2.57 \text{ m}^3$$

$$\text{Enthalpy change} = m(h_2 - h_1) = 28.628 \text{ MJ}$$

$$\text{Internal Energy change} = m(u_2 - u_1) = 26.0581 \text{ MJ}$$

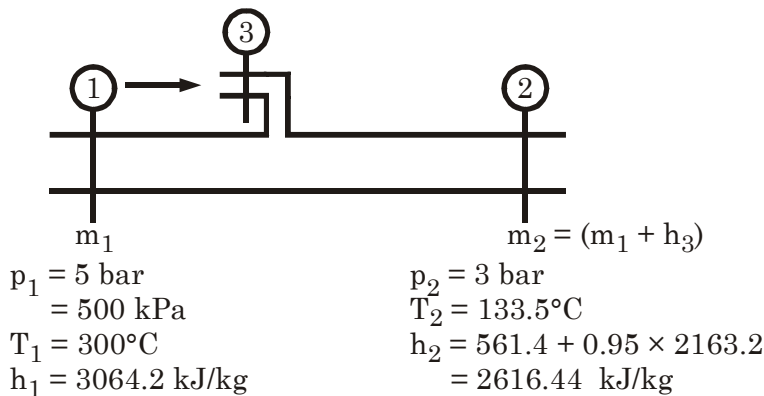
$$\text{Entropy change} = m (s_2 - s_1) = 64.3 \text{ kJ/K}$$

**Q9.4**

Water at 40°C is continuously sprayed into a pipeline carrying 5 tonnes of steam at 5 bar, 300°C per hour. At a section downstream where the pressure is 3 bar, the quality is to be 95%. Find the rate of water spray in kg/h.

(Ans. 912.67 kg/h)

**Solution:**



$$T_3 = 40^\circ \text{C}$$

$$h_3 = 167.6 \text{ kJ/kg}$$

$\therefore$  **For adiabatic steady flow**

$$\dot{m}_1 h_1 + \dot{m}_3 h_3 = \dot{m}_2 (h_2) = (\dot{m}_1 + \dot{m}_3) h_2$$

$$\therefore \dot{m}_1 (h_1 - h_2) = \dot{m}_3 (h_2 - h_3)$$

$$\therefore \dot{m}_3 = \dot{m}_1 \frac{(h_1 - h_2)}{(h_2 - h_3)}$$

$$= 5000 \times \left\{ \frac{3064.2 - 2616.44}{2616.44 - 167.6} \right\} \text{ kg/hr}$$

$$= 914.23 \text{ kg/hr.}$$

**Q9.5**

A rigid vessel contains 1 kg of a mixture of saturated water and saturated steam at a pressure of 0.15 MPa. When the mixture is heated, the state passes through the critical point. Determine

(a) The volume of the vessel

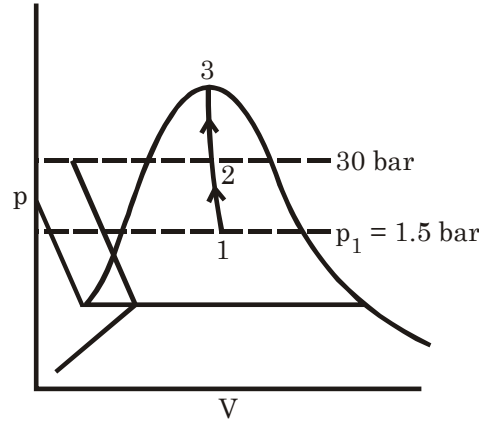
(b) The mass of liquid and of vapour in the vessel initially



- (c) The temperature of the mixture when the pressure has risen to 3 MPa  
 (d) The heat transfer required to produce the final state (c).

(Ans. (a) 0.003155 m<sup>3</sup>, (b) 0.9982 kg, 0.0018 kg,  
 (c) 233.9°C, (d) 581.46 kJ/kg)

Solution:



It is a rigid vessel so if we

- (a) Heat this then the process will be constant volume heating. So the volume of the vessel is critical volume of water = 0.00317 m<sup>3</sup>

$$(b) \quad v = v_f + x(v_g - v_{fg}) \quad \therefore x = \frac{v - v_f}{v_g - v_f} = \frac{0.00317 - 0.001053}{1.159 - 0.001053}$$

$$\therefore \text{Mass of vapour} = 0.0018282 \text{ kg}$$

$$\therefore \text{Mass of water} = 0.998172 \text{ kg}$$

- (c) As it passes through critical point then at 3 MPa i.e. 30 bar also it will be wet steam 50 temperatures will be 233.8°C.

- (d) Required heat (Q) = (u<sub>2</sub> - u<sub>1</sub>)

$$= (h_2 - h_1) - (p_2 v_2 - p_1 v_1)$$

$$= (h_{2f} + x_2 h_{fg_2}) - (h_{1f} + x_1 h_{fg_1}) - p_2 \{ v_f + x_2 (v_g - v_f) \}_2 + p_1 \{ v_f + x_1 (v_g - v_f) \}_1$$

$$v_2 = v_{f_2} + x_2 (v_{g_2} - v_{f_2})$$

$$\therefore x_2 = \frac{v_2 - v_{f_2}}{v_{g_2} - v_{f_2}} = \frac{0.00317 - 0.001216}{0.0666 - 0.001216} = 0.029885$$

$$\begin{aligned} \therefore Q &= (1008.4 + 0.029885 \times 1793.9) \\ &\quad - (467.1 + 0.0018282 \times 2226.2) - 3000 (0.001216 + \\ &\quad 0.029885 (0.0666 - 0.001216)) + 150(0.001053 + 0.001828 (1.159 - 0.0018282)) \\ &= 581.806 \text{ kJ/kg} \end{aligned}$$

Q9.6

A rigid closed tank of volume 3 m<sup>3</sup> contains 5 kg of wet steam at a pressure of 200 kPa. The tank is heated until the steam becomes dry saturated. Determine the final pressure and the heat transfer to the tank.

(Ans. 304 kPa, 3346 kJ)

**Solution:**  $V_1 = 3 \text{ m}^3$

$$m = 5 \text{ kg}$$

$$\therefore v_1 = \frac{3}{5} = 0.6 \text{ m}^3/\text{kg}$$

$$p_1 = 200 \text{ kPa} = 2 \text{ bar}$$

$$\therefore x_1 = \frac{v_1 - v_f}{v_g - v_f} = \frac{(0.6 - 0.001061)}{(0.885 - 0.001061)} = 0.67758$$

$$h_1 = h_f + x_1 h_{fg} = 504.7 + 0.67758 \times 2201.6 = 1996.5 \text{ kJ/kg}$$

$$u_1 = h_1 - p_1 v_1 = 1996.5 - 200 \times 0.6 = 1876.5 \text{ kJ/kg}$$

As rigid tank so heating will be cost vot heating.

$$\therefore v_{g_2} = 0.6 \text{ m}^3/\text{kg}$$

From Steam Table  $v_g = 0.606 \text{ m}^3/\text{kg}$  for  $p = 300 \text{ kPa}$   
 $v_g = 0.587 \text{ m}^3/\text{kg}$  for  $p = 310 \text{ kPa}$

$$\therefore \text{For } V = 0.6 \text{ m}^3 \quad p_2 = 300 \times \frac{10 \times 0.006}{0.019} = 303.16 \text{ kPa}$$

$$\therefore u_2 = h_2 - p_2 v_2 = 2725 - 303.16 \times 0.6 = 2543 \text{ kJ/kg}$$

$$\therefore \text{Heat supplied } Q = m(u_2 - u_1) = 3333 \text{ kJ}$$

**Q9.7**

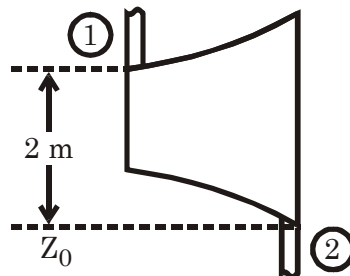
Steam flows through a small turbine at the rate of 5000 kg/h entering at 15 bar, 300°C and leaving at 0.1 bar with 4% moisture. The steam enters at 80 m/s at a point 2 in above the discharge and leaves at 40 m/s. Compute the shaft power assuming that the device is adiabatic but considering kinetic and potential energy changes. How much error would be made if these terms were neglected? Calculate the diameters of the inlet and discharge tubes.

(Ans. 765.6 kW, 0.44%, 6.11 cm, 78.9 cm)

**Solution:**  $\dot{m} = 5000 \text{ kg/hr} = \frac{5000}{3600} \text{ kg/s}$

$$p_1 = 15 \text{ bar}$$

$$t_1 = 300^\circ \text{C}$$



$\therefore$  From Steam Table

$$h_1 = 3037.6 \text{ kJ/kg}$$

$$V_1 = 80 \text{ m/s}$$

$$Z_1 = (Z_0 + 2) \text{ m}$$

$$p_2 = 0.1 \text{ bar}$$

$$x_2 = \frac{(100 - 4)}{100} = 0.96$$

$$t_2 = 45.8^\circ \text{C}$$

$$v_1 = 0.169 \text{ m}^3/\text{kg} \quad h_2 = h_f + x_2 h_{fg}$$

$$= 191.8 + 0.96 \times 2392.8 = 2489 \text{ kJ/kg}$$

$$V_2 = 40 \text{ m/s}, Z_2 = Z_0 \text{ m} \quad \therefore v_2 = 14.083 \text{ m}^3/\text{kg}$$

$$\therefore \text{Work output (W)} = \dot{m} \left[ (h_1 - h_2) + \frac{V_1^2 - V_2^2}{2000} + \frac{g(Z_1 - Z_2)}{2000} \right]$$

$$= \frac{5000}{3600} \left[ (3037.6 - 2489) + \frac{80^2 - 90^2}{2000} + \frac{9.81(2)}{2000} \right] \text{ kW}$$

$$= 765.45 \text{ kW}$$

If P.E. and K.E. is neglected the

$$W' = \dot{m}(h_1 - h_2) = 762.1 \text{ kW}$$

$$\therefore \text{Error} = \frac{W - W'}{W} \times 100\% = 0.44\%$$

$$\text{Area at inlet (A}_1\text{)} = \frac{\dot{m}v_1}{V_1} = 0.002934 \text{ m}^2 = 29.34 \text{ cm}^2$$

$$\therefore d_1 = 6.112 \text{ cm}$$

$$\text{Area at outlet (A}_2\text{)} = \frac{\dot{m}v_2}{V_2} = 0.489 \text{ m}^2 \quad \therefore d_2 = 78.9 \text{ cm}$$

**Q9.8** A sample of steam from a boiler drum at 3 MPa is put through a throttling calorimeter in which the pressure and temperature are found to be 0.1 MPa, 120°C. Find the quality of the sample taken from the boiler.

(Ans. 0.951)

**Solution :**  $p_1 = 3 \text{ MPa} = 30 \text{ bar}$

$p_2 = 0.1 \text{ MPa} = 1 \text{ bar}$

$t_2 = 120^\circ \text{ C}$

$$h_2 = 2676.2 + \frac{20}{50} (2776.4 - 2676.2)$$

$$= 2716.3 \text{ kJ/kg}$$

$$\therefore h_1 = h_2$$

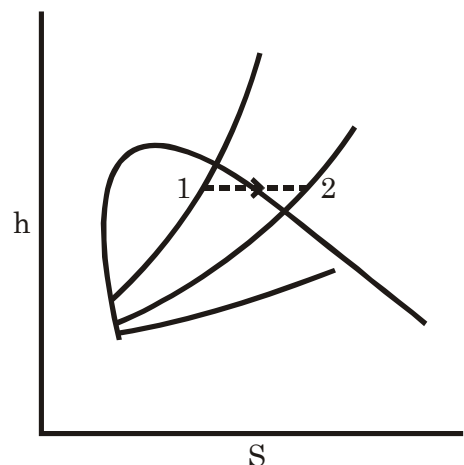
$$\therefore h_1 = 2716.3 \text{ at } 30 \text{ bar}$$

If dryness fraction is

$$\therefore h_1 = h_{g1} + x h_{fg1}$$

$$\therefore x = \frac{h_1 - h_{f1}}{h_{fg1}}$$

$$= \frac{2716.3 - 1008.4}{1793.9} = 0.952$$



**Q9.9** It is desired to measure the quality of wet steam at 0.5 MPa. The quality of steam is expected to be not more than 0.9.

(a) Explain why a throttling calorimeter to atmospheric pressure will not serve the purpose.

- (b) Will the use of a separating calorimeter, ahead of the throttling calorimeter, serve the purpose, if at best 5°C degree of superheat is desirable at the end of throttling? What is the minimum dryness fraction required at the exit of the separating calorimeter to satisfy this condition?

(Ans. 0.97)

**Solution:** (a) After throttling if pressure is atm. Then minimum temperature required  
 $t = t_{\text{sat}} + 5^\circ\text{C} = 100 + 5 = 105^\circ\text{C}$   
 Then Enthalpy required

$$= 2676 + \frac{5}{50}(2776.3 - 2676) \text{ kJ/kg} = 2686 \text{ kJ/kg}$$

If at 0.5 MPa = 5 bar dryness fraction is < 0.9

$$\therefore h_{\text{max}} = h_f + 0.9 h_{fg} = 640.1 + 0.9 \times 2107.4 = 2536.76 \text{ kJ/kg}$$

So it is not possible to give 5° super heat or at least saturation i.e. (2676 kJ/kg) so it is not correct.

- (b) Minimum dryness fraction required at the exit of the separating calorimeter (x) then

$$h = h_f + x h_{fg} \quad \therefore x = \frac{2686 - 640.1}{2107.4} = 0.971$$

**Q9.10**

The following observations were recorded in an experiment with a combined separating and throttling calorimeter:

Pressure in the steam main–15 bar

Mass of water drained from the separator–0.55 kg

Mass of steam condensed after passing through the throttle valve –4.20 kg

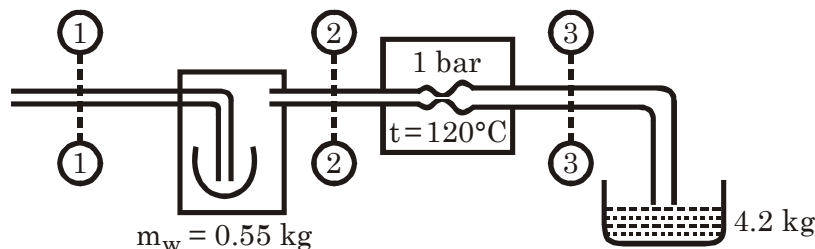
Pressure and temperature after throttling–1 bar, 120°C

Evaluate the dryness fraction of the steam in the main, and state with reasons, whether the throttling calorimeter alone could have been used for this test.

(Ans. 0.85)

**Solution:**  $p_1 = 15 \text{ bar} = p_2$   
 $T_1 = 198.3^\circ\text{C} = t_2$   
 $p_3 = 1 \text{ bar}, T_3 = 120^\circ\text{C}$

$$\therefore h_3 = 2716.3 \text{ kJ/kg}$$



$$h_2 = h_{2f} + x_2 \times h_{fg_2} = 844.7 + x_2 \times 1945.2$$

$$\therefore x_2 = 0.96216 \quad \therefore \text{dry steam} = x_2 \times 4.2$$

$$\therefore \text{Total dryness fraction (x)}$$

$$= \frac{x_2 \times 4.2}{4.2 + 0.55} = \frac{0.96216 \times 4.2}{4.2 + 0.55} = 0.85$$

$$h_1 = h_{f1} + x h_{fg1} = 844.7 + 0.85 \times 1945.2 = 2499.6 \text{ kJ/kg}$$

But at 1 bar minimum 5° super heat i.e. 105°C enthalpy is 2686 kJ/kg

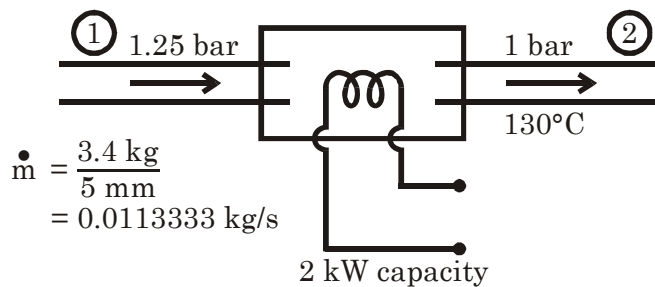
So it is not possible to calculate only by throttling calorimeter.

**Q9.11** Steam from an engine exhaust at 1.25 bar flows steadily through an electric calorimeter and comes out at 1 bar, 130°C. The calorimeter has two 1 kW heaters and the flow is measured to be 3.4 kg in 5 min. Find the quality in the engine exhaust. For the same mass flow and pressures, what is the maximum moisture that can be determined if the outlet temperature is at least 105°C?

(Ans. 0.944, 0.921)

**Solution:**  $h_2 = 2676.2 + \frac{30}{50} (2776.4 - 2676.2) = 2736.3 \text{ kJ/kg}$

$$\dot{m} h_1 = \dot{m} h_2 - \dot{Q}$$



$$h_1 = h_2 - \frac{\dot{Q}}{\dot{m}} = 2560 \text{ kJ/kg}$$

At 1.25 bar: from Steam Table

At 1.2 bar,  $h_f = 439.4 \text{ kJ/kg}$

At 1.3 bar,  $h_f = 449.2 \text{ kJ/kg}$

At 1.25 bar  $h_f = 444.3 \text{ kJ/kg}$ ;

If dryness fraction is  $x$

$$\text{Then } 2560 = 444.3 + x \times 2241$$

$$\text{or } x = 0.9441$$

If outlet temperature is 105° C then

$$h_2 = 2686 \text{ kJ/kg}$$

$$h_{fg} = 2244.1 \text{ kJ/kg}$$

$$h_{fg} = 2237.8 \text{ kJ/kg}$$

$$h_{fg} = 2241 \text{ kJ/kg}$$

(then from problem 9.9)

$$\therefore h_1 = h_2 - \frac{\dot{Q}}{\dot{m}} = 2509.53 \text{ kJ/kg}$$

Then if dryness fraction is  $x_2$  then

$$2509 = 444.3 + x_2 \times 2241$$

$$\therefore x_2 = 0.922 \text{ (min)}$$

**Q9.12** Steam expands isentropically in a nozzle from 1 MPa, 250°C to 10 kPa. The steam flow rate is 1 kg/s. Find the velocity of steam at the exit from the nozzle, and the exit area of the nozzle. Neglect the velocity of steam at the inlet to the nozzle.

The exhaust steam from the nozzle flows into a condenser and flows out as saturated water. The cooling water enters the condenser at 25°C and leaves at 35°C. Determine the mass flow rate of cooling water.

(Ans. 1224 m/s, 0.0101 m<sup>2</sup>, 47.81 kg/s)

**Solution:** At inlet

$$h_1 = 2942.6 \text{ kJ/kg}$$

$$t_2 = 45.8^\circ \text{C}$$

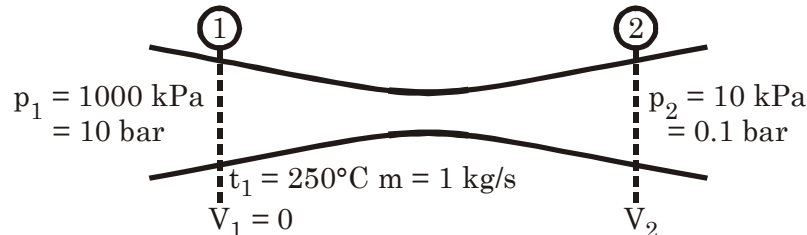
$$s_1 = 6.925 \text{ kJ/kg-K}$$

$$s_2 = 6.925 \text{ kJ/kg-K}$$

If dry fraction  $x$

$$h_{f2} = 191.8 \text{ kJ/kg}$$

$$v_2 = 12.274 \text{ m}^3/\text{kg}$$



$$\therefore s_1 = s_2 = 0.649 + x \times 7.501 \quad \therefore x = 0.8367$$

$$\therefore h_2 = 191.8 + 0.8367 \times 2392.8 = 2193.8 \text{ kJ/kg}$$

$$\therefore V_2 = \sqrt{2000(h_1 - h_2)} = 1224 \text{ m/s}$$

$$\therefore \text{Outlet Area} = \frac{\dot{m}v_2}{V_2} = 100.3 \text{ cm}^2$$

$\therefore$  If water flow rate is  $m$  kg/s

$$\therefore 1 \times (2193.8 - 191.8) = m \times 4.187 (35 - 25)$$

$$\therefore m = 47.815 \text{ kg/s}$$

**Q9.13** A reversible polytropic process, begins with steam at  $p_1 = 10 \text{ bar}$ ,  $t_1 = 200^\circ\text{C}$ , and ends with  $p_2 = 1 \text{ bar}$ . The exponent  $n$  has the value 1.15. Find the final specific volume, the final temperature, and the heat transferred per kg of fluid.

**Solution:**  $p_1 = 10 \text{ bar} = 1000 \text{ kPa}$

$$p_2 = 1 \text{ bar} = 100 \text{ kPa}$$

$$t_1 = 200^\circ \text{C} = 473 \text{ K}$$

From Steam Table

$$V_1 = 0.206 \text{ m}^3/\text{s}$$

$$h_1 = 2827.9 \text{ kJ/kg}$$

$$\therefore v_2 = \frac{p_1 v_1^n}{p_2} = \left( \frac{p_1}{p_2} \right)^{\frac{1}{n}} \cdot v_1 = \left( \frac{10}{1} \right)^{\frac{1}{1.15}} \times 0.206 = 1.5256 \text{ m}^3/\text{kg}$$

As at 1 bar  $v_g = 1.694 \text{ m}^3/\text{kg}$   $\therefore$  then steam is wet

$$\therefore 1.5256 = 0.001043 + x (1.694 - 0.001043)$$

$$\therefore x = 0.9$$

Final temperature =  $99.6^\circ \text{C}$

$$= u_1 - u_2$$

$$= (h_1 - h_2) - (p_1 v_1 - p_2 v_2)$$

$$= (2827.9 - 2450.8) - (1000 \times 0.206 - 100 \times 1.5256)$$

$$= 323.7 \text{ kJ/kg}$$

$$[h_2 = h_{f2} + x h_{fg2}]$$

$$= 417.5 + 0.9 \times 2257.9$$

$$= 2450.8$$

$$\text{Work done (W)} = \frac{P_1 V_1 - P_2 V_2}{n - 1} = 356.27 \text{ kJ/kg}$$

∴ From first law of thermo dynamics

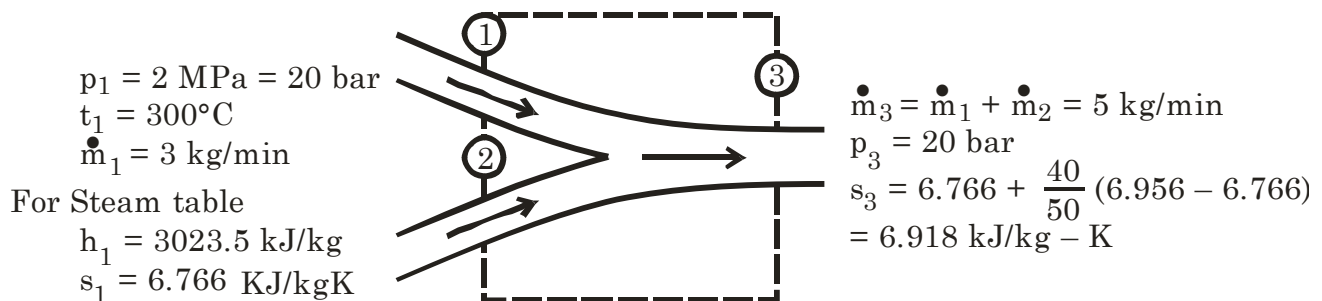
$$Q_2 = (u_2 - u_1) + W_{1-2}$$

$$= (-323.7 + 356.27) = 32.567 \text{ kJ/kg}$$

**Q9.14** Two streams of steam, one at 2 MPa, 300°C and the other at 2 MPa, 400°C, mix in a steady flow adiabatic process. The rates of flow of the two streams are 3 kg/min and 2 kg/min respectively. Evaluate the final temperature of the emerging stream, if there is no pressure drop due to the mixing process. What would be the rate of increase in the entropy of the universe? This stream with a negligible velocity now expands adiabatically in a nozzle to a pressure of 1 kPa. Determine the exit velocity of the stream and the exit area of the nozzle.

(Ans. 340°C, 0.042 kJ/K min, 1530 m/s, 53.77 cm<sup>2</sup>)

**Solution:**  $p_2 = 2 \text{ MPa} = 20 \text{ bar}$   
 $t_2 = 400^\circ \text{C}$   
 $\dot{m}_2 = 2 \text{ kg/min}$   
 $h_2 = 3247.6 \text{ kJ/kg}$   
 $s_2 = 7.127 \text{ kJ/kg-K}$



For adiabatic mixing process

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

$$\therefore h_3 = 3113.14 \text{ kJ/kg}$$

$$\therefore \text{Final temperature (t)} = 300 + \frac{3113.14 - 3023.5}{3137 - 3023.5} \times 50 = 340^\circ \text{C}$$

Rate of increase of the enthalpy of the universe

$$\dot{s}_{\text{gen}} = \dot{m}_3 S_3 - \dot{m}_1 S_1 - \dot{m}_2 S_2 = 0.038 \text{ kJ/K - min}$$

After passing through nozzle if dryness fraction is x then

$$S_3 = S_{\text{exit}} \text{ or } 6.918 = 0.106 + x \times 8.870 \quad \therefore x = 0.768$$

$$\therefore h_e = 29.3 + 0.768 \times 2484.9 = 1937.7 \text{ kJ/kg}$$

$$\therefore V = \sqrt{2000 (3113.14 - 1937.7)} = 1533.3 \text{ m/s}$$

$$\text{Exit area of the nozzle} = \frac{\dot{m}v}{V} = 0.0054 \text{ m}^2 = 54 \text{ cm}^2$$

Q9.15

Boiler steam at 8 bar, 250°C, reaches the engine control valve through a pipeline at 7 bar, 200°C. It is throttled to 5 bar before expanding in the engine to 0.1 bar, 0.9 dry. Determine per kg of steam

- The heat loss in the pipeline
- The temperature drop in passing through the throttle valve
- The work output of the engine
- The entropy change due to throttling
- The entropy change in passing through the engine.

(Ans. (a) 105.3 kJ/kg, (b) 5°C, (c) 499.35 kJ/kg, (d) 0.1433 kJ/kg K, (e) 0.3657 kJ/kg K)

Solution:

∴ From Steam Table

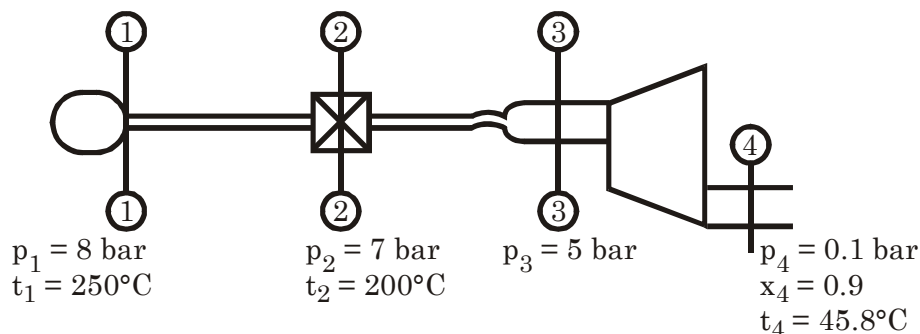
$$h_1 = 2950.1 \text{ kJ/kg}$$

$$h_2 = 2844.8$$

$$h_3 = 2844.8$$

$$h_4 = h_{fa} + x_a h_{fga} = 2345.3 \text{ kJ/kg}$$

∴ Heat loss in pipe line =  $(h_1 - h_2) = 105.3 \text{ kJ/kg}$



(b) In throttling process  $h_2 = h_3$

From Steam Table

5 bar 151.8° C

$$h_g = 2747.5$$

5 bar 200° C

$$h = 2855.4$$

$$\therefore t_3 = 200 - \frac{2855.4 - 2844.8}{2855.4 - 2747.5} \times (200 - 151.8)$$

$$= 200 - 4.74 = 195.26^\circ \text{ C}$$

∴  $\Delta t = 4.74^\circ \text{ C}$  (drop)

(c) Work output for the engine (W) =  $h_3 - h_4$

$$= (2844.8 - 2345.3) \text{ kJ/kg} = 499.48 \text{ kJ/kg}$$

(d) From Steam Table

$$s_2 = 6.886 \text{ kJ/kg} - \text{K}$$

$$s_3 = 6.8192 + \frac{(195.26 - 151.8)}{(200 - 151.8)} (7.059 - 6.8192) = 7.03542 \text{ kJ/kg} - \text{K}$$

$$\therefore \Delta s = s_3 - s_2 = 0.1494 \text{ kJ/kg} - \text{K}$$

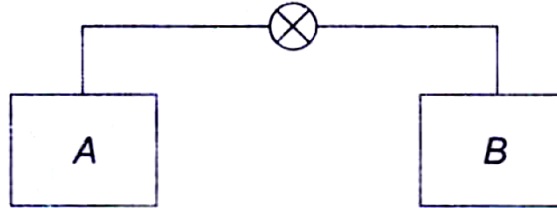
(e) For Steam Table

$$s_4 = s_{ga} + 0.9 s_{fga} = 0.649 + 0.9 \times 7.501 = 7.4 \text{ kJ/kg} - \text{K}$$

$$\Delta S = s_4 - s_3 = 0.3646 \text{ kJ/kg} - \text{K}$$



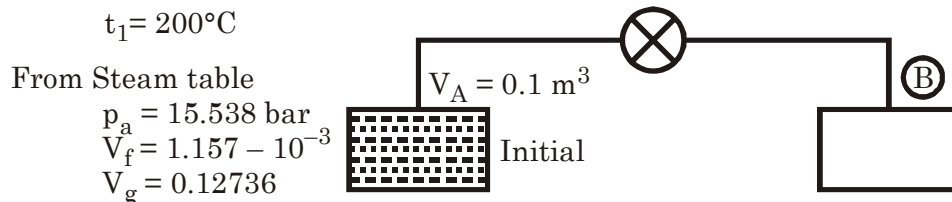
**Q9.16** Tank *A* (Figure) has a volume of  $0.1 \text{ m}^3$  and contains steam at  $200^\circ\text{C}$ , 10% liquid and 90% vapour by volume, while tank *B* is evacuated. The valve is then opened, and the tanks eventually come to the same pressure,



Which is found to be 4 bar. During this process, heat is transferred such that the steam remains at  $200^\circ\text{C}$ . What is the volume of tank *B*?

(Ans.  $4.89 \text{ m}^3$ )

**Solution:**



$$\therefore \text{Initial volume of liquid} = \frac{10}{100} \times 0.1 \text{ m}^3$$

$$m_f = 8.643 \text{ kg}$$

$$\text{Initial mass of steam} = (m_g)$$

$$= \frac{90}{100} \times 0.1$$

$$= \frac{0.09}{0.12736} \text{ kg} = 0.70666 \text{ kg}$$

$$\therefore \text{Total mass} = 9.3497 \text{ kg}$$

After open the valve when all over per = 4 bar at  $200^\circ\text{C}$

Then sp. Volume =  $0.534 \text{ m}^3/\text{kg}$

$$\therefore \text{Total volume (V)} = 9.3497 \times 0.534 \text{ m}^3 = 4.9927 \text{ m}^3$$

$$\therefore \text{Volume of Tank B} = V - V_A = 4.8927 \text{ m}^3$$

**Q9.17** Calculate the amount of heat which enters or leaves 1 kg of steam initially at 0.5 MPa and  $250^\circ\text{C}$ , when it undergoes the following processes:

- It is confined by a piston in a cylinder and is compressed to 1 MPa and  $300^\circ\text{C}$  as the piston does 200 kJ of work on the steam.
- It passes in steady flow through a device and leaves at 1 MPa and  $300^\circ\text{C}$  while, per kg of steam flowing through it, a shaft puts in 200 kJ of work. Changes in K.E. and P.E. are negligible.
- It flows into an evacuated rigid container from a large source which is maintained at the initial condition of the steam. Then 200 kJ of shaft work is transferred to the steam, so that its final condition is 1 MPa and  $300^\circ\text{C}$ .

(Ans. (a)  $-130 \text{ kJ}$  (b)  $-109 \text{ kJ}$ , and (c)  $-367 \text{ kJ}$ )

**Solution:** Initially:  $p_i = 0.5 \text{ MPa} = 5 \text{ bar}$ ; mass = 1 kg

$$t_i = 250^\circ\text{C}$$

$$\therefore \text{From Steam Table} \quad h_i = 2960.7 \text{ kJ/kg}$$

$$u_i = 2729.5 \text{ kJ/kg}$$

$$v_i = 0.474 \text{ m}^3/\text{kg}$$

(a) After compression  
 $p = 1 \text{ MPa} = 10 \text{ bar}$

$$T = 300^\circ\text{C}$$

$$\therefore \text{From S.T.} \quad u = 2793.2 \text{ kJ/kg}$$

$$h = 3051.2 \text{ kJ/kg}$$

$$\text{and} \quad W_{\text{input}} = 200 \text{ kJ}$$

$\therefore$  From first law of thermodynamics

$$\begin{aligned} Q_{1-2} &= m(u_2 - u_1) + W_{1-2} \\ &= [(2793.2 - 2729.5) - 200] \text{ kJ} \\ &= [63.7 - 200 \text{ kJ}] = -136.3 \text{ kJ} \\ &\text{i.e. heat rejection to atm.} \end{aligned}$$



(b) For steady flow process

$$\begin{aligned} h_1 + \frac{V_1^2}{2} + gz_1 + \frac{dQ}{dm} \\ = h_2 + \frac{V_2^2}{2} + gz_2 + \frac{dW}{dm} \end{aligned}$$

$h$  or  $V_1, V_2, Z_1, Z_2$  are negligible so

$$\begin{aligned} \frac{dQ}{dm} &= (h_2 - h_1) + \frac{dW}{dm} \\ &= (3051.2 - 2960.7) - 200 \\ &= -109.5 \text{ kJ/kg} \\ &\quad \text{[heat rejection]} \end{aligned}$$

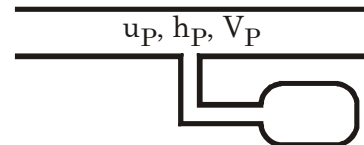
(c) Energy of the gas after filling

$$E_1 = u_i \text{ kJ/kg} = 2729.5 \text{ kJ/kg}$$

Energy of the gas after filling

$$E_2 = u_2 = 2793.2 \text{ kJ/kg}$$

$$\begin{aligned} \therefore \Delta E &= E_2 - E_1 \\ &= (2793.2 - 2729.5) \text{ kJ/kg} \\ &= 63.7 \text{ kJ/kg} \end{aligned}$$



There is a change in a volume of gas because of the collapse of the envelop to zero volume

$$W_1 = p_i(0 - v_i) = -p_i v_i = -500 \times 0.474 \text{ kJ/kg} = -237 \text{ kJ/kg}$$

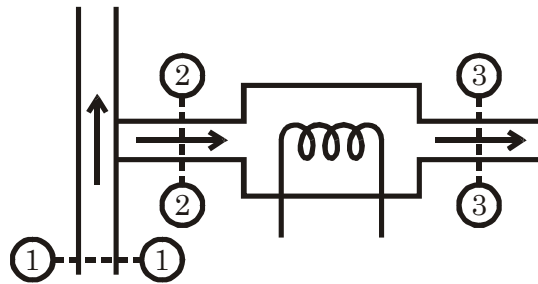
$\therefore$  From first law of thermodynamic

$$\begin{aligned} Q &= \Delta E + W_1 + W_2 \\ &= (63.7 - 237 - 200) \text{ kJ/kg} = -373.3 \text{ kJ/kg} \end{aligned}$$

- Q9.18** A sample of wet steam from a steam main flows steadily through a partially open valve into a pipeline in which is fitted an electric coil. The valve and the pipeline are well insulated. The steam mass flow rate is 0.008 kg/s while the coil takes 3.91 amperes at 230 volts. The main pressure is 4 bar, and the pressure and temperature of the steam downstream of the coil are 2 bar and 160°C respectively. Steam velocities may be assumed to be negligible.
- (a) Evaluate the quality of steam in the main.
- (b) State, with reasons, whether an insulated throttling calorimeter could be used for this test.

(Ans. (a) 0.97, (b) Not suitable)

**Solution:** (a)



$$\begin{aligned}\dot{m}_2 &= 0.008 \text{ kg/s} & \dot{Q} &= i^2 R = \frac{3.91 \times 230}{1000} \text{ kW} = 0.8993 \text{ kW} \\ p_2 &= 4 \text{ bar} & p_3 &= 2 \text{ bar}; t_2 = 160^\circ\text{C} \\ t_2 &= 143.6^\circ\text{C} & h_3 &= 2768.8 + \frac{10}{50} (2870.5 - 2768.8) \text{ kJ/kg} \\ & & &= 2789.14 \text{ kJ/kg}\end{aligned}$$

From steady flow energy equation

$$\dot{m} h_2 + \dot{Q} = \dot{m} h_3 + 0 : h_2 = h_3 - \frac{\dot{Q}}{\dot{m}} = 2676.73 \text{ kJ/kg}$$

If dryness fraction of steam  $x$  then

$$\begin{aligned}h_2 &= h_{f2} + x h_{fg2} \\ \text{or } 2676.73 &= 604.7 + x \times 2133 \quad \therefore x = 0.9714\end{aligned}$$

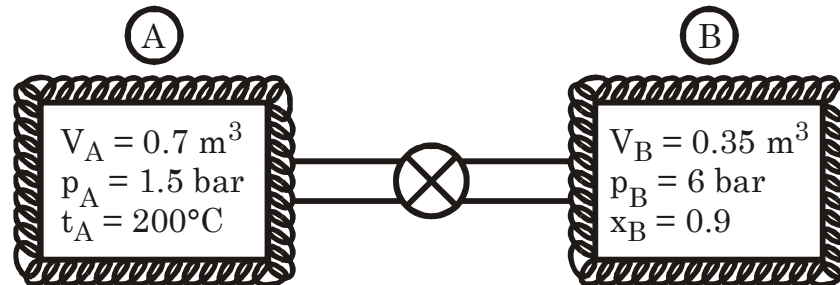
- (b) For throttling minimum enthalpy required 2686 kJ/kg if after throttling 5°C super heat and atm. Pressure is maintained as here enthalpy is less so it is not possible in **throttling calorimeter**.

- Q9.19** Two insulated tanks, *A* and *B*, are connected by a valve. Tank *A* has a volume of 0.70 m<sup>3</sup> and contains steam at 1.5 bar, 200°C. Tank *B* has a volume of 0.35 m<sup>3</sup> and contains steam at 6 bar with a quality of 90%. The valve is then opened, and the two tanks come to a uniform state. If there is no heat transfer during the process, what is the final pressure? Compute the entropy change of the universe.

(Ans. 322.6 KPa, 0.1985 kJ/K)

**Solution:** From Steam Table from Steam Table  
Sp. Enthalpy ( $h_A$ ) = 2872.9 kJ/kg  $t_B = 158.8^\circ\text{C}$   
Sp. Vol ( $v_A$ ) = 1.193 m<sup>3</sup>/kg Sp. Enthalpy ( $h_B$ )

$$= (670.4 + 0.9 \times 2085) = 2547 \text{ kJ/kg}$$



Sp. Internal energy ( $u$ ) = 2656.2 kJ/kg =

Sp. Vol. ( $v_B$ ) =  $v_{Bf} + x(v_{Bg} - v_{Bf}) = 0.2836 \text{ m}^3/\text{kg}$

Sp. entropy ( $s$ ) = 7.643 kJ/kg – K

Sp. in energy ( $u_B$ ) =  $u_f + x \times u_{fg} = 2376.7 \text{ kJ/kg}$

Sp. entropy ( $s_B$ ) = 6.2748 kJ/kg – K

$$u_{f_B} = h_{f_B} - p_{f_B} v_{f_B} = 670.4 - 600 \times 0.001101 = 669.74 \text{ kJ/kg}$$

$$u_{fg} = h_{fg} - p_{f_B} (v_g - v_f) \quad m_B = \frac{V_B}{v_B} = 1.2341 \text{ kg}$$

$$= 1896.7 \text{ kJ/kg}$$

$$\therefore m_A = \frac{V_A}{v_A} = 0.61242 \text{ kg}$$

$\therefore$  From First Law of thermodynamics

$$U_1 = U_2$$

$$\therefore m_A u_A + m_B u_B = (m_A + m_B) u \quad \therefore u = 2469.4 \text{ kJ/kg}$$

$$\text{And sp. volume of gas after mixing} = \frac{V_A + V_B}{m_A + m_B} = 0.5686 \text{ m}^3/\text{kg}$$

**Q9.20**

A spherical aluminum vessel has an inside diameter of 0.3 m and a 0.62 cm thick wall. The vessel contains water at 25°C with a quality of 1%. The vessel is then heated until the water inside is saturated vapour. Considering the vessel and water together as a system, calculate the heat transfer during this process. The density of aluminum is 2.7 g/cm<sup>3</sup> and its specific heat is 0.896 kJ/kg K.

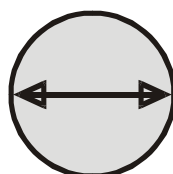
(Ans. 2682.82 kJ)

**Solution:** Volume of water vapour mixture ( $V$ ) =  $\frac{4}{3} \pi d_i^3 = 0.113097 \text{ m}^3$

$$\text{Ext. volume} = \frac{4}{3} \pi d_o^3 = 0.127709 \text{ m}^3$$

$$\therefore \text{Volume of } A_1 = 0.0146117 \text{ m}^3$$

$$\therefore \text{Mass} = 39.451 \text{ kg}$$



$$d_i = 0.3 \text{ m}$$

$$d_o = d_i + 2t = 0.3 + \frac{0.62}{100} \times 2 \text{ m} = 0.3124 \text{ m}$$

At 25° C; 1% quality

From Steam Table

$$p_1 = 0.0317 \text{ bar}$$

$$v_1 = 0.001003 + \frac{1}{100}(43.36) = 0.434603 \text{ m}^3/\text{kg}$$

$$h_1 = 104.9 + \frac{1}{100} \times 24212.3 = 129.323 \text{ kJ/kg}$$

$$u_1 = h_1 - p_1 v_1 = 127.95 \text{ kJ/kg}$$

$$\text{Mass of water and water vapour} = \frac{0.113097}{0.434603} \text{ kg} = 0.26023 \text{ kg}$$

Carnot volume heating until dry saturated

So then Sp. volume  $v_g = 0.434603 \text{ m}^3/\text{kg}$

For Steam Table

At 4.2 bar  $v_g = 0.441$

At 4.4 bar  $v_g = 0.423$

$$(p_f) = 4.2 + 0.2 \times \frac{0.441 - 0.434603}{0.441 - 0.423} = 4.27 \text{ bar}$$

$$\text{Then } h_f = 2739.8 + \frac{0.07}{0.2} (2741.9 - 2739.8) = 2740.55 \text{ kJ/kg}$$

$$t_f = 146^\circ \text{ C}$$

$$u_f = h_f - p_f v_f = 2555 \text{ kJ/kg}$$

$$\begin{aligned} \therefore \text{Heat required to water} &= m(u_f - u_1) \\ &= 0.26023(2555 - 127.95) \text{ kJ} \\ &= 631.58 \text{ kJ} \end{aligned}$$

$$\begin{aligned} \text{Heat required for } A_1 &= 39.451 \times 0.896 \times (146 - 25) \\ &= 4277.2 \text{ kJ} \end{aligned}$$

$$\text{Total heat required} = 4908.76 \text{ kJ}$$

**Q9.21**

**Steam at 10 bar, 250°C flowing with negligible velocity at the rate of 3 kg/min mixes adiabatically with steam at 10 bar, 0.75 quality, flowing also with negligible velocity at the rate of 5 kg/min. The combined stream of steam is throttled to 5 bar and then expanded isentropically in a nozzle to 2 bar. Determine**

- (a) The state of steam after mixing**
- (b) The state of steam after throttling**
- (c) The increase in entropy due to throttling**
- (d) The velocity of steam at the exit from the nozzle**
- (e) The exit area of the nozzle. Neglect the K.E. of steam at the inlet to the nozzle.**

(Ans. (a) 10 bar, 0.975 dry, (b) 5 bar, 0.894 dry, (c) 0.2669 kJ/kg K, (d) 540 m/s, (e) 1.864 cm<sup>2</sup>)

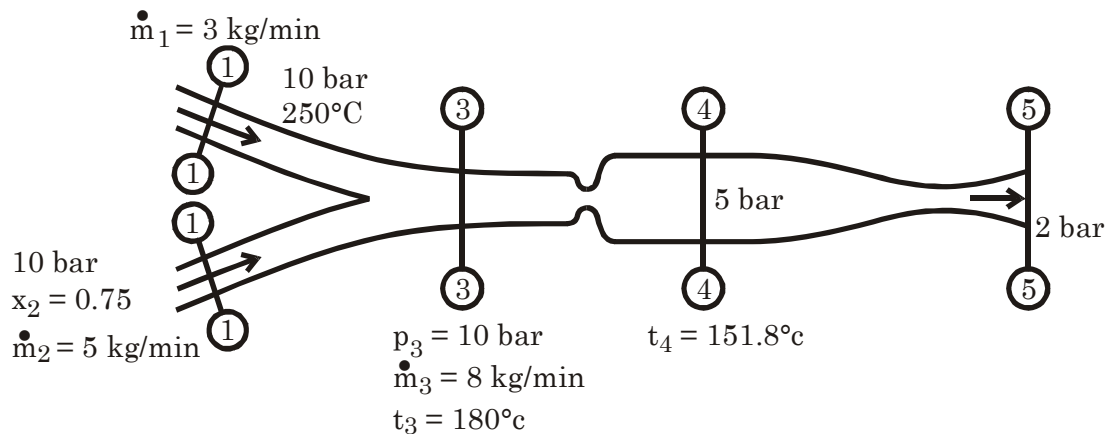
**Solution:**

From Steam Table

$$h_1 = 2942.6 \text{ kJ/kg}$$

$$h_2 = 762.6 + 0.75 \times 2013.6 = 2272.8 \text{ kJ/kg}$$

$$\begin{aligned} \therefore h_3 &= \frac{3 \times 2942.6 + 5 \times 2272.8}{8} \\ &= 2524 \text{ kJ/kg} \end{aligned}$$



$$(a) \quad h_3 = 762.6 + x_3 \times 2013.6 \text{ or } x_3 = 0.87474$$

$$(b) \quad h_4 = 2524 \text{ kJ/kg} = 640.1 + x_4 \times 2107.4$$

$$x_4 = 0.89395$$

$$s_4 = 1.8604 + x_4 \times 4.9588 = 6.2933 \text{ kJ/kg} - \text{K}$$

$$\therefore s_5 = s_4$$

$$\therefore \text{at 2 bar quality of steam}$$

$$6.2933 = 1.5301 + x_5 \times 5.5967$$

$$x_5 = 0.851$$

$$\therefore h_5 = 504.7 + 0.851 \times 2201.6 = 2378.4 \text{ kJ/kg}$$

$$(c) \quad s_3 = s_f + x_3 s_{fg} = 2.1382 + 0.89395 \times 4.4446 = 6.111451 \text{ kJ/kg}$$

$$\Delta s = s_4 - s_3 = 6.2933 - 6.11145 = 0.18185 \text{ kJ/kg} - \text{K}$$

$$(d) \quad V = \sqrt{2000(2524 - 2378.4)} = 540 \text{ m/s}$$

$$(e) \quad A = \frac{m v}{V}$$

$$\Rightarrow \frac{m \times x_5 \cdot 0.885}{V} = \frac{8}{60} \times 0.851 \times \frac{0.885}{540} \text{ m}^2 = 1.86 \text{ cm}^2$$

**Q9.22**

Steam of 65 bar, 400°C leaves the boiler to enter a steam turbine fitted with a throttle governor. At a reduced load, as the governor takes action, the pressure of steam is reduced to 59 bar by throttling before it is admitted to the turbine. Evaluate the availabilities of steam before and after the throttling process and the irreversibility due to it.

(Ans.  $I = 21 \text{ kJ/kg}$ )

**Solution:**

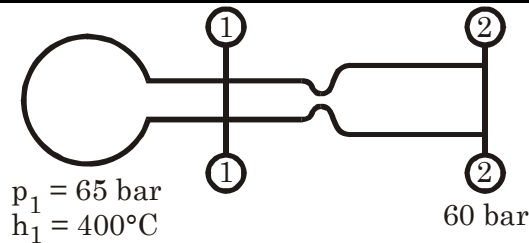
From Steam Table

$$h_1 = 3167.65 \text{ kJ/kg}$$

$$s_1 = 6.4945 \text{ kJ/kg-K}$$

$$h_2 = 3167.65 \text{ kJ/kg}$$

$$t_2 = 396.6^\circ \text{C}$$



$$s_2 = 6.333 + \frac{46.6}{50} (6.541 - 6.333)$$

$$= 6.526856$$

$$\therefore \Delta s = s_4 - s_3 = 0.032356 \text{ kJ/kg} - \text{K}$$

$$\text{Atmospheric Pressure } p_0 = 1 \text{ bar}$$

$$T_0 = 25^\circ \text{C}$$

$\therefore$  Availability before throttling

$$\psi = (h_1 - h_0) - T_0 (s_1 - s_0) + \frac{V_1^2}{2} + g (2 Z_0)$$

Same as example 9.14

**Q9.23**

A mass of wet steam at temperature  $165^\circ\text{C}$  is expanded at constant quality 0.8 to pressure 3 bar. It is then heated at constant pressure to a degree of superheat of  $66.5^\circ\text{C}$ . Find the enthalpy and entropy changes during expansion and during heating. Draw the  $T$ - $s$  and  $h$ - $s$  diagrams.

(Ans. – 59 kJ/kg, 0.163 kJ/kg K during expansion and 676 kJ/kg, 1.588 kJ/kg K during heating)

**Solution:**

$$p_1 = 7 \text{ bar}$$

$$t_1 = 165^\circ \text{C}$$

For Steam Table

$$h_1 = h_f + 0.8 h_{fg}$$

$$= 2349 \text{ kJ/kg}$$

$$s_1 = s_f + 0.8 \times s_{fg}$$

$$= 5.76252 \text{ kJ/kg} - \text{K}$$

For Steam Table at 3 bar

$$t_2 = 133.5^\circ \text{C}$$

$$h_2 = 561.4 + 0.8 \times 2163.2 = 2292 \text{ kJ}$$

$$s_2 = 1.6716 + 0.8 \times 5.3193$$

$$= 5.92704 \text{ kJ/kg} - \text{K}$$

$\therefore$  temperature of (3)

$$t_3 = 200^\circ\text{C}$$

$\therefore$   $h_3 = 2865.6 \text{ kJ/K}$

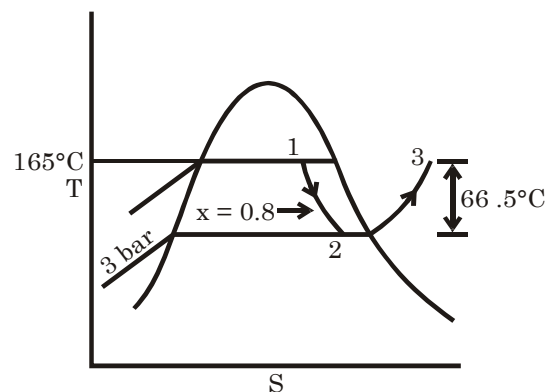
$$s_3 = 7.311 \text{ kJ/kg-K}$$

$\therefore$  Enthalpy charge in expansion =  $(h_1 - h_2) = 57 \text{ kJ/kg}$

Entropy charge in expansion =  $(s_2 - s_1) = 0.16452 \text{ kJ/kg-K}$

Enthalpy charge in heating =  $h_3 - h_2 = 573.6 \text{ kJ/kg}$

Entropy charge in heating =  $s_3 - s_2 = 1.38396 \text{ kJ/kg} - \text{K}$







# 10.

## Properties of Gases and Gas Mixture

### Some Important Notes

1. As  $p \rightarrow 0$ , or  $T \rightarrow \infty$ , the real gas approaches the **ideal gas behaviour**.

$$\bar{R} = 8.3143 \text{ kJ/kmole-K}$$

2.  $Tds = du + pdv$

$$Tds = dh - vdp$$

3.  $\gamma = 1 + \frac{2}{N}$

For mono-atomic gas  $N = 3$

For di -atomic gas  $N = 5$

For Tri-atomic gas  $N = 6$

[N = **degrees of freedom**]

4. **Reversible adiabatic process**

$$pv^\gamma = C; \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$$

5. **For isentropic process**

$$u_2 - u_1 = \frac{RT_1}{\gamma - 1} \left[ \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right]; h_2 - h_1 = \frac{\gamma}{\gamma - 1} (RT_1) \left[ \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

(i) **For closed system**  $\int_1^2 pdv = \frac{p_1 v_1 - p_2 v_2}{\gamma - 1}$

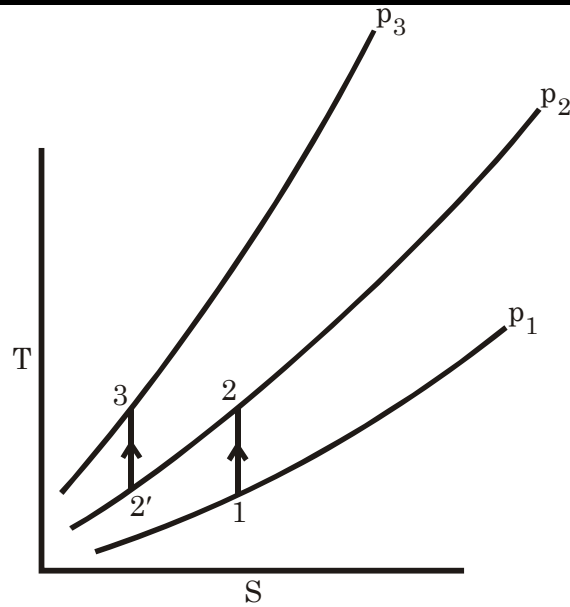
**For steady flow**  $\int_1^2 v dp = \frac{\gamma}{\gamma - 1} (p_1 v_1 - p_2 v_2)$

6. Isobaric process ( $p = C$ ),  $n = 0$ ,  $pv^0 = C$   
 Isothermal process ( $T = C$ ),  $n = 1$ ,  $pv^1 = RT$   
 Isentropic process ( $s = C$ ),  $n = \gamma$ ,  $pv^\gamma = C$   
 Isometric or isobaric process ( $V = C$ ),  $n = \infty$

7. For minimum work in multistage compressor,  $p_2 = \sqrt{p_1 p_3}$

(i) Equal pressure ratio i.e.  $\frac{p_2}{p_1} = \frac{p_3}{p_2}$

(ii) Equal discharge temperature ( $T_2 = T_3$ )



And (iii) Equal work for the two stages.

8. **Volumetric Efficiency**  $(\eta_{vol}) = 1 + C - C \left( \frac{p_2}{p_1} \right)^{\frac{1}{n}}$

Where,  $C = \frac{\text{Clearance volume}}{\text{Piston displacement volume}}$

9. **Equation of states for real gas**

(i) **Van der Waals equation:**  $\left( p + \frac{a}{v^2} \right) (v - b) = RT$

or  $p = \frac{RT}{v - b} - \frac{a}{v^2}$

or  $\left( p_r + \frac{3}{v_r^2} \right) (3v_r - 1) = 8T_r$

(ii) **Beattie Bridge man equation**

$$p = \frac{RT(1-e)}{v^2} (v+B) - \frac{A}{v^2}$$

Where  $A = A_0 \left( 1 - \frac{a}{v} \right)$ ;  $B = B_0 \left( 1 - \frac{b}{v} \right)$ ;  $e = \frac{C}{vT^3}$

'Does not' give satisfactory **results in the critical point region.**

(iii) **Virial Expansions:**

$$\frac{p\bar{v}}{RT} = 1 + B'p + C'p^2 + D'p^3 + \dots$$

Or  $\frac{p\bar{v}}{RT} = 1 + \frac{B}{\bar{v}} + \frac{C}{\bar{v}^2} + \frac{D}{\bar{v}^3} + \dots \alpha$

10.  $a = 3 p_c v_c^2$ ;  $b = \frac{v_c}{3}$ ;  $R = \frac{8}{3} \frac{p_c v_c}{T_c}$ ; values of  $Z$  at critical point **0.375** for Van der Waal gas.

11. **Boyle temperature**  $(T_B) = \frac{a}{bR}$

12.  $\mu = x_1 \mu_1 + x_2 \mu_2 + \dots + x_c \mu_c$

$$R_m = \frac{m_1 R_1 + m_2 R_2 + \dots + m_c R_c}{m_1 + m_2 + \dots + m_c}$$

$$u_m = \frac{m_1 u_1 + m_2 u_2 + \dots + m_c u_c}{m_1 + m_2 + \dots + m_c};$$

$$h_m = \frac{m_1 h_1 + m_2 h_2 + \dots + m_c h_c}{m_1 + m_2 + \dots + m_c}$$

$$C_{pm} = \frac{m_1 c_{p1} + m_2 c_{p2} + \dots + m_c c_{pc}}{m_1 + m_2 + \dots + m_c}$$

$$C_{vm} = \frac{m_1 c_{v1} + m_2 c_{v2} + \dots + m_c c_{vc}}{m_1 + m_2 + \dots + m_c}$$

$$s_f - s_i = - \left[ m_1 R_1 \ln \frac{p_1}{p} + m_2 R_2 \ln \frac{p_2}{p} + \dots + m_c R_c \ln \frac{p_c}{p} \right]$$

13. **Gibbs function**  $G = \bar{R}T \sum n_x (\phi_k + \ln p + \ln x_k)$

## Questions with Solution P. K. Nag

**Q.10.1** What is the mass of air contained in a room  $6 \text{ m} \times 9 \text{ m} \times 4 \text{ m}$  if the pressure is **101.325 kPa** and the temperature is **25°C**?

(Ans. 256 kg)

**Solution:** Given pressure (p) = 101.325 kPa  
 Temperature (T) = 25°C = (25 + 273) K = 298 K  
 Volume (V) =  $6 \times 9 \times 4 \text{ m}^3 = 216 \text{ m}^3$   
 From equation of states

$$pV = mRT \quad \text{For air } R = 0.287 \text{ kJ/kg} - \text{K, Gas constant mass is } m \text{ kg}$$

$$\therefore m = \frac{pV}{RT} = \frac{101.325 \times 216}{0.287 \times 298} \text{ kg} = 255.9 \text{ kg}$$

**Q.10.2** The usual cooking gas (mostly methane) cylinder is about **25 cm** in diameter and **80 cm** in height. It is changed to **12 MPa** at room temperature (**27°C**).

(a) Assuming the ideal gas law, find the mass of gas filled in the cylinder.

(b) Explain how the actual cylinder contains nearly **15 kg** of gas.

(c) If the cylinder is to be protected against excessive pressure by means of a fusible plug, at what temperature should the plug melt to limit the maximum pressure to **15 MPa**?

**Solution:** Given diameter (D) = 25 cm = 0.25 m  
 Height (H) = 80 cm = 0.8 m

$$\therefore \text{Volume of the cylinder } (V) = \frac{\pi D^2}{4} \times H = 0.03927 \text{ m}^3$$

Gas pressure (p) = 12 MPa = 12000 kPa

Temperature

$$(T) = 27^\circ \text{C} = 300 \text{ K}$$

(a) Mass of gas filled in the cylinder

$$m = \frac{pV}{RT} \left[ \text{Here } R = \text{Gas constant} = \frac{\bar{R}}{M} = \frac{8.3143}{16} \text{ kJ/kg} - \text{K} = 0.51964 \right]$$

$$= 3.023 \text{ kg}$$

(b) In cooking gas main component is ethen and it filled in 18 bar pressure. At that pressure it is not a gas it is liquid form in atmospheric temperature so its weight is amount 14 kg.

(c) Let temperature be T K, then pressure,  $p = 15 \text{ MPa} = 15000 \text{ kPa}$

$$\therefore T = \frac{pV}{mR} = \frac{15000 \times 0.03927}{3.023 \times 0.51964} = 375 \text{ K} = 102^\circ \text{C}$$

**Q.10.3** A certain gas has  $c_p = 0.913$  and  $c_v = 0.653 \text{ kJ/kg} - \text{K}$ . Find the molecular weight and the gas constant  $R$  of the gas.

**Solution:** Gas constant,  $R = c_p - c_v = (0.913 - 0.653) \text{ kJ/kg} - \text{K} = 0.26 \text{ kJ/kg} - \text{K}$

If molecular weight, ( $M$ )  $\text{kJ/kg} - \text{mole}$

$$\text{Then } \bar{R} = MR \quad \therefore M = \frac{\bar{R}}{R} = \frac{8.3143}{0.26} \text{ kJ/kg} - \text{mole} = 31.98 \text{ kJ/kg} - \text{mole}$$

**Q.10.4** From an experimental determination the specific heat ratio for acetylene ( $\text{C}_2\text{H}_2$ ) is found to 1.26. Find the two specific heats.

**Solution:** Gas constant of acetylene ( $\text{C}_2\text{H}_2$ ) ( $R$ ) =  $\frac{\bar{R}}{M} = \frac{8.3143}{26} \text{ kJ/kg} - \text{K} = 0.3198 \text{ kJ/kg} - \text{K}$

As adiabatic index ( $\gamma$ ) = 1.26 then

$$c_p = \frac{\gamma}{\gamma - 1} R = 1.55 \text{ kJ/kg} - \text{K}$$

and 
$$c_v = \frac{R}{\gamma - 1} = 1.23 \text{ kJ/kg} - \text{K}$$

**Q.10.5** Find the molal specific heats of monatomic, diatomic, and polyatomic gases, if their specific heat ratios are respectively 5/3, 7/5 and 4/3.

**Solution:** Mono-atomic:  $c_p = \frac{\gamma}{\gamma - 1} \bar{R} = 20.79 \text{ kJ/kg} - \text{mole} - \text{K};$

$$c_v = \frac{\bar{R}}{\gamma - 1} = 12.47 \text{ kJ/kg} - \text{mole} - \text{K}$$

Di-atomic:  $c_p = \frac{\gamma}{\gamma - 1} \bar{R} = 29.1 \text{ kJ/kg} - \text{mole} - \text{K};$

$$c_v = \frac{\bar{R}}{\gamma - 1} = 20.79 \text{ kJ/kg} - \text{mole} - \text{K}$$

Polyatomic:  $c_p = \frac{\gamma}{\gamma - 1} \bar{R} = 33.26 \text{ kJ/kg - mole - K};$

$c_v = 24.94 \text{ kJ/kg - mole - K}$

**Q.10.6** A supply of natural gas is required on a site 800 m above storage level. The gas at  $-150^\circ\text{C}$ , 1.1 bar from storage is pumped steadily to a point on the site where its pressure is 1.2 bar, its temperature  $15^\circ\text{C}$ , and its flow rate  $1000 \text{ m}^3/\text{hr}$ . If the work transfer to the gas at the pump is 15 kW, find the heat transfer to the gas between the two points. Neglect the change in K.E. and assume that the gas has the properties of methane ( $\text{CH}_4$ ) which may be treated as an ideal gas having  $\gamma = 1.33$  ( $g = 9.75 \text{ m/s}^2$ ).

(Ans. 63.9 kW)

**Solution:** Given: At storage  $(p_1) = 1.1 \text{ bar} = 110 \text{ kPa}$   
 $(T_1) = -150^\circ\text{C} = 123 \text{ K}$   
 $p_3 = 1.2 \text{ bar} = 120 \text{ kPa}$   
 $T_3 = 288 \text{ K}$

Flow rate  $(\dot{V}_3) = 1000 \text{ m}^3/\text{hr} = \frac{5}{18} \text{ m}^3/\text{s}$

Gas constant ( $R$ ) =  $\frac{\bar{R}}{16} = 0.51964 \text{ kJ/kg - K}$

$\therefore c_p = \frac{\gamma}{\gamma - 1} R = 2.0943 \text{ kJ/kg}$

$\therefore p_3 \dot{V}_3 = \dot{m} R T_3 \quad \therefore \dot{m} = \frac{p_3 \dot{V}_3}{R T_3} = 0.22273 \text{ kg/s}$

Pump work  $\left(\frac{dW}{dt}\right) = -15 \text{ kW}$

$\therefore$  From steady flow energy equation

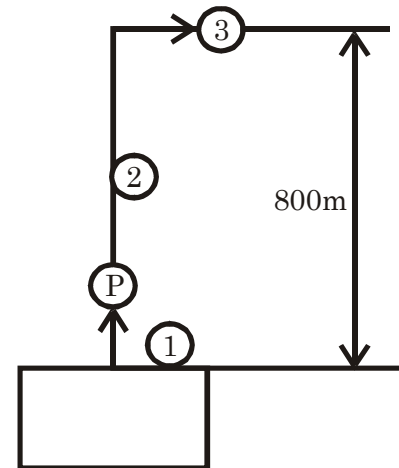
$$\dot{m}(h_1 + 0 + gZ_1) + \frac{dQ}{dt} = \dot{m}(h_3 + 0 + gZ_3) + \frac{dW}{dt}$$

$$\therefore \frac{dQ}{dt} = \dot{m} \left[ (h_3 - h_1) + g \frac{(Z_3 - Z_1)}{1000} \right] + \frac{dW}{dt}$$

$$= \dot{m} \left[ c_p(T_3 - T_1) + g \frac{\Delta Z}{1000} \right] + \frac{dW}{dt}$$

$$= 0.22273 \left[ 2.0943 \times (288 - 123) + \frac{9.75 \times 800}{1000} \right] + (-15)$$

$$= 63.7 \text{ kJ/s} = 63.7 \text{ kW (heat given to the system)}$$



**Q.10.7** A constant volume chamber of  $0.3 \text{ m}^3$  capacity contains 1 kg of air at  $5^\circ\text{C}$ . Heat is transferred to the air until the temperature is  $100^\circ\text{C}$ . Find the work done, the heat transferred, and the changes in internal energy, enthalpy and entropy.

**Solution:** Constant volume ( $V$ ) =  $0.3 \text{ m}^3$

$$T_2 = 100^\circ\text{C} = 373 \text{ K}$$

Mass ( $m$ ) =  $1 \text{ kg}$

$$\therefore p_2 = \frac{p_1}{T_1} \times T_2 = 357 \text{ kPa}$$

$$T_1 = 5^\circ\text{C} = 278 \text{ K}$$

$$\therefore p_1 = \frac{mRT_1}{V} = 265.95 \text{ kPa}$$

$$\text{Work done} = \int p dV = 0$$

$$\text{Heat transferred } Q = \int du + \int dW = \int dW = m c_v \int dT = m c_v (T_2 - T_1) = 68.21 \text{ kJ}$$

$$\text{Change in internal Energy} = \int du = 68.21 \text{ kJ}$$

$$\text{Change in Enthalpy} = \int dh = m c_p (T_2 - T_1) = 95.475 \text{ kJ}$$

$$\text{Change in Entropy} = \int ds = s_2 - s_1 = m c_p \ln \frac{V_2}{V_1} + m c_v \ln \frac{p_2}{p_1}$$

$$= m c_v \ln \frac{p_2}{p_1} = 1 \times 0.718 \times \ln \frac{357}{265.95}$$

$$= 0.2114 \text{ kJ/kg} - \text{K}$$

**Q.10.8** One kg of air in a closed system, initially at  $5^\circ\text{C}$  and occupying  $0.3 \text{ m}^3$  volume, undergoes a constant pressure heating process to  $100^\circ\text{C}$ . There is no work other than  $p dv$  work. Find (a) the work done during the process, (b) the heat transferred, and (c) the entropy change of the gas.

**Solution:**

$$T_1 = 278 \text{ K}$$

$$V_1 = 0.3 \text{ m}^3$$

$$m = 1 \text{ kg}$$

$$\therefore p_1 = 265.95 \text{ kPa}$$

$$T_2 = 100^\circ\text{C} = 373 \text{ K}$$

$$p_2 = 265.95 \text{ kPa}$$

$$\therefore V_2 = \frac{mRT_2}{p_2} = 0.40252 \text{ m}^3$$

(a) Work during the process

$$(W_{12}) = \int_1^2 p dV = p(V_2 - V_1) = 27.266 \text{ kJ}$$

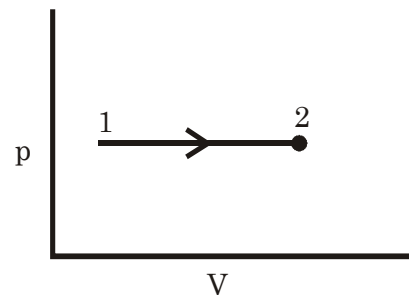
(b) Heat transferred  $Q_{1-2} = u_2 - u_1 + W_{12}$

$$= m c_v (T_2 - T_1) + W_{1-2} = 95.476 \text{ kJ}$$

(c) Entropy change of the gas

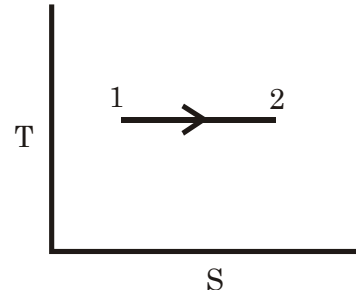
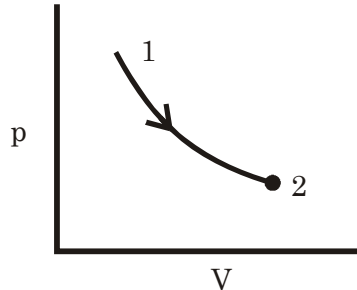
$$s_2 - s_1 = m c_p \ln \frac{V_2}{V_1} + m c_v \ln \frac{p_2}{p_1}$$

$$= m c_p \ln \frac{V_2}{V_1} = 0.29543 \text{ kJ/kg} - \text{K}$$



**Q.10.9**  $0.1 \text{ m}^3$  of hydrogen initially at  $1.2 \text{ MPa}$ ,  $200^\circ\text{C}$  undergoes a reversible isothermal expansion to  $0.1 \text{ MPa}$ . Find (a) the work done during the process, (b) the heat transferred, and (c) the entropy change of the gas.

**Solution:**  $V_1 = 0.1 \text{ m}^3$   
 $p_1 = 1.2 \text{ MPa} = 1200 \text{ kPa}$   
 $T_1 = 473 \text{ K}$



$$R = \frac{\bar{R}}{M} = \frac{8.3143}{2} \text{ kJ/kg} - \text{K} = 4.157 \text{ kJ/kg} - \text{K}$$

$$m = \frac{p_1 V_1}{RT_1} = 0.06103 \text{ kg}$$

Reversible isothermal expansion

So  $T_2 = T_1 = 473 \text{ K}$

Enthalpy change ( $\Delta h$ ) =  $m c_p (T_2 - T_1) = 0$

And  $p_2 = 0.1 \text{ MPa} = 100 \text{ kPa}$

Heat transferred ( $Q$ ) =  $\Delta u + \Delta W$

$$\begin{aligned} \therefore V_2 &= \frac{mRT_2}{p_2} = 1.2 \text{ m}^3 & = u_2 - u_1 + \int_1^2 p \, dV \\ pV &= RT & = 0 + RT \int \frac{dV}{V} \\ \therefore p &= \frac{RT}{V} & = RT \ln \left( \frac{V_2}{V_1} \right) \\ & & = 4.157 \times 473 \times \ln \left( \frac{1.2}{0.1} \right) \\ & & = 4886 \text{ kJ} \end{aligned}$$

$$\text{Work done (W)} = \int_1^2 p \, dV = 4886 \text{ kJ}$$

$$\begin{aligned} \text{Entropy change, } s_2 - s_1 &= mc_p \ln \frac{V_2}{V_1} + mc_v \ln \frac{p_2}{p_1} \\ &= 0.06103 \left[ 14.55 \ln \left( \frac{1.2}{0.1} \right) + 10.4 \ln \left( \frac{100}{1200} \right) \right] \\ &= 0.6294 \text{ kJ/kg} - \text{K} \end{aligned}$$

For  $\text{H}_2$  diatomic gas ( $\gamma = 1.4$ )

$$c_p = \frac{\gamma}{\gamma - 1} R = 14.55 \text{ kJ/kg} - \text{K}; \quad c_v = \frac{R}{\gamma - 1} = 10.4 \text{ kJ/kg} - \text{K}$$

**Q.10.10**

**Air in a closed stationary system expands in a reversible adiabatic process from 0.5 MPa, 15°C to 0.2 MPa. Find the final temperature, and per kg of air, the change in enthalpy, the heat transferred, and the work done.**

**Solution:**  $p_1 = 0.5 \text{ MPa} = 500 \text{ kPa}$

$$T_1 = 15^\circ\text{C} = 288 \text{ K}$$

Let mass is 1 kg

$$\therefore V_1 = \frac{1 \times R \times T_1}{p_1} = 0.1653 \text{ m}^3/\text{kg}$$

$$p_2 = 0.2 \text{ MPa} = 200 \text{ kPa}$$

$$\therefore p_1 v_1^\gamma = p_2 v_2^\gamma :$$

$$v_2 = v_1 \times \left( \frac{p_1}{p_2} \right)^{\frac{1}{\gamma}} = 0.31809 \text{ m}^3/\text{kg}$$

$$\therefore \frac{T_1}{T_2} = \left( \frac{p_1}{p_2} \right)^{\frac{\gamma-1}{\gamma}} \therefore T_2 = T_1 \times \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} = 222 \text{ K}$$

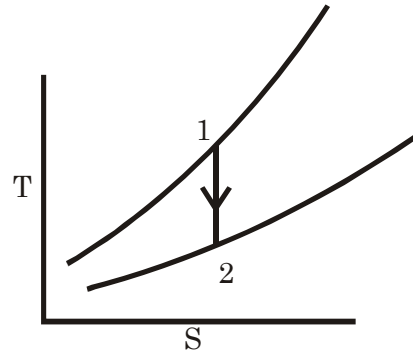
Change of Enthalpy

$$(\Delta H) = mc_p(T_2 - T_1) = -66.33 \text{ kJ/kg}$$

The Heat transferred ( $Q$ ) = 0

The work done

$$(W) = \int_1^2 p \, dv = \frac{p_1 v_1 - p_2 v_2}{\gamma - 1} = 47.58 \text{ kJ/kg}$$



**Q.10.11** If the above process occurs in an open steady flow system, find the final temperature, and per kg of air, the change in internal energy, the heat transferred, and the shaft work. Neglect velocity and elevation changes.

**Solution:** Final temperature will be same because then also  $\frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}}$  valid.

$$\text{i.e. } T_2 = 222 \text{ K}$$

$$\text{Change in internal energy} = \Delta u = mc_v(T_2 - T_1) = -47.4 \text{ kJ/kg}$$

$$\text{Shaft work} \quad (W) = -\int_1^2 v \, dp = \frac{\gamma}{\gamma-1} [p_1 v_1 - p_2 v_2] = +66.33 \text{ kJ/kg}$$

Heat transferred:

$$h_1 + 0 + 0 + \frac{dQ}{dm} = h_2 + 0 + 0 + \frac{dW}{dm}$$

$$\therefore \frac{dQ}{dm} = (h_2 - h_1) + \frac{dW}{dm} = -66.33 + 66.33 = 0$$

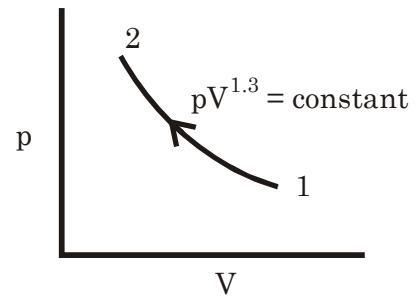
[As it is reversible adiabatic so  $dQ = 0$ ]

**Q.10.12** The indicator diagram for a certain water-cooled cylinder and piston air compressor shows that during compression  $pv^{1.3} = \text{constant}$ . The compression starts at 100 kPa, 25°C and ends at 600 kPa. If the process is reversible, how much heat is transferred per kg of air?



**Solution:**

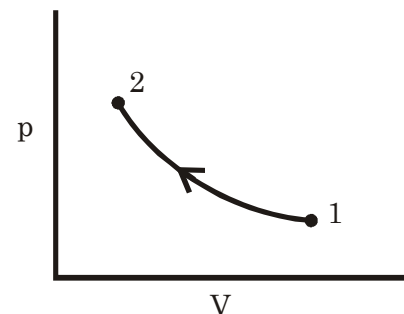
$$\begin{aligned}
 p_1 &= 100 \text{ kPa} \\
 T_1 &= 298 \text{ K} \\
 \therefore v_1 &= \frac{RT_1}{p_1} = 0.8553 \text{ m}^3/\text{kg} \\
 p_2 &= 600 \text{ kPa} \\
 v_2 &= v_1 \left( \frac{p_1}{p_2} \right)^{\frac{1}{n}} = 0.21554 \text{ m}^3/\text{kg} \\
 T_2 &= T_1 \times \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} = 451 \text{ K} \\
 h_1 + 0 + 0 + \frac{dQ}{dm} &= h_2 + 0 + 0 + \frac{dW}{dm} \\
 \therefore \frac{dQ}{dm} &= (h_2 - h_1) + \frac{dW}{dm} \\
 \therefore \frac{dW}{dm} &= \frac{n[p_1 v_1 - p_2 v_2]}{n-1} \\
 &= -189.774 \text{ kJ/kg} \\
 &= C_p(T_2 - T_1) - 189.774 \\
 &= 153.765 - 189.774 \\
 &= -36 \text{ kJ/kg} \\
 &\text{[Heat have to be rejected]}
 \end{aligned}$$



**Q.10.13** An ideal gas of molecular weight 30 and  $\gamma = 1.3$  occupies a volume of  $1.5 \text{ m}^3$  at  $100 \text{ kPa}$  and  $77^\circ\text{C}$ . The gas is compressed according to the law  $p v^{1.25} = \text{constant}$  to a pressure of  $3 \text{ MPa}$ . Calculate the volume and temperature at the end of compression and heating, work done, heat transferred, and the total change of entropy.

**Solution:**

$$\begin{aligned}
 R &= \frac{\bar{R}}{30} = 0.27714 \text{ kJ/kg} - \text{K} \\
 \gamma &= 1.3; n = 1.25 \\
 \therefore c_v &= \frac{R}{\gamma - 1} = 0.9238 \text{ kJ/kg} - \text{K} \\
 c_p &= \gamma \frac{R}{\gamma - 1} = 1.2 \text{ kJ/kg} - \text{K} \\
 p_1 &= 100 \text{ kPa}; V_1 = 1.5 \text{ m}^3; T_1 = 350 \text{ K} \\
 p_2 &= 3 \text{ MPa} = 3000 \text{ kPa} \\
 V_2 &= V_1 \left( \frac{p_1}{p_2} \right)^{\frac{1}{n}} = 0.09872 \text{ m}^3 \\
 m &= \frac{p_1 V_1}{RT_1} = 1.5464 \text{ kg} \\
 \therefore T_2 &= \frac{p_2 V_2}{mR} = 691 \text{ K} \\
 \therefore
 \end{aligned}$$



$$\begin{aligned}
 \text{Work done (W}_{1-2}) &= \int_1^2 p dV \\
 \therefore p_1 V_1^n &= p V^n = p_2 V_2^n \\
 &= p_1 V_1^n \int_1^2 \frac{dV}{V^n} = \frac{p_1 V_1^n}{-n+1} \left[ \frac{1}{V_2^{n-1}} - \frac{1}{V_1^{n-1}} \right] \\
 &= \frac{p_2 V_2 - p_1 V_1}{1-n} = \frac{p_1 V_1 - p_2 V_2}{n-1} \\
 &= \frac{100 \times 1.5 - 3000 \times 0.09872}{1.25 - 1} \text{ kJ} = -584.64 \text{ kJ}
 \end{aligned}$$

$$\begin{aligned}
 \text{Heat transfer } Q &= u_2 - u_1 + W_{1-2} \\
 &= mc_v(T_2 - T_1) + W_{1-2} \\
 &= [1.5464 \times 0.9238 (691 - 350) - 584.64] \text{ kJ} \\
 &= -97.5 \text{ kJ}
 \end{aligned}$$

$$\begin{aligned}
 \Delta S = S_2 - S_1 &= \left[ mc_v \ln \frac{p_2}{p_1} + mc_p \ln \frac{V_2}{V_1} \right] \\
 &= -0.19 \text{ kJ/K}
 \end{aligned}$$

**Q.10.14** Calculate the change of entropy when 1 kg of air changes from a temperature of 330 K and a volume of 0.15 m<sup>3</sup> to a temperature of 550 K and a volume of 0.6 m<sup>3</sup>.

If the air expands according to the law,  $pv^n = \text{constant}$ , between the same end states, calculate the heat given to, or extracted from, the air during the expansion, and show that it is approximately equal to the change of entropy multiplied by the mean absolute temperature.

**Solution:** Try please.

**Q.10.15** 0.5 kg of air, initially at 25°C, is heated reversibly at constant pressure until the volume is doubled, and is then heated reversibly at constant volume until the pressure is doubled. For the total path, find the work transfer, the heat transfer, and the change of entropy.

**Solution:** Try please.

**Q.10.16** An ideal gas cycle of three processes uses Argon (Mol. wt. 40) as a working substance. Process 1-2 is a reversible adiabatic expansion from 0.014 m<sup>3</sup>, 700 kPa, 280°C to 0.056 m<sup>3</sup>. Process 2-3 is a reversible isothermal process. Process 3-1 is a constant pressure process in which heat transfer is zero. Sketch the cycle in the  $p$ - $v$  and  $T$ - $s$  planes, and find (a) the work transfer in process 1-2, (b) the work transfer in process 2-3, and (c) the net work of the cycle. Take  $\gamma = 1.67$ .

**Solution:** Try please.

**Q.10.17** A gas occupies 0.024 m<sup>3</sup> at 700 kPa and 95°C. It is expanded in the non-flow process according to the law  $pv^{1.2} = \text{constant}$  to a pressure of 70 kPa

after which it is heated at constant pressure back to its original temperature. Sketch the process on the  $p$ - $v$  and  $T$ - $s$  diagrams, and calculate for the whole process the work done, the heat transferred, and the change of entropy. Take  $c_p = 1.047$  and  $c_v = 0.775$  kJ/kg K for the gas.

**Solution:** Try please.

**Q.10.18** 0.5 kg of air at 600 kPa receives an addition of heat at constant volume so that its temperature rises from 110°C to 650°C. It then expands in a cylinder poly tropically to its original temperature and the index of expansion is 1.32. Finally, it is compressed isothermally to its original volume. Calculate (a) the change of entropy during each of the three stages, (b) the pressures at the end of constant volume heat addition and at the end of expansion. Sketch the processes on the  $p$ - $v$  and  $T$ - $s$  diagrams.

**Solution:** Try please.

**Q.10.19** 0.5 kg of helium and 0.5 kg of nitrogen are mixed at 20°C and at a total pressure of 100 kPa. Find (a) the volume of the mixture, (b) the partial volumes of the components, (c) the partial pressures of the components, (d) the mole fractions of the components, (e) the specific heats  $c_p$  and  $c_v$  of the mixture, and (f) the gas constant of the mixture.

**Solution:** Try please.

**Q.10.20** A gaseous mixture consists of 1 kg of oxygen and 2 kg of nitrogen at a pressure of 150 kPa and a temperature of 20°C. Determine the changes in internal energy, enthalpy and entropy of the mixture when the mixture is heated to a temperature of 100°C (a) at constant volume, and (b) at constant pressure.

**Solution:** Try please.

**Q.10.21** A closed rigid cylinder is divided by a diaphragm into two equal compartments, each of volume 0.1 m<sup>3</sup>. Each compartment contains air at a temperature of 20°C. The pressure in one compartment is 2.5 MPa and in the other compartment is 1 MPa. The diaphragm is ruptured so that the air in both the compartments mixes to bring the pressure to a uniform value throughout the cylinder which is insulated. Find the net change of entropy for the mixing process.

**Solution:** Try please.

**Q.10.22** A vessel is divided into three compartments (a), (b), and (c) by two partitions. Part (a) contains oxygen and has a volume of 0.1 m<sup>3</sup>, (b) has a volume of 0.2 m<sup>3</sup> and contains nitrogen, while (c) is 0.05 m<sup>3</sup> and holds CO<sub>2</sub>. All three parts are at a pressure of 2 bar and a temperature of 13°C. When the partitions are removed and the gases mix, determine the change of entropy of each constituent, the final pressure in the vessel and the partial pressure of each gas. The vessel may be taken as being completely isolated from its surroundings.

(Ans. 0.0875, 0.0783, 0.0680 kJ/K; 2 bar; 0.5714, 1.1429, 0.2857 bar.)

# Properties of Gases and Gas Mixtures

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**Solution:**

(a)	(b)	(c)
0.1 m <sup>3</sup>	0.2 m <sup>3</sup>	0.05 m <sup>3</sup>
O <sub>2</sub>	N <sub>2</sub>	W <sub>2</sub>

$$p = 2 \text{ bar} = 200 \text{ kPa}$$

$$T = 13^\circ \text{C} = 286 \text{ K}$$

After mixing temperature of the mixture will be same as before  $13^\circ\text{C} = 286 \text{ K}$  and also pressure will be same as before  $2 \text{ bar} = 200 \text{ kPa}$ . But total volume will be

$$V = V_a + V_b + V_c$$

$$= (0.1 + 0.2 + 0.05) = 0.35 \text{ m}^3$$

$$\therefore m_a = \frac{pV_a}{R_a T} = \frac{200 \times 0.1}{\frac{8.3143}{32} \times 286} \text{ kg} = 0.26915 \text{ kg}$$

$$m_b = \frac{pV_b}{R_b T} = \frac{200 \times 0.2}{\frac{8.3143}{28} \times 286} \text{ kg} = 0.471 \text{ kg}$$

$$m_c = \frac{pV_c}{R_c T} = \frac{200 \times 0.05}{\frac{8.319}{44} \times 286} \text{ kg} = 0.18504 \text{ kg}$$

$$\therefore \Delta S = S_2 - S_1 = mc_p \ln \frac{T_2}{T_1} - mR \ln \frac{p_2}{p_1} \quad \text{Here } T_2 = T_1 \text{ so } \left[ \therefore \frac{p_2}{p_1} = \frac{v_1}{v_2} \right]$$

$$(S_2 - S_1)_{O_2} = m_{O_2} R_{O_2} \ln \frac{V_o}{V_{O_2}} = 0.26915 \times \frac{8.3143}{32} \times \ln \left( \frac{0.35}{0.1} \right)$$

$$= 0.087604 \text{ kJ/K}$$

$$(S_2 - S_1)_{N_2} = m_{N_2} R_{N_2} \ln \left( \frac{V}{V_{N_2}} \right) = 0.471 \times \frac{8.3143}{32} \times \ln \left( \frac{0.35}{0.2} \right) = 0.078267 \text{ kJ/K}$$

$$(S_2 - S_1)_{CO_2} = m_{CO_2} R_{CO_2} \ln \left( \frac{V}{V_{CO_2}} \right) = 0.18504 \times \frac{8.3143}{44} \times \ln \left( \frac{0.35}{0.05} \right) = 0.06804 \text{ kJ/K}$$

Partial pressure after mixing

$$\text{Mole fraction of } O_2 (x_{O_2}) = \frac{0.1}{0.35}$$

$$\text{Mole fraction of } N_2 (x_{N_2}) = \frac{0.2}{0.35}$$

$$\text{Mole fraction of } CO_2, (x_{CO_2}) = \frac{0.05}{0.35}$$

[ $\therefore$  At same temperature and pressure same mole of gas has same]

$$\therefore \text{Partial pressure of } O_2; (p_{O_2}) = x_{O_2} \times p = \frac{0.1}{0.35} \times 200 = 57.143 \text{ kPa}$$

$$\text{Partial pressure of } N_2; (p_{N_2}) = x_{N_2} \times p = \frac{0.2}{0.35} \times 200 = 114.29 \text{ kPa}$$

$$\text{Partial pressure of } CO_2; (p_{CO_2}) = x_{CO_2} \times p = \frac{0.05}{0.35} \times 200 = 28.514 \text{ kPa}$$

**Q.10.23** A Carnot cycle uses 1 kg of air as the working fluid. The maximum and minimum temperatures of the cycle are 600 K and 300 K. The maximum pressure of the cycle is 1 MPa and the volume of the gas doubles during the isothermal heating process. Show by calculation of net work and heat supplied that the efficiency is the maximum possible for the given maximum and minimum temperatures.

**Solution:** Try please.

**Q.10.24** An ideal gas cycle consists of three reversible processes in the following sequence: (a) constant volume pressure rise, (b) isentropic expansion to  $r$  times the initial volume, and (c) constant pressure decrease in volume. Sketch the cycle on the  $p$ - $v$  and  $T$ - $s$  diagrams. Show that the efficiency of the cycle is

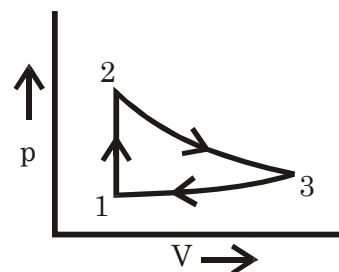
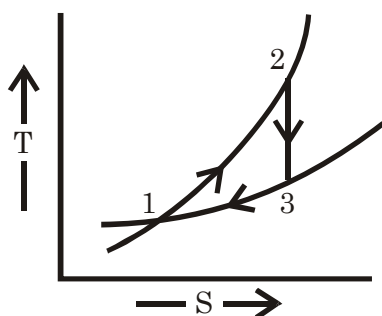
$$\eta_{\text{cycle}} = \frac{r^\gamma - 1 - \gamma(r - 1)}{r^\gamma - 1}$$

Evaluate the cycle efficiency when  $\gamma = \frac{4}{3}$  and  $r = 8$ .

(Ans. ( $\eta = 0.378$ ))

**Solution:** For process 1 – 2 constant volume heating

$$\begin{aligned} Q_{1-2} &= \Delta u + p dv \\ &= mc_v \Delta T + p dv \\ &= mc_v \Delta T = mc_v (T_2 - T_1) \end{aligned}$$



$Q_{2-3} = 0$  as isentropic expansion.

$$Q_{3-1} = mc_p \Delta T = mc_p (T_3 - T_1)$$

$$\therefore \text{Efficiency} = 1 - \frac{\text{heat rejection}}{\text{heat addition}}$$

$$= 1 - \frac{mc_p (T_3 - T_1)}{mc_v (T_2 - T_1)} = 1 - \gamma \frac{\left(\frac{T_3}{T_1} - 1\right)}{\left(\frac{T_2}{T_1} - 1\right)}$$

$$\text{Here } \frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \text{ as } V_1 = V_2$$

$$\therefore \frac{T_2}{T_1} = \frac{p_2}{p_1} = r^\gamma$$

$$\text{And } p_2 V_2^\gamma = p_3 V_3^\gamma$$

$$\text{or } \frac{p_2}{p_3} = \left(\frac{V_3}{V_2}\right)^\gamma = r^\gamma \text{ as } p_3 = p_1 \text{ then}$$

$$\text{And } \frac{p_3 V_3}{T_3} = \frac{p_2 V_2}{T_1}$$

$$\frac{p_2}{p_1} = r^\gamma$$

$$\text{or } \frac{T_3}{T_1} = \frac{v_3}{v_1} = \frac{v_3}{v_2} = r \quad \therefore \eta = 1 - \frac{\gamma(\gamma - 1)}{r^\gamma - 1} = \frac{r^\gamma - 1 - \gamma(r - 1)}{r^\gamma - 1} \text{ Proved}$$

$$\text{If } \gamma = \frac{4}{3} \text{ and } r = 8 \text{ then} \quad \eta_{\text{cycle}} = \frac{r^\gamma - 1 - \gamma(r - 1)}{r^\gamma - 1}$$

$$\eta = 1 - \frac{\frac{4}{3}(8 - 1)}{(8^{\frac{4}{3}} - 1)} = 0.37778$$

**Q.10.25** Using the Dietetic equation of state

$$P = \frac{RT}{v - b} \cdot \exp\left(-\frac{a}{RT_v}\right)$$

(a) Show that

$$p_c = \frac{a}{4e^2 b^2}, v_c = 2b, T_c = \frac{a}{4Rb}$$

(b) Expand in the form

$$pv = RT \left( 1 + \frac{B}{v} + \frac{C}{v^2} + \dots \right)$$

(c) Show that

$$T_B = \frac{a}{bR}$$

**Solution :** Try please.

**Q.10.26** The number of moles, the pressures, and the temperatures of gases *a*, *b*, and *c* are given below

Gas	m (kg mol)	P (kPa)	t (°C)
N <sub>2</sub>	1	350	100
CO	3	420	200
O <sub>2</sub>	2	700	300

If the containers are connected, allowing the gases to mix freely, find (a) the pressure and temperature of the resulting mixture at equilibrium, and (b) the change of entropy of each constituent and that of the mixture.

**Solution :** Try please.

**Q.10.27** Calculate the volume of 2.5 kg moles of steam at 236.4 atm. and 776.76 K with the help of compressibility factor versus reduced pressure graph. At this volume and the given pressure, what would the temperature be in K, if steam behaved like a van der Waals gas?  
The critical pressure, volume, and temperature of steam are 218.2 atm., 57 cm<sup>3</sup>/g mole, and 647.3 K respectively.

**Solution :** Try please.

**Q.10.28** Two vessels, *A* and *B*, each of volume 3 m<sup>3</sup> may be connected together by a tube of negligible volume. Vessel *a* contains air at 7 bar, 95°C while *B*

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contains air at 3.5 bar, 205°C. Find the change of entropy when A is connected to B. Assume the mixing to be complete and adiabatic.

(Ans. 0.975 kJ/kg K)

**Solution:**

$$V_A = V_B = 3\text{m}^3$$

$$p_A = 7\text{ bar} = 700\text{ kPa}$$

$$T_A = 95^\circ\text{C} = 368\text{ K}$$

$$p_B = 3.5\text{ bar} = 350\text{ kPa}$$

$$T_B = 205^\circ\text{C} = 478\text{ K}$$

$$\therefore m_A = \frac{p_A V_A}{RT_A} = 19.883\text{ kg}$$

$$m_B = \frac{p_B V_B}{RT_B} = 7.6538\text{ kg}$$

In case of Adiabatic mixing for closed system  
Internal energy remains constant.

$$\therefore U_A + U_B = U$$

$$\text{or } m_A c_v \cdot T_A + m_B c_v \cdot T_B = (m_A + m_B) c_v T$$

$$\therefore T = \frac{m_A T_A + m_B T_B}{m_A + m_B} = 398.6\text{ K}$$

After mixing partial for of A

$$\left[ \begin{array}{l} \text{Total pressure} \\ \therefore p = \frac{mRT}{V} = 525.03\text{ kPa} \end{array} \right]$$

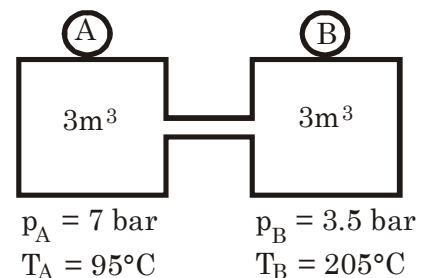
$$p_{Af} = \frac{m_A RT}{V} = 379.1\text{ kPa}$$

$$p_{Bf} = \frac{m_B RT}{V} = 145.93\text{ kPa}$$

$$\begin{aligned} \Delta S_A = S_{Af} - S_A &= m_A c_p \ln \frac{T}{T_A} - m_A R \ln \frac{p_{Af}}{p_A} \\ &= 5.0957\text{ kJ/K} \end{aligned}$$

$$\begin{aligned} \Delta S_{Bf} - S_B &= m_B c_p \ln \frac{T}{T_B} - m_B R \ln \frac{p_{Bf}}{p_B} \\ &= 0.52435\text{ kJ/kg} \end{aligned}$$

$$\therefore \Delta S_{\text{univ}} = \Delta S_A + \Delta S_B = 5.62\text{ kJ/K}$$

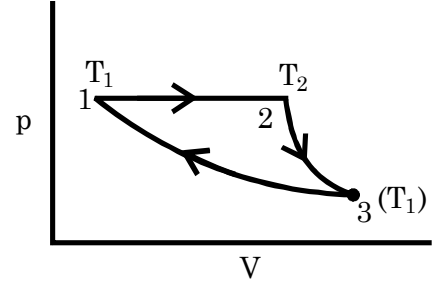
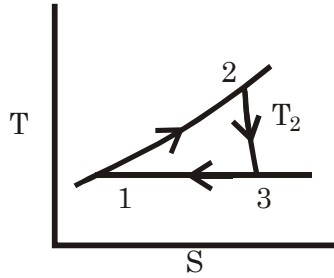


**Q.10.29**

An ideal gas at temperature  $T_1$  is heated at constant pressure to  $T_2$  and then expanded reversibly, according to the law  $pv^n = \text{constant}$ , until the temperature is once again  $T_1$ . What is the required value of  $n$  if the changes of entropy during the separate processes are equal?

$$\left( \text{Ans.} \left( n = \frac{2\gamma}{\gamma + 1} \right) \right)$$

**Solution:** Let us mass of gas is 1 kg



Then

$$s_2 - s_1 = 1 \times c_p \ln \frac{v_2}{v_1} + c_v \ln \frac{p_2}{p_1} = \left[ c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \right] = c_p \ln \frac{T_2}{T_1} = \frac{\gamma R}{\gamma - 1} \times \ln \frac{T_2}{T_1}$$

$$\text{or } s_3 - s_2 = c_p \ln \frac{T_3}{T_2} - R \ln \frac{p_3}{p_2}$$

$$\text{Hence } T_3 = T_1 \text{ and } \frac{T_3}{T_2} = \left( \frac{p_3}{p_2} \right)^{\frac{n-1}{n}} \quad \therefore \left( \frac{p_3}{p_2} \right) = \left( \frac{T_3}{T_2} \right)^{\frac{n}{n-1}}$$

$$= c_p \ln \frac{T_1}{T_2} - R \frac{n}{n-1} \ln \left( \frac{T_1}{T_2} \right) = \left[ \frac{\gamma R}{\gamma - 1} - \frac{nR}{n-1} \right] \ln \frac{T_1}{T_2} = R \left( \ln \frac{T_2}{T_1} \right) \left[ \frac{n}{n-1} - \frac{\gamma}{\gamma - 1} \right]$$

$$\therefore \text{As } s_2 - s_1 = s_3 - s_2$$

$$\therefore \frac{\gamma R}{\gamma - 1} \ln \frac{T_2}{T_1} = R \left( \ln \frac{T_2}{T_1} \right) \times \left[ \frac{n}{n-1} - \frac{\gamma}{\gamma - 1} \right]$$

$$\text{or } 2 \frac{\gamma}{\gamma - 1} = \frac{n}{n-1}$$

$$\text{or } 2n\gamma - 2\gamma = n\gamma - n$$

$$\text{or } n(\gamma n) = 2\gamma$$

$$\text{or } n = \left( \frac{2\gamma}{\gamma + 1} \right) \text{ proved}$$

**Q.10.30** A certain mass of sulphur dioxide ( $\text{SO}_2$ ) is contained in a vessel of  $0.142 \text{ m}^3$  capacity, at a pressure and temperature of 23.1 bar and  $18^\circ\text{C}$  respectively. A valve is opened momentarily and the pressure falls immediately to 6.9 bar. Sometimes later the temperature is again  $18^\circ\text{C}$  and the pressure is observed to be 9.1 bar. Estimate the value of specific heat ratio.

(Ans. 1.29)

**Solution:** Mass of  $\text{SO}_2$  before open the valve  $S = 32$   $O \rightarrow 16 \times 2 = 64$

$$m_1 = \frac{pV}{R_{\text{SO}_2} T} = \frac{2310 \times 0.142}{\frac{8.3143}{64} \times 291} = 8.6768 \text{ kg}$$

$$R_{\text{SO}_2} = 0.12991 \text{ kJ/kg-K}$$

Mass of  $\text{SO}_2$  after closing the valve



$$m_2 = \frac{910 \times 0.142}{R_{SO_2} \times 291} = 3.4181 \text{ kg}$$

If intermediate temperature is T then

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \quad \text{or} \quad \frac{9.1 \times 0.142}{291} = \frac{6.9 \times 0.142}{T}$$

$$\text{or } T = 220.65 \text{ K}$$

As valve is opened momentarily term process is adiabatic

$$\text{So } \frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \quad \text{or} \quad \frac{220.65}{291} = \left( \frac{6.9}{23.1} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\text{or} \quad \left( 1 - \frac{1}{\gamma} \right) = \frac{\ln \left( \frac{220.65}{291} \right)}{\ln \left( \frac{6.9}{23.1} \right)} = 0.22903$$

$$\text{or} \quad \frac{1}{\gamma} = 1 - 0.22903 = 0.77097$$

$$\therefore \gamma = 1.297$$

**Q.10.31**

**A gaseous mixture contains 21% by volume of nitrogen, 50% by volume of hydrogen, and 29% by volume of carbon-dioxide. Calculate the molecular weight of the mixture, the characteristic gas constant  $R$  for the mixture and the value of the reversible adiabatic index  $\gamma$ . (At 10°C, the  $c_p$  values of nitrogen, hydrogen, and carbon dioxide are 1.039, 14.235, and 0.828 kJ/kg K respectively.)**

**A cylinder contains 0.085 m<sup>3</sup> of the mixture at 1 bar and 10°C. The gas undergoes a reversible non-flow process during which its volume is reduced to one-fifth of its original value. If the law of compression is  $p v^{1.2} = \text{constant}$ , determine the work and heat transfer in magnitude and sense and the change in entropy.**

**(Ans. 19.64 kg/kg mol, 0.423 kJ/kg K, 1.365, -16 kJ, -7.24 kJ, -0.31 kJ/kg K)**

**Solution :**

Volume ratio = 21: 50: 29

$\therefore$  Mass ratio = 21  $\times$  28: 50  $\times$  2: 29  $\times$  44

$$\text{Let } m_{N_2} = 21 \times 28 \text{ kg}, \quad m_{H_2} = 50 \times 2 \text{ kg}, \quad m_{N_2} = 29 \times 44 \text{ kg}$$

$$\quad \quad \quad = 588 \text{ kg} \quad \quad \quad = 100 \text{ kg} \quad \quad \quad = 1276 \text{ kg}$$

$$\therefore R_{\text{mix}} = \frac{m_{N_2} R_{N_2} + m_{H_2} R_{H_2} + m_{CO_2} R_{CO_2}}{m_{N_2} + m_{H_2} + m_{CO_2}}$$

$$= \frac{21 \times \bar{R} + 50 \times \bar{R} + 29 \bar{R}}{21 \times 28 + 50 \times 2 + 29 \times 44}$$

$$= 0.42334 \text{ kJ/kg} - \text{K}$$

$$\left[ \begin{array}{l} R_{N_2} = \frac{\bar{R}}{28} \\ R_{H_2} = \frac{\bar{R}}{2}, R_{CO_2} = \frac{\bar{R}}{44} \end{array} \right]$$

$$c_{p \text{ Mix}} = \frac{m_{N_2} C_{p_{N_2}} + m_{H_2} C_{p_{H_2}} + m_{CO_2} C_{p_{CO_2}}}{m_{N_2} + m_{H_2} + m_{CO_2}} \quad [m_{N_2} + m_{H_2} + m_{CO_2} = 1964]$$

$$= \frac{21 \times 28 \times 1.039 + 100 \times 14.235 + 0.828 \times 1276}{588 + 100 + 1276} = 1.5738 \text{ kJ/kg} - \text{K}$$

$$c_{vN_2} = 1.039 - \frac{\bar{R}}{28} = 0.74206$$

$$c_{vH_2} = 14.235 - \frac{\bar{R}}{2} = 10.078$$

$$c_{vCO_2} = 0.828 - \frac{\bar{R}}{44} = 0.63904$$

$$\therefore c_{v \text{ Mix}} = \frac{588 \times 0.74206 + 100 \times 10.078 + 1276 \times 0.63904}{588 + 100 + 1276} = 1.1505 \text{ kJ/kg} - \text{K}$$

$$c_{v \text{ mix}} = c_{p \text{ mix}} - R_{\text{mix}} = 1.5738 - 0.42334 = 1.1505 \text{ kJ/kg} - \text{K}$$

$$\therefore \gamma_{\text{mix}} = \frac{c_{p \text{ mix}}}{c_{v \text{ mix}}} = 1.368$$

Given

$$\Rightarrow p_1 = 1 \text{ bar} = 100 \text{ kPa}$$

$$p_2 = 690 \text{ kPa (Calculated)}$$

$$V_2 = 0.085 \text{ m}^3$$

$$V_2 = \frac{V_1}{5} = 0.017 \text{ m}^3$$

$$T_1 = 10^\circ \text{C} = 283 \text{ K}$$

$$T_2 = 390.5 \text{ K (Calculated)}$$

$$\therefore \frac{p_2}{p_1} = \left( \frac{V_1}{V_2} \right)^n = 5^{1.2}$$

$$\therefore p_2 = 100 \times 5^{1.2} \text{ kPa}$$

$$\therefore \frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \quad \therefore T_2 = 390.5 \text{ K}$$

$$W = \frac{p_1 V_1 - p_2 V_2}{n-1}$$

$$\left[ \because W = \int_1^2 p dV = C \int_1^2 \frac{dV}{V^4} \right]$$

$$= \frac{100 \times 0.085 - 690 \times 0.017}{1.2 - 1}$$

$$= -16.15 \text{ kJ}$$

[i.e. work have to be given to the system]

$$Q = u_2 - u_1 + W$$

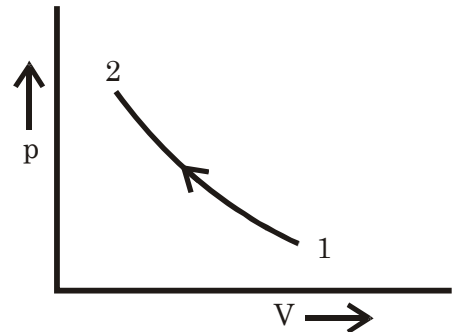
$$m = \frac{p_1 V_1}{RT_1} = 0.070948 \text{ kg}$$

$$= mc_v(T_2 - T_1) + W$$

$$= (8.7748 - 16.15) \text{ kJ}$$

$$= -7.3752 \text{ kJ}$$

[i.e. Heat flow through system]



**Charge of entropy**

$$\begin{aligned}\Delta S &= S_2 - S_1 = m c_p \ln \left( \frac{T_2}{T_1} \right) - m R \ln \left( \frac{p_2}{p_1} \right) \\ &= m \left[ 1.5738 \ln \left( \frac{390.5}{283} \right) - 0.42334 \times \ln \left( \frac{690}{100} \right) \right] \text{ kJ/K} \\ &= -0.022062 \text{ kJ/K} = -22.062 \text{ J/K}\end{aligned}$$

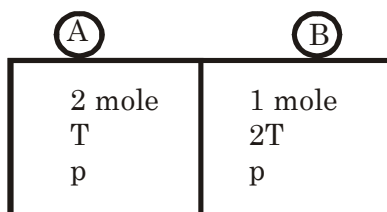
**Q.10.32** Two moles of an ideal gas at temperature  $T$  and pressure  $p$  are contained in a compartment. In an adjacent compartment is one mole of an ideal gas at temperature  $2T$  and pressure  $p$ . The gases mix adiabatically but do not react chemically when a partition separating the compartments are withdrawn. Show that the entropy increase due to the mixing process is given by

$$R \left( \ln \frac{27}{4} + \frac{\gamma}{\gamma - 1} \ln \frac{32}{27} \right)$$

Provided that the gases are different and that the ratio of specific heat  $\gamma$  is the same for both gases and remains constant.

What would the entropy change be if the mixing gases were of the same Species?

**Solution:**  $V_A = \frac{n \bar{R} T}{p} = \frac{2 \bar{R} T}{p} \quad V_B = \frac{n \bar{R} 2T}{p} = \frac{2 \bar{R} T}{p}$



After mixing if final temperature is  $T_f$  then

$$T_f = \frac{2 \times T + 1 \times 2T}{2 + 1} = \frac{4}{3} T \quad p_f = \frac{n \bar{R} T_f}{V_f} = \frac{3 \times \bar{R} \times \frac{4}{3} T \times p}{4 \bar{R} T}$$

$\therefore$  Final pressure =  $p$

$$\text{Temperature} = \frac{4}{3} T \quad \text{and} \quad \text{Volume} = V_A + V_B = \frac{4 \bar{R} T}{p}$$

$$\text{After mixing Partial Pressure of A} = p_{fA} = \frac{2}{3} p \quad c_{pA} = \frac{\gamma}{\gamma - 1} \bar{R}$$

$$\text{Partial pressure of B} = p_{fB} = \frac{1}{3} p$$

$$\begin{aligned}\therefore (\Delta S)_A &= n_A \left[ c_{pA} \ln \frac{T_f}{T_A} - \bar{R} \ln \frac{p_{fA}}{p_A} \right] \\ &= 2 \bar{R} \left[ \frac{\gamma}{\gamma - 1} \ln \frac{4}{3} - \ln \frac{2}{3} \right]\end{aligned}$$

$$\begin{aligned}
 (\Delta S)_B &= n_B \left[ c_{p_B} \ln \frac{T_f}{T_B} - \bar{R} \ln \frac{p_{fB}}{p_B} \right] \\
 &= \bar{R} \left[ \frac{\gamma}{\gamma-1} \ln \frac{2}{3} - \ln \frac{1}{3} \right]
 \end{aligned}$$

$$\therefore (\Delta S)_{\text{univ}} = (\Delta S)_A + (\Delta S)_B$$

$$\begin{aligned}
 &= \bar{R} \left[ \left( \ln \frac{9}{4} + \ln 3 \right) + \frac{\gamma}{\gamma-1} \left( \ln \frac{16}{9} + \ln \frac{2}{3} \right) \right] \\
 &= \bar{R} \left[ \ln \frac{27}{4} + \frac{\gamma}{\gamma-1} \ln \frac{32}{27} \right] \text{ Proved.}
 \end{aligned}$$

**Q.10.33**  $n_1$  moles of an ideal gas at pressure  $p_1$  and temperature  $T$  are in one compartment of an insulated container. In an adjoining compartment, separated by a partition, are  $n_2$  moles of an ideal gas at pressure  $p_2$  and temperature  $T$ . When the partition is removed, calculate (a) the final pressure of the mixture, (b) the entropy change when the gases are identical, and (c) the entropy change when the gases are different. Prove that the entropy change in (c) is the same as that produced by two independent free expansions.

**Solution:** Try please.

**Q.10.34** Assume that 20 kg of steam are required at a pressure of 600 bar and a temperature of 750°C in order to conduct a particular experiment. A 140-litre heavy duty tank is available for storage. Predict if this is an adequate storage capacity using:

- The ideal gas theory,
- The compressibility factor chart,
- The van der Waals equation with  $a = 5.454 \text{ (litre)}^2 \text{ atm/ (g mol)}^2$ ,  $b = 0.03042 \text{ litres/gmol}$  for steam,
- The Mollier chart
- The steam tables.

Estimate the error in each.

**Solution:** Try please.

**Q.10.35** Estimate the pressure of 5 kg of  $\text{CO}_2$  gas which occupies a volume of 0.70  $\text{m}^3$  at 75°C, using the Beattie-Bridgeman equation of state. Compare this result with the value obtained using the generalized compressibility chart. Which is more accurate and why? For  $\text{CO}_2$  with units of atm, litres/g mol and K,  $A_0 = 5.0065$ ,  $a = 0.07132$ ,  $B_0 = 0.10476$ ,  $b = 0.07235$ ,  $C \times 10^{-4} = 66.0$ .

**Solution:** Try please.

**Q.10.36** Measurements of pressure and temperature at various stages in an adiabatic air turbine show that the states of air lie on the line  $pv^{1.25} =$

constant. If kinetic and gravitational potential energy is neglected, prove that the shaft work per kg as a function of pressure is given by the following relation

$$W = 3.5 p_1 v_1 \left[ 1 - \left( \frac{p_2}{p_1} \right)^{1/5} \right]$$

Take  $\gamma$  for air as 1.4.

**Solution:** Using S.F.E.E.

$$Q - W + \Delta \left[ \frac{V_2^2}{2} + gZ \right] = h_2 - h_1$$

or  $Q - W = mc_p(T_2 - T_1)$

$$= \frac{\gamma}{\gamma - 1} mR(T_2 - T_1) \quad \therefore p_1 v_1 = mRT_1 \quad p_2 v_2 = mRT_2$$

$$= \frac{\gamma p_1 v_1}{\gamma - 1} \left[ \frac{p_2 v_2}{p_1 v_1} - 1 \right]$$

$$= \frac{\gamma}{\gamma - 1} p_1 v_1 \left[ \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

Here adiabatic process  $\therefore Q \rightarrow 0$  and as

So  $W = \frac{\gamma}{\gamma - 1} \times p_1 v_1 \left[ 1 - \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right]$

$$\gamma = 1.4 \text{ and } n = 1.25$$

$$W = 3.5 p_1 v_1 \left[ 1 - \left( \frac{p_2}{p_1} \right)^{\frac{1}{5}} \right] \text{ proved}$$

**Q.10.37** A mass of an ideal gas exists initially at a pressure of 200 kPa, temperature 300 K, and specific volume 0.5 m<sup>3</sup>/kg. The value of  $r$  is 1.4. (a) Determine the specific heats of the gas. (b) What is the change in entropy when the gas is expanded to pressure 100 kPa according to the law  $pv^{1.3} = \text{const}$ ? (c) What will be the entropy change if the path is  $pv^{1.5} = \text{const}$ . (by the application of a cooling jacket during the process)? (d) What is the inference you can draw from this example?

(Ans. (a) 1.166, 0.833 kJ/kg K, (b) 0.044 kJ/kg K (c) - 0.039 kJ/kg K  
(d) Entropy increases when  $n < \gamma$  and decreases when  $n > \gamma$ )

**Solution:** Given

$$p_1 = 200 \text{ kPa}$$

$$T_1 = 300 \text{ K}$$

$$v_1 = 0.5 \text{ m}^3/\text{kg}$$

$$\gamma = 1.4$$

(a) Gas constant(  $R$ ) =  $\frac{p_1 v_1}{T_1} = \frac{200 \times 0.5}{300} = 0.33333 \text{ kJ/kg} - \text{K}$

∴ Super heat at constant Pressure

$$c_p = \frac{\gamma}{\gamma - 1} R = \frac{1.4}{1.4 - 1} \times 0.33333 = 1.1667 \text{ kJ/kg} - \text{K}$$

$$c_v = c_p - R = 0.83333 \text{ kJ/kg} - \text{K}$$

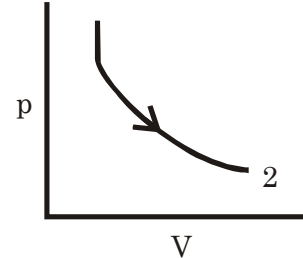
(b) Given  $p_2 = 100 \text{ kPa}$

$$v_2 = v_1 \times \left( \frac{p_1}{p_2} \right)^{\frac{1}{1.3}} = 0.85218 \text{ m}^3/\text{kg}$$

$$\therefore s_2 - s_1 = c_p \ln \frac{V_2}{V_1} + c_v \ln \frac{p_2}{p_1}$$

$$= 1.1667 \times \ln \left( \frac{0.85218}{0.5} \right) + 0.83333 \times \ln \left( \frac{100}{200} \right) \text{ kJ/kg} - \text{K}$$

$$= 0.044453 \text{ kJ/kg} - \text{K} = 44.453 \text{ J/kg} - \text{K}$$



(c) If path is  $pv^{1.5} = C$ . Then

$$v_2 = v_1 \times \left( \frac{p_1}{p_2} \right)^{\frac{1}{1.5}} = 0.7937 \text{ m}^3/\text{kg}$$

$$\therefore s_2 - s_1 = 1.1667 \times \ln \left( \frac{0.7937}{0.5} \right) + 0.83333 \ln \left( \frac{150}{200} \right) \text{ kJ/kg} - \text{K}$$

$$= -0.03849 \text{ kJ/kg} - \text{K}$$

(d)  $n > \gamma$  is possible if cooling arrangement is used and  $\Delta S$  will be -ve

**Q.10.38**

(a) A closed system of 2 kg of air initially at pressure 5 atm and temperature  $227^\circ\text{C}$ , expands reversibly to pressure 2 atm following the law  $pv^{1.25} = \text{const}$ . Assuming air as an ideal gas, determine the work done and the heat transferred.

(Ans. 193 kJ, 72 kJ)

(b) If the system does the same expansion in a steady flow process, what is the work done by the system?

(Ans. 241 kJ)

**Solution:**

Given

$$m = 2 \text{ kg}$$

$$p_1 = 5 \text{ atm} = 506.625 \text{ kPa}$$

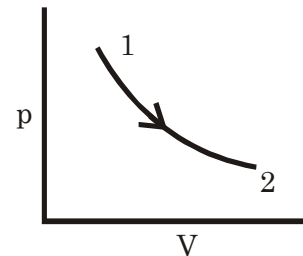
$$T_1 = 227^\circ\text{C} = 500 \text{ K}$$

$$p_2 = 2 \text{ atm} = 202.65 \text{ kPa}$$

$$T_2 = T_1 \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} = 457.9 \text{ K}$$

$$W_{1-2} = \frac{p_1 V_1 - p_2 V_2}{n - 1} = \frac{mR(T_1 - T_2)}{n - 1}$$

$$= \frac{2 \times 0.287(500 - 457.9)}{1.25 - 1} = 211.46 \text{ kJ}$$



Reversible polytropic process

Heat transfer

$$Q_{1-2} = u_2 - u_1 + W_{1-2}$$

$$= mc_v(T_2 - T_1) + W_{1-2}$$

$$= 2 \times 0.718 (457.9 - 550) + W = -132.26 + 211.46 = 79.204 \text{ kJ}$$

(b) For steady flow reversible polytropic process

$$W = h_1 - h_2$$

$$= \frac{n}{n-1} [p_1 V_1 - p_2 V_2] = \frac{mR}{n-1} [T_1 - T_2] = 264.33 \text{ kJ}$$

**Q.10.39** Air contained in a cylinder fitted with a piston is compressed reversibly according to the law  $pv^{1.25} = \text{const.}$  The mass of air in the cylinder is 0.1 kg. The initial pressure is 100 kPa and the initial temperature 20°C. The final volume is 1/8 of the initial volume. Determine the work and the heat transfer.

(Ans. – 22.9 kJ, –8.6 kJ)

**Solution:** It is a reversible polytropic process

$$m = 0.1 \text{ kg}$$

$$p_1 = 100 \text{ kPa}$$

$$\therefore V_1 = \frac{mRT_1}{P_1}$$

$$= 0.084091 \text{ m}^3$$

$$\therefore p_2 = p_1 \left( \frac{V_1}{V_2} \right)^{1.25} = 100 \times 8^{1.25}$$

$$T_2 = T_1 \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}}$$

$$\begin{aligned} \therefore W_{1-2} &= \frac{p_1 V_1 - p_2 V_2}{n-1} \\ &= \frac{100 \times 0.084091 - 1345.4 \times 0.010511}{1.25 - 1} \end{aligned}$$

$$= -22.93 \text{ kJ}$$

$$Q_{1-2} = u_2 - u_1 + W_{1-2}$$

$$= mc_v (T_2 - T_1) + W_{1-2}$$

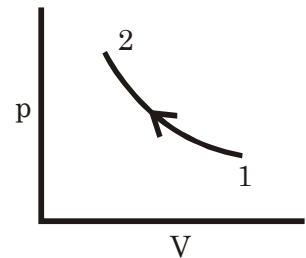
$$= 0.2 \times 0.718 \times (492.77 - 293) - 22.93$$

$$= -8.5865 \text{ kJ}$$

$$p_2 = 1345.4 \text{ kPa}$$

$$T_2 = 492.77 \text{ K}$$

$$V_2 = 0.010511 \text{ m}^3$$



**Q.10.40** Air is contained in a cylinder fitted with a frictionless piston. Initially the cylinder contains 0.5 m<sup>3</sup> of air at 1.5 bar, 20°C. The air is Then compressed reversibly according to the law  $pv^n = \text{constant}$  until the final pressure is 6 bar, at which point the temperature is 120°C. Determine: (a) the polytropic index  $n$ , (b) the final volume of air, (c) the work done on the air and the heat transfer, and (d) the net change in entropy.

(Ans. (a) 1.2685, (b) 0.1676 m<sup>3</sup> (c) –95.3 kJ, –31.5 kJ, (d) 0.0153 kJ/K)

**Solution:** Given

$$p_1 = 1.5 \text{ bar} = 150 \text{ kPa}$$

$$T_1 = 20^\circ\text{C} = 293 \text{ K}$$

$$V_1 = 0.5 \text{ m}^3$$

$$\therefore m = \frac{p_1 V_1}{RT_1} = 0.89189 \text{ kg}$$

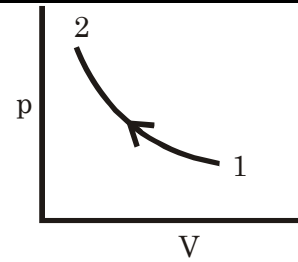
$$p_2 = 6 \text{ bar} = 600 \text{ kPa}$$

$$T_2 = 120^\circ \text{C} = 393 \text{ K}$$

$$\therefore \frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}}$$

$$\text{or} \left( 1 - \frac{1}{n} \right) = \frac{\ln \left( \frac{T_2}{T_1} \right)}{\ln \left( \frac{p_2}{p_1} \right)} = 0.2118$$

$$\therefore n = 1.2687$$



(a) The polytropic index,  $n = 1.2687$

$$(b) \text{ Final volume of air } (V_2) = \frac{mRT_2}{p_2} = \frac{0.189 \times 0.287 \times 393}{600} \text{ m}^3 = 0.16766 \text{ m}^3$$

$$(c) \quad W_{1-2} = \int_1^2 p dV = \frac{p_1 V_1 - p_2 V_2}{n - 1}$$

$$= \frac{150 \times 0.5 - 600 \times 0.16766}{1.2687 - 1} \text{ kJ} = -95.263 \text{ kJ}$$

$$Q_{1-2} = u_2 - u_1 + W_{1-2}$$

$$= mc_v(T_2 - T_1) + W_{1-2}$$

$$= 0.89189 \times 0.718(393 - 293) + W_{1-2}$$

$$= -31.225 \text{ kJ}$$

$$(d) \quad \Delta s = s_2 - s_1 = m \left[ c_v \ln \frac{p_2}{p_1} + c_p \ln \frac{V_2}{V_1} \right]$$

$$= 0.89189 \left[ 0.718 \ln \left( \frac{600}{150} \right) + 1.005 \ln \left( \frac{0.16766}{0.5} \right) \right] = -0.091663 \text{ kJ/K}$$

**Q.10.41** The specific heat at constant pressure for air is given by

$$c_p = 0.9169 + 2.577 \times 10^{-4} T - 3.974 \times 10^{-8} T^2 \text{ kJ/kg K}$$

Determine the change in internal energy and that in entropy of air when it undergoes a change of state from 1 atm and 298 K to a temperature of 2000 K at the same pressure.

( Ans. 1470.4 kJ/kg, 2.1065 kJ/kg K )

**Solution:**  $p_1 = p_2 = 1 \text{ atm} = 101.325 \text{ kPa}$

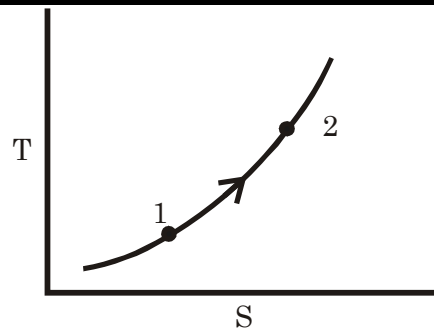
$$T_1 = 298 \text{ K}; T_2 = 2000 \text{ K}$$

$$c_p = 0.9169 + 2.577 \times 10^{-4} T - 3.974 \times 10^{-8} T^2 \text{ kJ/kg K}$$

$$\Delta u = u_2 - u_1 = \int m c_v dT$$



$$\begin{aligned}
 &= \int m(c_p - R) dT \\
 &= \int mc_p dT - mR \int dT \\
 &= 1 \times \int_{298}^{2000} (0.9169 + 2.577 \times 10^{-4} T - 3.974 \\
 &\quad \times 10^{-8} T^2) dT - 1 \times 0.287 \int_{298}^{2000} dT \text{ kJ/kg}
 \end{aligned}$$



$$= (1560.6 + 503.96 - 105.62 - 488.47) \text{ kJ/kg}$$

$$= 1470.5 \text{ kJ/kg}$$

$$\therefore Tds = dh - vdp$$

$$\text{or } Tds = mc_p dT - v dp$$

$$\therefore \int_1^2 dS = m \int_{298}^{2000} c_p \frac{dT}{T}$$

$$\therefore s_2 - s_1 = 0.9169 \times \ln \frac{2000}{298} + 2.577 \times 10^{-4} (2000 - 298)$$

$$- 3.974 \times 10^{-8} \times \left( \frac{2000^2 - 298^2}{2} \right)$$

$$= 2.1065 \text{ kJ/kg} - \text{K}$$

**Q.10.42**

A closed system allows nitrogen to expand reversibly from a volume of  $0.25 \text{ m}^3$  to  $0.75 \text{ m}^3$  along the path  $pv^{1.32} = \text{const}$ . The original pressure of the gas is  $250 \text{ kPa}$  and its initial temperature is  $100^\circ\text{C}$ .

(a) Draw the p-v and T-s diagrams.

(b) What are the final temperature and the final pressure of the gas?

(c) How much work is done and how much heat is transferred?

(d) What is the Entropy change of nitrogen?

(Ans. (b)  $262.44 \text{ K}$ ,  $58.63 \text{ kPa}$ ,  
(c)  $57.89 \text{ kJ}$ ,  $11.4 \text{ kJ}$ , (d)  $0.0362 \text{ kJ/K}$ )

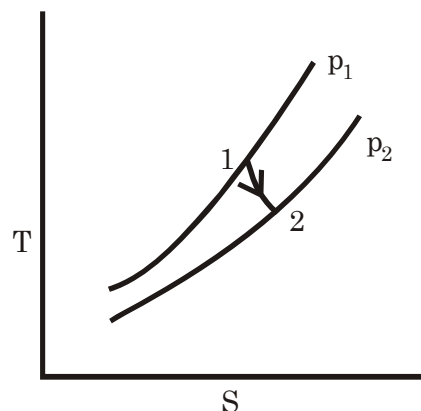
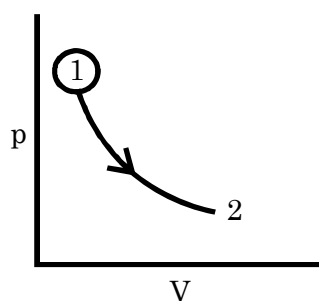
**Solution:**

Given

$$p_1 = 250 \text{ kPa}$$

$$V_1 = 0.25 \text{ m}^3$$

$$T_1 = 100^\circ\text{C} = 373 \text{ K}$$



$$\therefore m = \frac{p_1 V_1}{RT_1} = 0.563 \text{ kg} = 0.5643 \text{ kg}$$

$$R_{N_2} = \frac{8.3143}{28} = 0.29694 \text{ kJ/kg}$$

$$p_2 = p_1 \times \left( \frac{V_1}{V_2} \right)^n = 58.633 \text{ kPa}$$

$$V_2 = 0.75 \text{ m}^3$$

$$T_2 = T_1 \times \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} = 262.4 \text{ K}$$

$$W = \frac{p_1 V_1 - p_2 V_2}{n-1} = \frac{250 \times 0.25 - 58.633 \times 0.75}{(1.32-1)} = 57.891 \text{ kJ}$$

$$Q = u_2 - u_1 + W = mc_v(T_2 - T_1) + W$$

$$= 0.5643 \times 0.7423 (262.4 - 373) + W$$

$$c_v = \frac{R}{\gamma - 1} = 0.7423$$

$$= 11.56 \text{ kJ} \qquad \gamma = 1 + \frac{2}{5} = 1.4$$

$$c_p = \frac{\gamma}{\gamma - 1} R = \frac{1.4}{1.4 - 1} \times 0.29694 = 1.04 \text{ kJ/kg-K}$$

$$\therefore \Delta s = s_2 - s_1 = m \left[ c_p \ln \frac{V_2}{V_1} + c_v \ln \frac{p_2}{p_1} \right]$$

$$= 0.5643 \left[ 1.04 \times \ln \left( \frac{0.75}{0.25} \right) + 0.7423 \times \ln \left( \frac{58.633}{250} \right) \right] \text{ kJ/K}$$

$$= 0.0373 \text{ kJ/kg-K}$$

**Q.10.43** Methane has a specific heat at constant pressure given by  $c_p = 17.66 + 0.06188 T$  kJ/kg mol K when 1 kg of methane is heated at constant volume from 27 to 500°C. If the initial pressure of the gas is 1 atm, calculate the final pressure, the heat transfer, the work done and the change in entropy.

(Ans. 2.577 atm, 1258.5 kJ/kg, 2.3838 kJ/kg K)

**Solution:** Given  $p_1 = 1 \text{ atm} = 101.325 \text{ kPa}$

$$T_1 = 27^\circ\text{C} = 300 \text{ K}$$

$$m = 1 \text{ kg}$$

$$\therefore V_1 = \frac{mRT_1}{p_1}$$

$$= 1.5385 \text{ m}^3 = V_2$$

(i) Find pressure ( $p_2$ ) =  $\frac{mRT_2}{V_2}$

$$= 261 \text{ kPa} \approx 2.577 \text{ atm}$$

(ii) Heat transfer  $Q = \int mc_v dT$

$$= m \int [c_p - R] dT$$

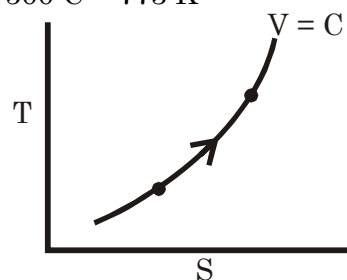
$$R = \frac{\bar{R}}{16}$$

$$= 0.51964 \text{ kJ/kg-K}$$

$$p_2 = 261 \text{ kPa}$$

$$V_2 = 1.5385$$

$$T_2 = 500^\circ\text{C} = 773 \text{ K}$$



$$= 1 \times \int_{300}^{773} (1.1038 + 3.8675 \times 10^{-3} - 0.51964) dT$$

$$= 0.58411(773 - 300) + 3.8675 \times 10^{-3} \frac{(773^2 - 300^2)}{2}$$

$$c_p = \frac{17.66}{16} + \frac{0.06188}{16} T \text{ kJ/kg} - \text{K}$$

$$= 1.1038 + 3.8675 \times 10^{-3} T = 1257.7 \text{ kJ/kg}$$

(iii) Work done =  $\int_1^2 p dV = 0$

$$\therefore T ds = du = mc_v dT$$

$$ds = mc_v \frac{dT}{T} = m(c_p - R) \frac{dT}{T}$$

$$\therefore \int_1^2 dS = \int_{300}^{773} \left( \frac{1 \times 0.58411 + 3.8675 \times 10^{-3} T}{T} \right) dT$$

$$s_2 - s_1 = 0.58411 \ln \frac{773}{300} + 3.8675 \times 10^{-3} (773 - 300) = 2.3822 \text{ kJ/kg} - \text{K}$$

**Q.10.44** Air is compressed reversibly according to the law  $pv^{1.25} = \text{const.}$  from an initial pressures of 1 bar and volume of  $0.9 \text{ m}^3$  to a final volume of  $0.6 \text{ m}^3$ . Determine the final pressure and the change of entropy per kg of air.

(Ans. 1.66 bar,  $-0.0436 \text{ kJ/kg K}$ )

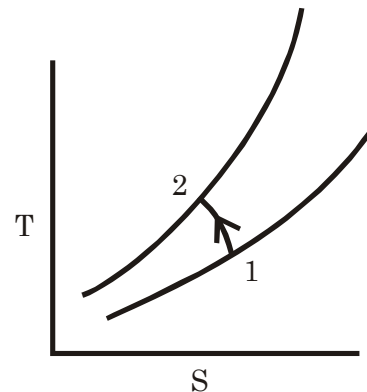
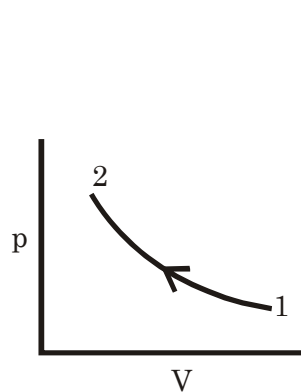
**Solution:**

$$p_1 = 1 \text{ bar}$$

$$V_1 = 0.9 \text{ m}^3$$

$$V_2 = 0.6 \text{ m}^3$$

$$\therefore p_2 = p_1 \left( \frac{V_1}{V_2} \right)^{1.25} = 1.66 \text{ bar}$$



$$\Delta s = s_2 - s_1 = \left( c_p \ln \frac{V_2}{V_1} + c_v \ln \frac{p_2}{p_1} \right)$$

$$= 1.005 \times \ln \left( \frac{0.6}{0.9} \right) + 0.718 \times \ln \left( \frac{1.66}{1} \right) \text{ kJ/kg} - \text{K}$$

$$= -0.043587 \text{ kJ/kg} - \text{K}$$

**Q.10.45** In a heat engine cycle, air is isothermally compressed. Heat is then added at constant pressure, after which the air expands isentropically to

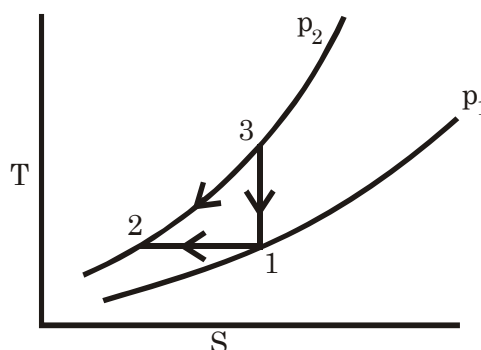
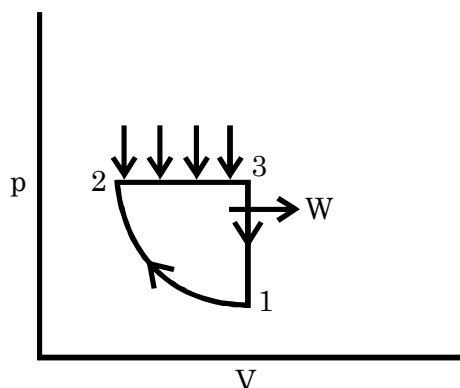
its original state. Draw the cycle on p-v and T'-s coordinates. Show that the cycle efficiency can be expressed in the following form

$$\eta = 1 - \frac{(\gamma - 1) \ln r}{\gamma [r^{\gamma-1/\gamma} - 1]}$$

Where  $r$  is the pressure ratio,  $p_2/p_1$ . Determine the pressure ratio and the cycle efficiency if the initial temperature is  $27^\circ\text{C}$  and the maximum temperature is  $327^\circ\text{C}$ .

(Ans. 13.4, 32.4%)

**Solution:** Heat addition ( $Q_1$ ) =  $Q_{2-3} = mc_p(T_3 - T_2)$



$$\text{Heat rejection } (Q_2) = mRT_1 \ln \left( \frac{p_2}{p_1} \right)$$

$$\begin{aligned} \therefore \eta &= 1 - \frac{Q_2}{Q_1} = 1 - \frac{RT_1 \ln \left( \frac{p_2}{p_1} \right)}{C_p(T_3 - T_2)} \\ &= 1 - \frac{\gamma - 1}{\gamma} \frac{\ln \left( \frac{p_2}{p_1} \right)}{\left( \frac{T_3}{T_1} - 1 \right)} \end{aligned}$$

$$\text{Here, } \frac{p_2}{p_1} = r$$

$$\therefore \frac{T_3}{T_1} = \left( \frac{p_3}{p_1} \right)^{\frac{\gamma-1}{\gamma}} = r^{\frac{\gamma-1}{\gamma}}$$

$$c_p = \frac{\gamma R}{\gamma - 1}$$

$$= 1 - \frac{\gamma - 1}{\gamma} \frac{\ln r}{(r^{\frac{\gamma-1}{\gamma}} - 1)} \quad \text{Proved}$$

$\Rightarrow$  If initial temperature ( $T_1$ ) =  $27^\circ\text{C} = 300 \text{ K} = T_2$

And  $T_3 = 327^\circ\text{C} = 600 \text{ K}$

$$\therefore r = \left( \frac{T_3}{T_1} \right)^{\frac{\gamma}{\gamma-1}} = \left( \frac{600}{300} \right)^{\frac{1.4}{1.4-1}} = 11.314$$

$$\therefore \eta = 1 - \frac{(1.4 - 1)}{(1.4)} \times \frac{\ln(11.314)}{\left[ (11.314)^{\frac{1.4-1}{1.4}} - 1 \right]} = 0.30686$$

**Q.10.46** What is the minimum amount of work required to separate 1 mole of air at 27°C and 1 atm pressure (assumed composed of 1/5 O<sub>2</sub> and 4/5 N<sub>2</sub>) into oxygen and nitrogen each at 27°C and 1 atm pressure?

( Ans. 1250 J )

**Solution:** Total air is 1 mole

So  $O_2 = \frac{1}{5} \text{ mole} = 0.0064 \text{ kg}$

$N_2 = \frac{4}{5} \text{ mole} = 0.0224 \text{ kg}$

Mixture, pressure = 1 atm, temperature = 300 K

Partial pressure of O<sub>2</sub> =  $\frac{1}{5}$  atm

Partial pressure of N<sub>2</sub> =  $\frac{4}{5}$  atm

Minimum work required is isothermal work

$$= m_{O_2} R_{O_2} T_{1_{O_2}} \ln \frac{p_{f_{O_2}}}{p_{1_{O_2}}} + m_{N_2} R_{N_2} T_{1_{N_2}} \ln \left( \frac{p_{f_{N_2}}}{p_{1_{N_2}}} \right)$$

$$= 0.0064 \times \frac{8.3143}{32} \times 300 \ln(5) + 0.0224 \times \frac{8.3143}{28} \times 300 \ln\left(\frac{5}{4}\right)$$

$$= 1.248 \text{ kJ} = 1248 \text{ J}$$

**Q.10.47** A closed adiabatic cylinder of volume 1 m<sup>3</sup> is divided by a partition into two compartments 1 and 2. Compartment 1 has a volume of 0.6 m<sup>3</sup> and contains methane at 0.4 MPa, 40°C, while compartment 2 has a volume of 0.4 m<sup>3</sup> and contains propane at 0.4 MPa, 40°C. The partition is removed and the gases are allowed to mix.

(a) When the equilibrium state is reached, find the entropy change of the universe.

(b) What are the molecular weight and the specific heat ratio of the mixture?

The mixture is now compressed reversibly and adiabatically to 1.2 MPa. Compute

(c) the final temperature of the mixture,

(d) The work required per unit mass, and

(e) The specific entropy change for each gas. Take c<sub>p</sub> of methane and propane as 35.72 and 74.56 kJ/kg mol K respectively.

(Ans. (a) 0.8609 kJ/K, (b) 27.2, 1.193 (c) 100.9°C,  
(d) 396 kJ, (e) 0.255 kJ/kg K)

**Solution:** After mixing  $p_f = 400 \text{ kPa}$   
 $T_f = 313 \text{ K}$

But partial pressure of (p<sub>1f</sub>)

$$CH_4 = \frac{0.6}{1} \times 400 = 240 \text{ kPa}$$

∴  $p_{2f} = 0.4 \times 400 = 160 \text{ kPa}$

①	②
$V_1 = 0.6 \text{ m}^3$	$V_2 = 0.4 \text{ m}^3$
$p_1 = 400 \text{ kPa}$	$p_2 = 400 \text{ kPa}$
$T_1 = 313 \text{ K}$	$T_2 = 313 \text{ K}$
CH <sub>4</sub>	C <sub>3</sub> H <sub>8</sub>

$$\begin{aligned}
 (a) \quad (\Delta S)_{CH_4} &= m_{CH_4} \left[ c_{p_{CH_4}} \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \right] \\
 &= m_{CH_4} R_{CH_4} \ln \left( \frac{p_i}{p_f} \right) \\
 &= \frac{p_1 V_1}{R_{CH_4} T_1} \times R_{CH_4} \ln \left( \frac{p_i}{p_f} \right) \\
 &= \frac{p_1 V_1}{T_1} \times R_{CH_4} \ln \left( \frac{p_i}{p_{f_1}} \right) \\
 (\Delta S)_{C_3H_8} &= \frac{p_2 V_2}{T_2} \times \ln \frac{p_i}{p_{f_2}} \\
 (\Delta S)_{Univ} &= (\Delta S)_{CH_4} + (\Delta S)_{C_3H_8} \\
 &= \frac{400 \times 0.6}{313} \ln \left( \frac{400}{240} \right) + \frac{400 \times 0.4}{313} \ln \left( \frac{400}{160} \right) \text{ kJ/K} \\
 &= 0.86 \text{ kJ/K}
 \end{aligned}$$

(b) Molecular weight

$$\begin{aligned}
 xM &= x_1 M_1 + x_2 M_2 \\
 \therefore M &= \frac{x_1}{x} M_1 + \frac{x_2}{x} M_2 = 0.6 \times 16 + 0.4 \times 44 = 27.2 \\
 c_{p \text{ mix}} &= \frac{n_1 c_{p_1} + n_2 c_{p_2}}{n_1 + n_2} = \frac{0.6 \times 35.72 + 0.4 \times 74.56}{1} = 51.256 \text{ kJ/kg} \\
 R_{\text{mix}} &= \bar{R} = 8.3143 \\
 \therefore c_{v \text{ mix}} &= c_{p \text{ mix}} - \bar{R} = 42.9417 \\
 \therefore \gamma_{\text{mix}} &= \frac{c_{p \text{ mix}}}{c_{v \text{ mix}}} = \frac{51.256}{42.9417} = 1.1936
 \end{aligned}$$

**Q.10.48** An ideal gas cycle consists of the following reversible processes: (i) isentropic compression, (ii) constant volume heat addition, (iii) isentropic expansion, and (iv) constant pressure heat rejection. Show that the efficiency of this cycle is given by

$$\eta = 1 - \frac{1}{r_k^{\gamma-1}} \left[ \frac{\gamma(a^{1/\gamma} - 1)}{a - 1} \right]$$

Where  $r_k$  is the compression ratio and  $a$  is the ratio of pressures after and before heat addition.

An engine operating on the above cycle with a compression ratio of 6 starts the compression with air at 1 bar, 300 K. If the ratio of pressures after and before heat addition is 2.5, calculate the efficiency and the m.e.p. of the cycle. Take

$$\gamma = 1.4 \text{ and } c_v = 0.718 \text{ kJ/kg K.}$$

( Ans. 0.579, 2.5322 bar)

**Solution:**  $Q_{2-3} = u_3 - u_2 + p dV = m c_v (T_3 - T_2)$

$$Q_{1-4} = mc_p(T_4 - T_1)$$

$$\therefore \eta = 1 - \frac{mc_p(T_4 - T_1)}{mc_v(T_3 - T_2)}$$

$$= 1 - \gamma \left( \frac{T_4 - T_1}{T_3 - T_2} \right)$$

$$\frac{T_2}{T_1} = \left( \frac{v_1}{v_2} \right)^{\gamma-1} = r_k^{\gamma-1}$$

$$\therefore T_2 = T_1 \times r_k^{\gamma-1}$$

$$\frac{T_3}{T_4} = \left( \frac{p_3}{p_4} \right)^{\frac{\gamma-1}{\gamma}} = \left( \frac{p_3}{p_1} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \frac{p_3}{p_2} = 9$$

$$\frac{p_2 v_2}{T_2} = \frac{p_3 v_3}{T_3}$$

$$\therefore \frac{T_3}{T_4} = (a \times r)^{\frac{\gamma-1}{\gamma}}$$

$$\left( \frac{p_2}{p_1} = \frac{v_1}{v_2} \right)^{\gamma} = a^{\gamma} \quad \therefore \frac{p_3}{p_2} = \frac{T_3}{T_2} = a,$$

$$\therefore T_3 = aT_2 = aT_1 r_k^{\gamma-1}$$

$$\frac{T_4}{T_3} = \left( \frac{p_4}{p_3} \right)^{\frac{\gamma-1}{\gamma}} = \left( \frac{p_1}{p_3} \right)^{\frac{\gamma-1}{\gamma}} = \left( \frac{p_1}{p_2} \times \frac{p_2}{p_3} \right)^{\frac{\gamma-1}{\gamma}}$$

$$= \left( \left( \frac{V_2}{V_1} \right)^{\gamma} \times \frac{1}{a} \right)^{\frac{\gamma-1}{\gamma}}$$

$$= (r_k^{-\gamma} a^{-1})^{\frac{\gamma-1}{\gamma}}$$

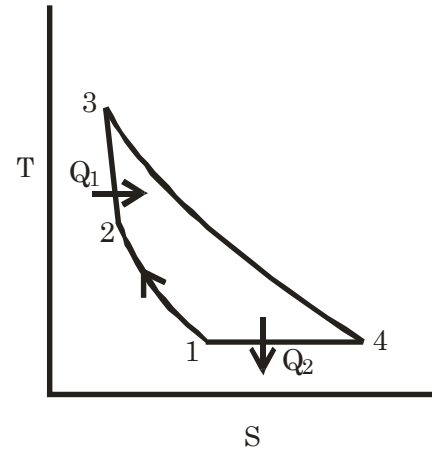
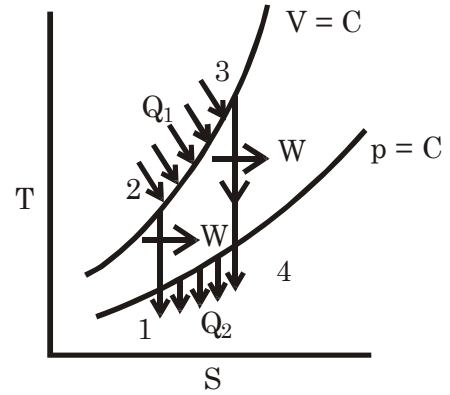
$$\begin{aligned} \therefore T_4 &= r_k^{1-\gamma} \cdot a^{\frac{1-\gamma}{\gamma}} \cdot T_3 \\ &= r_k^{1-\gamma} \cdot a^{\frac{1}{\gamma}-1} \cdot a \times T_1 \cdot r_k^{\gamma-1} \\ &= a^{\frac{1}{\gamma}} \cdot T_1 \end{aligned}$$

$$\begin{aligned} \therefore \eta &= 1 - \frac{\gamma(a^{\frac{1}{\gamma}} T_1 - T_1)}{(aT_1 r_k^{\gamma-1} - T_1 r_k^{\gamma-1})} \\ &= 1 - \frac{[\gamma(a^{\frac{1}{\gamma}} - 1)]}{r_k^{\gamma-1}(a - 1)} \text{ Proved.} \end{aligned}$$

Given  $p_1 = 1 \text{ bar} = 100 \text{ kPa}$

$T_1 = 300\text{K}, \quad r_k = 6,$

$a = 2.5, \quad \gamma = 1.4$



$$\therefore \eta = 1 - \frac{1.4(2.5^{\frac{1}{1.4}} - 1)}{6^{1.4-1}(2.5 - 1)} = 0.57876$$

$$\begin{aligned} Q_1 &= mc_v(T_3 - T_2) \\ &= mc_v(aT_1r_k^{\gamma-1} - T_1r_k^{\gamma-1}) \\ &= mc_v T_1r_k^{\gamma-1}(a - 1) \\ &= m \times 0.718 \times 300 \times 6^{0.4} \times (2.5 - 1) \\ &= 661.6 \text{ m kJ} \end{aligned}$$

$$\therefore W = \eta Q_1 = 382.9 \text{ m kJ}$$

$$\text{For } V_4 = ; T_4 = 2.5^{\frac{1}{1.4}} \times 300 = 577.25 \text{ K}$$

$$p_4 = p_1 = 100 \text{ kPa}$$

$$V_4 = \frac{mRT_4}{p_4} = \frac{m \times 0.287 \times 577.25}{100} \text{ m}^3$$

$$= 1.6567 \text{ m m}^3$$

$$V_1 = \frac{mRT_1}{p_1} = \frac{m \times 0.287 \times 300}{100}$$

$$= 0.861 \text{ m m}^3$$

$$\therefore V_4 - V_1 = 0.7957 \text{ m}^3$$

Let m.e.p. is  $p_m$  then

$$p_m (V_4 - V_1) = W$$

$$p_m = \frac{382.9 \times m}{0.7957 \text{ m}} \text{ kPa}$$

$$= 481.21 \text{ kPa} = 4.8121 \text{ bar}$$

**Q10.49** The relation between  $u$ ,  $p$  and  $v$  for many gases is of the form  $u = a + bpv$  where  $a$  and  $b$  are constants. Show that for a reversible adiabatic process  $pv^\gamma = \text{constant}$ , where

$$\gamma = (b + 1)/b.$$

**Solution:** Try please.

**Q10.50** (a) Show that the slope of a reversible adiabatic process on  $p$ - $v$  coordinates is

$$\frac{dp}{dv} = \frac{1}{kv} \frac{c_p}{c_v} \text{ where } k = -\frac{1}{v} \left( \frac{\partial v}{\partial p} \right)_T$$

(b) Hence, show that for an ideal gas,  $pv^\gamma = \text{constant}$ , for a reversible adiabatic process.

**Solution:** Try please.

**Q10.51** A certain gas obeys the Clausius equation of state  $p(v - b) = RT$  and has its internal energy given by  $u = c_v T$ . Show that the equation for a reversible adiabatic process is  $p(v - b)^\gamma = \text{constant}$ , where  $\gamma = c_p / c_v$ .

**Solution:** Try please.



- Q10.52** (a) Two curves, one representing a reversible adiabatic process undergone by an ideal gas and the other an isothermal process by the same gas, intersect at the same point on the p-v diagram. Show that the ratio of the slope of the adiabatic curve to the slope of the isothermal curve is equal to  $\gamma$ .
- (b) Determine the ratio of work done during a reversible adiabatic process to the work done during an isothermal process for a gas having  $\gamma = 1.6$ . Both processes have a pressure ratio of 6.

**Solution:** Try please.

- Q10.53** Two containers  $p$  and  $q$  with rigid walls contain two different monatomic gases with masses  $m_p$  and  $m_q$ , gas constants  $R_p$  and  $R_q$ , and initial temperatures  $T_p$  and  $T_q$  respectively, are brought in contact with each other and allowed to exchange energy until equilibrium is achieved. Determine:
- (a) the final temperature of the two gases and
- (b) the change of entropy due to this energy exchange.

**Solution:** Try please.

- Q10.54** The pressure of a certain gas (photon gas) is a function of temperature only and is related to the energy and volume by  $p(T) = (1/3) (U/V)$ . A system consisting of this gas confined by a cylinder and a piston undergoes a Carnot cycle between two pressures  $P_1$  and  $P_2$ .
- (a) Find expressions for work and heat of reversible isothermal and adiabatic processes.
- (b) Plot the Carnot cycle on p-v and T- s diagrams.
- (c) Determine the efficiency of the cycle in terms of pressures.
- (d) What is the functional relation between pressure and temperature?

**Solution:** Try please.

- Q10.55** The gravimetric analysis of dry air is approximately: oxygen = 23%, nitrogen = 77%. Calculate:
- (a) The volumetric analysis,
- (b) The gas constant,
- (c) The molecular weight,
- (d) the respective partial pressures,
- (e) The specific volume at 1 atm, 15°C, and
- (f) How much oxygen must be added to 2.3 kg air to produce . A mixture which is 50% oxygen by volume?

(Ans. (a) 21%  $O_2$ , 79%  $N_2$ , (b) 0.288 kJ/kg K,  
(d) 21 kPa for  $O_2$  ' (e) 0.84 m<sup>3</sup>/kg, (f) 1.47 kg)

**Solution:** By gravimetric analysis  $O_2: N_2 = 23: 77$

$$\begin{aligned} \text{(a) } \therefore \text{ By volumetric analysis } O_2: N_2 &= \frac{23}{32} : \frac{77}{28} \\ &= 0.71875: 2.75 \\ &= 0.71875 \times \frac{(100)}{(0.71875 - 2.75)} : \frac{2.75 \times 100}{2.75} \\ &= 20.72: 79.28 \end{aligned}$$

(b) Let total mass = 100 kg

$$\therefore \text{O}_2 = 23 \text{ kg}, \text{N}_2 = 77 \text{ kg}$$

$$\begin{aligned} \therefore R &= \frac{23 \times R_{\text{O}_2} + 77 \times R_{\text{N}_2}}{23 + 77} \\ &= \frac{23 \times \frac{8.3143}{32} + 77 \times \frac{8.3143}{28}}{23 + 77} \\ &= 0.2884 \text{ kJ/kg} - \text{K} \end{aligned}$$

(c) For molecular weight ( $\mu$ )

$$x\mu = x_1 \mu_1 + x_2 \mu_2$$

$$\begin{aligned} \text{or } \mu &= \frac{x_1}{x} \times \mu_1 + \frac{x_2}{x} \mu_2 \\ &= 0.2072 \times 32 + 0.7928 \times 28 = 28.83 \end{aligned}$$

(d) Partial pressure of  $\text{O}_2 = x_{\text{O}_2} \times p$

$$= 0.2072 \times 101.325 \text{ kPa} = 20.995 \text{ kPa}$$

$$\text{Partial pressure of } \text{N}_2 = x_{\text{N}_2} \times p = 0.7928 \times 101.325 \text{ kPa} = 80.33 \text{ kPa}$$

(e) Sp. volume,  $v = \frac{RT}{\rho} = \frac{0.2884 \times 288}{101.325} \text{ m}^3/\text{kg} = 0.81973 \text{ m}^3/\text{kg}$

Density  $\rho = \rho_1 + \rho_2$

$$\begin{aligned} \therefore \frac{1}{v} &= \frac{1}{v_1} + \frac{1}{v_2} = \frac{p_{\text{O}_2}}{R_{\text{O}_2} \times 288} + \frac{p_{\text{N}_2}}{R_{\text{N}_2} \times 288} \\ &= \frac{0.2072 \times 101.325 \times 32}{8.3143 \times 288} + \frac{0.7928 \times 101.325 \times 28}{8.3143 \times 288} \end{aligned}$$

$$\therefore v = 0.81974 \text{ m}^3/\text{kg}$$

(f) In 2.3 kg of air  $\text{O}_2 = 2.3 \times 0.23 \text{ kg} = 0.529 \text{ kg}$

$$\therefore \text{N}_2 = 2.3 \times 0.77 = 1.771 \text{ kg} = 63.25 \text{ mole}$$

For same volume we need same mole  $\text{O}_2$

$$\text{Total } \text{O}_2 = 63.25 \times \frac{32}{1000} \text{ kg} = 2.024 \text{ kg}$$

$$\therefore \text{Oxygen must be added} = (2.024 - 0.529) \text{ kg} = 1.495 \text{ kg}$$

# 11. Thermodynamic Relations

## Some Important Notes

### Some Mathematical Theorem

**Theorem 1.** If a relation exists among the variables  $x$ ,  $y$  and  $z$ , then  $z$  may be expressed as a function of  $x$  and  $y$ , or

$$dz = \left( \frac{\partial z}{\partial x} \right)_y dx + \left( \frac{\partial z}{\partial y} \right)_x dy$$

then  $dz = M dx + N dy$ .

Where  $z$ ,  $M$  and  $N$  are functions of  $x$  and  $y$ . Differentiating  $M$  partially with respect to  $y$ , and  $N$  with respect to  $x$ .

$$\left( \frac{\partial M}{\partial y} \right)_x = \frac{\partial^2 z}{\partial x \partial y}$$

$$\left( \frac{\partial N}{\partial x} \right)_y = \frac{\partial^2 z}{\partial y \partial x}$$

$$\left( \frac{\partial M}{\partial y} \right)_x = \left( \frac{\partial N}{\partial x} \right)_y$$

This is the condition of exact (or perfect) differential.

**Theorem 2.** If a quantity  $f$  is a function of  $x$ ,  $y$  and  $z$ , and a relation exists among  $x$ ,  $y$  and  $z$ , then  $f$  is a function of any two of  $x$ ,  $y$  and  $z$ . Similarly any one of  $x$ ,  $y$  and  $z$  may be regarded to be a function of  $f$  and any one of  $x$ ,  $y$  and  $z$ . Thus, if

$$x = x(f, y)$$

$$dx = \left( \frac{\partial x}{\partial f} \right)_y df + \left( \frac{\partial x}{\partial y} \right)_f dy$$

Similarly, if

$$y = y(f, z)$$

$$dy = \left( \frac{\partial y}{\partial f} \right)_z df + \left( \frac{\partial y}{\partial z} \right)_f dz$$

Substituting the expression of  $dy$  in the preceding equation

$$dx = \left( \frac{\partial x}{\partial f} \right)_y df + \left( \frac{\partial x}{\partial y} \right)_f \left[ \left( \frac{\partial y}{\partial f} \right)_z df + \left( \frac{\partial y}{\partial z} \right)_f dz \right]$$

$$= \left[ \left( \frac{\partial x}{\partial f} \right)_y + \left( \frac{\partial x}{\partial y} \right)_f \left( \frac{\partial y}{\partial f} \right)_z \right] df + \left( \frac{\partial x}{\partial y} \right)_f \left( \frac{\partial y}{\partial z} \right)_f dz$$

Again

$$dx = \left( \frac{\partial x}{\partial f} \right)_z df + \left( \frac{\partial x}{\partial z} \right)_f dz$$

$$\left( \frac{\partial x}{\partial z} \right)_f = \left( \frac{\partial x}{\partial y} \right)_f \left( \frac{\partial y}{\partial z} \right)_f$$

$$\left( \frac{\partial x}{\partial y} \right)_f \left( \frac{\partial y}{\partial z} \right)_f \left( \frac{\partial z}{\partial x} \right)_f = 1$$

**Theorem 3.** Among the variables x, y, and z any one variable may be considered as a function of the other two. Thus

$$x = x(y, z)$$

$$dx = \left( \frac{\partial x}{\partial y} \right)_z dy + \left( \frac{\partial x}{\partial z} \right)_y dz$$

Similarly,

$$dz = \left( \frac{\partial z}{\partial x} \right)_y dx + \left( \frac{\partial z}{\partial y} \right)_x dy$$

$$dx = \left( \frac{\partial x}{\partial y} \right)_z dy + \left( \frac{\partial x}{\partial z} \right)_y \left[ \left( \frac{\partial z}{\partial x} \right)_y dx + \left( \frac{\partial z}{\partial y} \right)_x dy \right]$$

$$= \left[ \left( \frac{\partial x}{\partial y} \right)_z + \left( \frac{\partial x}{\partial z} \right)_y \left( \frac{\partial z}{\partial y} \right)_x \right] dy + \left( \frac{\partial x}{\partial z} \right)_y \left( \frac{\partial z}{\partial x} \right)_y dx$$

$$= \left[ \left( \frac{\partial x}{\partial y} \right)_z + \left( \frac{\partial x}{\partial z} \right)_y \left( \frac{\partial z}{\partial y} \right)_x \right] dy + dx$$

$$\therefore \left( \frac{\partial x}{\partial y} \right)_z + \left( \frac{\partial x}{\partial z} \right)_y \left( \frac{\partial z}{\partial y} \right)_x = 0$$

$$\left( \frac{\partial x}{\partial y} \right)_z \left( \frac{\partial z}{\partial x} \right)_y \left( \frac{\partial y}{\partial z} \right)_x = -1$$

Among the thermodynamic variables p, V and T. The following relation holds good

$$\left( \frac{\partial p}{\partial V} \right)_T \left( \frac{\partial V}{\partial T} \right)_p \left( \frac{\partial T}{\partial p} \right)_V = -1$$

## Maxwell's Equations

A pure substance existing in a single phase has only two independent variables. Of the eight quantities p, V, T, S, U, H, F (Helmholtz function), and G (Gibbs function) any one may be expressed as a function of any two others.

For a pure substance undergoing an infinitesimal reversible process

(a)  $dU = TdS - pdV$

(b)  $dH = dU + pdV + Vdp = TdS + Vdp$

(c)  $dF = dU - TdS - SdT = -pdT - SdT$

(d)  $dG = dH - TdS - SdT = Vdp - SdT$

Since U, H, F and G are thermodynamic properties and exact differentials of the type

$$dz = M dx + N dy, \text{ then}$$

$$\left( \frac{\partial M}{\partial y} \right)_x = \left( \frac{\partial N}{\partial x} \right)_y$$

Applying this to the four equations

$$\left(\frac{\partial T}{\partial V}\right)_s = -\left(\frac{\partial p}{\partial S}\right)_v$$

$$\left(\frac{\partial T}{\partial P}\right)_s = \left(\frac{\partial V}{\partial S}\right)_p$$

$$\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial S}{\partial V}\right)_T$$

$$\left(\frac{\partial V}{\partial T}\right)_p = -\left(\frac{\partial S}{\partial p}\right)_T$$

These four equations are known as **Maxwell's equations**.

### Questions with Solution (IES & IAS)

**(i) Derive:**  $dS = C_v \frac{dT}{T} + \left(\frac{\partial p}{\partial T}\right)_v dV$  [IAS - 1986]

Let entropy  $S$  be imagined as a function of  $T$  and  $V$ .  
Then  $S = S(T, V)$

$$\text{or } dS = \left(\frac{\partial S}{\partial T}\right)_v dT + \left(\frac{\partial S}{\partial V}\right)_T dV$$

multiplying both side by  $T$

$$TdS = T\left(\frac{\partial S}{\partial T}\right)_v dT + T\left(\frac{\partial S}{\partial V}\right)_T dV$$

Since  $T\left(\frac{\partial S}{\partial T}\right)_v = C_v$ , heat capacity at constant volume

and  $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_v$  by Maxwell's equation

$$\therefore TdS = C_v dT + T\left(\frac{\partial p}{\partial T}\right)_v dV$$

dividing both side by  $T$

$$dS = C_v \frac{dT}{T} + \left(\frac{\partial p}{\partial T}\right)_v dV \text{ proved}$$

**(ii) Derive:**

$$TdS = C_p dT - T\left(\frac{\partial V}{\partial T}\right)_p dp$$
[IES-1998]

Let entropy  $S$  be imagined as a function of  $T$  and  $p$ .

Then  $S = S(T, p)$

$$\text{or } dS = \left( \frac{\partial S}{\partial T} \right)_p dT + \left( \frac{\partial S}{\partial p} \right)_T dp$$

multiplying both side by T

$$TdS = T \left( \frac{\partial S}{\partial T} \right)_p dT + T \left( \frac{\partial S}{\partial p} \right)_T dp$$

Since  $T \left( \frac{\partial S}{\partial T} \right)_p = C_p$ , heat capacity at constant pressure

and  $\left( \frac{\partial S}{\partial p} \right)_T = - \left( \frac{\partial V}{\partial T} \right)_p$  by Maxwell's equation

$$\therefore TdS = C_p dT - T \left( \frac{\partial V}{\partial T} \right)_p dp \quad \text{proved.}$$

**(iii) Derive:**

$$TdS = C_v dT + T \frac{\beta}{k} dV = C_p dT - TV\beta dp = \frac{k C_v dp}{\beta} + \frac{C_p}{\beta V} dV$$

[IES-2001]

We know that volume expansivity  $(\beta) = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_p$

and isothermal compressibility  $(k) = - \frac{1}{V} \left( \frac{\partial V}{\partial p} \right)_T$

$\therefore$  From first TdS equation

$$TdS = C_v dT + T \left( \frac{\partial p}{\partial T} \right)_v dV$$

$$\frac{\beta}{k} = - \frac{\left( \frac{\partial V}{\partial T} \right)_p}{\left( \frac{\partial V}{\partial p} \right)_T} = - \left( \frac{\partial V}{\partial T} \right)_p \cdot \left( \frac{\partial p}{\partial V} \right)_T$$

$$\text{As } \left( \frac{\partial V}{\partial T} \right)_p \cdot \left( \frac{\partial T}{\partial p} \right)_v \cdot \left( \frac{\partial p}{\partial V} \right)_T = -1$$

$$\therefore - \left( \frac{\partial V}{\partial T} \right)_p \cdot \left( \frac{\partial p}{\partial V} \right)_T = \left( \frac{\partial p}{\partial T} \right)_v$$

$$\text{or } \frac{\beta}{k} = \left( \frac{\partial p}{\partial T} \right)_v$$

$$\therefore TdS = C_v dT + T \cdot \frac{\beta}{k} \cdot dV \quad \text{proved}$$

From second TdS relation

$$TdS = C_p dT - T \left( \frac{\partial V}{\partial T} \right)_p dp$$

$$\text{as } \beta = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_p$$

$$\therefore \left( \frac{\partial V}{\partial T} \right)_p = V\beta$$

$$\therefore TdS = C_p dT - TV\beta dp \quad \text{proved}$$

Let S is a function of p, V

$$\therefore S = S(p, V)$$

$$\therefore dS = \left( \frac{\partial S}{\partial p} \right)_V dp + \left( \frac{\partial S}{\partial V} \right)_p dV$$

Multiply both side by T

$$TdS = T \left( \frac{\partial S}{\partial p} \right)_V dp + T \left( \frac{\partial S}{\partial V} \right)_p dV$$

$$\text{or } TdS = T \left( \frac{\partial S}{\partial T} \cdot \frac{\partial T}{\partial p} \right)_V dp + T \left( \frac{\partial S}{\partial T} \cdot \frac{\partial T}{\partial V} \right)_p dV$$

$$\text{or } TdS = T \left( \frac{\partial S}{\partial T} \right)_V \cdot \left( \frac{\partial T}{\partial p} \right)_V dp + T \left( \frac{\partial S}{\partial T} \right)_p \cdot \left( \frac{\partial T}{\partial V} \right)_p dV$$

$$C_p = T \left( \frac{\partial S}{\partial T} \right)_p \quad \text{and} \quad C_V = T \left( \frac{\partial S}{\partial T} \right)_V$$

$$\therefore TdS = C_V \left( \frac{\partial T}{\partial p} \right)_V dp + C_p \left( \frac{\partial T}{\partial V} \right)_p dV$$

$$\text{From first } \frac{\beta}{k} = \left( \frac{\partial p}{\partial T} \right)_V \quad \text{or} \quad \frac{k}{\beta} = \left( \frac{\partial T}{\partial p} \right)_V$$

$$\therefore TdS = C_V \frac{k}{\beta} dp + C_p \left( \frac{\partial T}{\partial V} \right)_p dV$$

$$\therefore \beta = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_p$$

$$\therefore \left( \frac{\partial T}{\partial V} \right)_p = \frac{1}{\beta V}$$

$$\therefore TdS = \frac{C_V k dp}{\beta} + \frac{C_p}{\beta V} dV \quad \text{proved.}$$

**(iv) Prove that**

$$C_p - C_V = -T \left( \frac{\partial V}{\partial T} \right)_p^2 \cdot \left( \frac{\partial p}{\partial V} \right)_T$$

**[IAS-1998]**

We know that

$$TdS = C_p dT - T \left( \frac{\partial V}{\partial T} \right)_p dp = C_v dT + T \left( \frac{\partial p}{\partial T} \right)_v dV$$

$$\text{or} \quad (C_p - C_v) dT = T \left( \frac{\partial V}{\partial T} \right)_p dp + T \left( \frac{\partial p}{\partial T} \right)_v dV$$

$$\text{or} \quad dT = \frac{T \left( \frac{\partial V}{\partial T} \right)_p dp}{C_p - C_v} + \frac{T \left( \frac{\partial p}{\partial T} \right)_v dV}{C_p - C_v} \quad \text{--- (i)}$$

since  $T$  is a function of  $p, V$

$$T = T(p, V)$$

$$\text{or} \quad dT = \left( \frac{\partial T}{\partial p} \right)_v dp + \left( \frac{\partial T}{\partial V} \right)_p dV \quad \text{--- (ii)}$$

comparing (i) & (ii) we get

$$\frac{T \left( \frac{\partial V}{\partial T} \right)_p}{C_p - C_v} = \left( \frac{\partial T}{\partial p} \right)_v \quad \text{and} \quad \frac{T \left( \frac{\partial p}{\partial T} \right)_v}{C_p - C_v} = \left( \frac{\partial T}{\partial V} \right)_p$$

both these give

$$C_p - C_v = T \left( \frac{\partial V}{\partial T} \right)_p \left( \frac{\partial p}{\partial T} \right)_v$$

$$\text{Here} \quad \left( \frac{\partial p}{\partial T} \right)_v \cdot \left( \frac{\partial T}{\partial V} \right)_p \cdot \left( \frac{\partial V}{\partial p} \right)_T = -1 \quad \text{or} \quad \left( \frac{\partial p}{\partial T} \right)_v = - \left( \frac{\partial V}{\partial T} \right)_p \cdot \left( \frac{\partial p}{\partial V} \right)_T$$

$$\therefore \quad C_p - C_v = - T \left( \frac{\partial V}{\partial T} \right)_p^2 \cdot \left( \frac{\partial p}{\partial V} \right)_T \quad \text{proved.} \quad \text{.....Equation(A)}$$

This is a very important equation in thermodynamics. It indicates the following important facts.

(a) Since  $\left( \frac{\partial V}{\partial T} \right)_p^2$  is always positive, and  $\left( \frac{\partial p}{\partial V} \right)_T$  for any substance is negative.  $(C_p - C_v)$  is always

positive. Therefore,  $C_p$  is always greater than  $C_v$ .

(b) As  $T \rightarrow 0K, C_p \rightarrow C_v$  or at absolute zero,  $C_p = C_v$ .

(c) When  $\left( \frac{\partial V}{\partial T} \right)_p = 0$  (e.g for water at  $4^\circ\text{C}$ , when density is maximum. Or specific volume minimum).  $C_p = C_v$ .

(d) For an ideal gas,  $pV = mRT$

$$\left( \frac{\partial V}{\partial T} \right)_p = \frac{mR}{P} = \frac{V}{T}$$

$$\text{and} \quad \left( \frac{\partial p}{\partial V} \right)_T = - \frac{mRT}{V^2}$$

$$\therefore \quad C_p - C_v = mR$$

$$\text{or } c_p - c_v = R$$

Equation (A) may also be expressed in terms of **volume expansivity ( $\beta$ )** defined as



$$\beta = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_p$$

and **isothermal compressibility ( $k_T$ )** defined as

$$k_T = -\frac{1}{V} \left( \frac{\partial V}{\partial p} \right)_T$$

$$C_p - C_v = \frac{TV \left[ \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_p \right]^2}{-\frac{1}{V} \left( \frac{\partial V}{\partial p} \right)_T}$$

$$C_p - C_v = \frac{TV\beta^2}{k_T}$$

**(v) Prove that**

$$\frac{\beta}{k} = \left( \frac{\partial p}{\partial T} \right)_v \quad \text{and} \quad C_p - C_v = \left\{ p + \left( \frac{\partial U}{\partial V} \right)_T \right\} \left( \frac{\partial V}{\partial T} \right)_p$$

Hence show that

$$C_p - C_v = \frac{\beta^2 TV}{k} \quad \text{[IES-2003]}$$

$$\text{Here } \beta = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_p$$

$$k = -\frac{1}{V} \left( \frac{\partial V}{\partial p} \right)_T$$

$$\therefore \frac{\beta}{k} = -\frac{\left( \frac{\partial V}{\partial T} \right)_p}{\left( \frac{\partial V}{\partial p} \right)_T} = -\left( \frac{\partial V}{\partial T} \right)_p \cdot \left( \frac{\partial p}{\partial V} \right)_T$$

$$\text{we know that} \quad \left( \frac{\partial V}{\partial T} \right)_p \cdot \left( \frac{\partial T}{\partial p} \right)_v \cdot \left( \frac{\partial p}{\partial V} \right)_T = -1$$

$$\text{or} \quad -\left( \frac{\partial V}{\partial T} \right)_p \cdot \left( \frac{\partial p}{\partial V} \right)_T = \left( \frac{\partial p}{\partial T} \right)_v$$

$$\therefore \frac{\beta}{k} = \left( \frac{\partial p}{\partial T} \right)_v \quad \text{proved.}$$

$\Rightarrow$  From Tds relations

$$TdS = C_p dT - T \left( \frac{\partial V}{\partial T} \right)_p dP = C_v dT + T \left( \frac{\partial p}{\partial T} \right)_v dV$$

$$\therefore (C_p - C_v) dT = T \left( \frac{\partial V}{\partial T} \right)_p dP + T \left( \frac{\partial p}{\partial T} \right)_v dV$$

$$\text{or} \quad dT = \frac{T \left( \frac{\partial V}{\partial T} \right)_p}{C_p - C_v} dP + \frac{T \left( \frac{\partial p}{\partial T} \right)_v}{C_p - C_v} dV \quad \text{--- (i)}$$

Since T is a function of (p, V)

$$T = T(p, V)$$

$$\therefore dT = \left( \frac{\partial T}{\partial p} \right)_v dp + \left( \frac{\partial T}{\partial V} \right)_p dV \quad \text{--- (ii)}$$

Comparing (i) & (ii) we get

$$\frac{T \left( \frac{\partial V}{\partial T} \right)_p}{C_p - C_v} = \left( \frac{\partial T}{\partial p} \right)_v \quad \text{and} \quad \frac{T \left( \frac{\partial p}{\partial T} \right)_v}{C_p - C_v} = \left( \frac{\partial T}{\partial V} \right)_p$$

$$\therefore C_p - C_v = T \left( \frac{\partial V}{\partial T} \right)_p \cdot \left( \frac{\partial p}{\partial T} \right)_v$$

$$\text{as} \quad dU = dQ - pdV$$

$$\therefore dU = TdS - pdV$$

$$\text{or} \quad \left( \frac{\partial U}{\partial V} \right)_T = T \left( \frac{\partial S}{\partial V} \right)_T - p$$

$$\text{or} \quad \left( \frac{\partial U}{\partial V} \right)_T + p = T \left( \frac{\partial S}{\partial V} \right)_T$$

From Maxwell's Third relations

$$\left( \frac{\partial p}{\partial T} \right)_v = \left( \frac{\partial S}{\partial V} \right)_T$$

$$\therefore C_p - C_v = T \left( \frac{\partial V}{\partial T} \right)_p \cdot \left( \frac{\partial p}{\partial T} \right)_v = \left\{ p + \left( \frac{\partial U}{\partial V} \right)_T \right\} \left( \frac{\partial V}{\partial T} \right)_p$$

## (vi) Prove that

$$\text{Joule - Thomson co-efficient} \quad \mu = \left( \frac{\partial T}{\partial p} \right)_h = \frac{T^2}{C_p} \left[ \frac{\partial}{\partial T} \left( \frac{V}{T} \right) \right]_p \quad [\text{IES-2002}]$$

The numerical value of the slope of an isenthalpic on a T - p diagram at any point is called the Joule - Kelvin coefficient.

$$\therefore \mu = \left( \frac{\partial T}{\partial p} \right)_h$$

$$\text{Here} \quad dH = TdS + Vdp$$

$$\text{as } TdS = C_p dT - T \left( \frac{\partial V}{\partial T} \right)_p dp$$

$$\therefore dH = C_p dT - T \left( \frac{\partial V}{\partial T} \right)_p dp + V dp = C_p dT - \left[ T \left( \frac{\partial V}{\partial T} \right)_p - V \right] dp$$

$$\text{if } H = \text{const.} \quad \therefore dH = 0$$

$$\text{so } C_p (dT)_h - \left[ T \left( \frac{\partial V}{\partial T} \right)_p - V \right] (dp)_h = 0$$

$$\therefore \left( \frac{\partial T}{\partial p} \right)_h = \frac{1}{C_p} \left[ T \left( \frac{\partial V}{\partial T} \right)_p - V \right] = \frac{T^2}{C_p} \left[ \frac{1}{T} \cdot \left( \frac{\partial V}{\partial T} \right)_p - \frac{V}{T^2} \right] = \frac{T^2}{C_p} \left[ \frac{\partial}{\partial T} \left( \frac{V}{T} \right) \right]_p$$

### (vii) Derive Clausius – Clapeyron equation

$$\left( \frac{dp}{dT} \right) = \frac{h_{fg}}{T(v_g - v_f)} \quad \text{and} \quad \frac{dp}{p} = \frac{h_{fg}}{RT^2} dT \quad [\text{IES-2000}]$$

$$\left( \frac{\partial p}{\partial T} \right)_v = \left( \frac{\partial S}{\partial V} \right)_T \quad \text{Maxwells equation}$$

When saturated liquid convert to saturated vapour at constant temperature. During the evaporation, the pr. & T is independent of volume.

$$\therefore \left( \frac{dp}{dT} \right)_{\text{sat}} = \frac{s_g - s_f}{v_g - v_f}$$

$$s_g - s_f = s_{fg} = \frac{h_{fg}}{T}$$

$$\text{or } \left( \frac{dp}{dT} \right)_{\text{sat}} = \frac{h_{fg}}{T(v_g - v_f)}$$

→ It is useful to estimate properties like h from other measurable properties.

→ At a change of phase we may find  $h_{fg}$  i.e. latent heat.

At very low pressure  $v_g \approx v_{fg}$  as  $v_f$  very small

$$pv_g = RT \quad \text{or} \quad v_g = \frac{RT}{p}$$

$$\therefore \frac{dp}{dT} = \frac{h_{fg}}{T \cdot v_g} = \frac{h_{fg}}{T \cdot \frac{RT}{p}} = \frac{h_{fg} \cdot p}{RT^2}$$

$$\text{or } \frac{dp}{p} = \frac{h_{fg}}{R} \cdot \frac{dT}{T^2}$$

$$\text{or } \ln \left( \frac{p_2}{p_1} \right) = \frac{h_{fg}}{R} \left( \frac{1}{T_1} - \frac{1}{T_2} \right)$$

→ Knowing vapour pressure  $p_1$  at temperature  $T_1$ , we may find out  $p_2$  at temperature  $T_2$ .

## Joule-Kelvin Effect or Joule-Thomson coefficient

The value of the specific heat  $c_p$  can be determined from  $p$ - $v$ - $T$  data and the Joule-Thomson coefficient. The **Joule-Thomson coefficient**  $\mu_J$  is defined as

$$\mu_J = \left( \frac{\partial T}{\partial p} \right)_h$$

Like other partial differential coefficients introduced in this section, the Joule-Thomson coefficient is defined in terms of thermodynamic properties only and thus is itself a property. The units of  $\mu_J$  are those of temperature divided by pressure.

A relationship between the specific heat  $c_p$  and the Joule-Thomson coefficient  $\mu_J$  can be established to write

$$\left( \frac{\partial T}{\partial p} \right)_h \left( \frac{\partial p}{\partial h} \right)_T \left( \frac{\partial h}{\partial T} \right)_p = -1$$

The first factor in this expression is the Joule-Thomson coefficient and the third is  $c_p$ . Thus

$$c_p = \frac{-1}{\mu_J \left( \partial p / \partial h \right)_T}$$

With  $(\partial h / \partial p)_T = 1 / (\partial p / \partial h)_T$  this can be written as

$$c_p = -\frac{1}{\mu_J} \left( \frac{\partial h}{\partial p} \right)_T$$

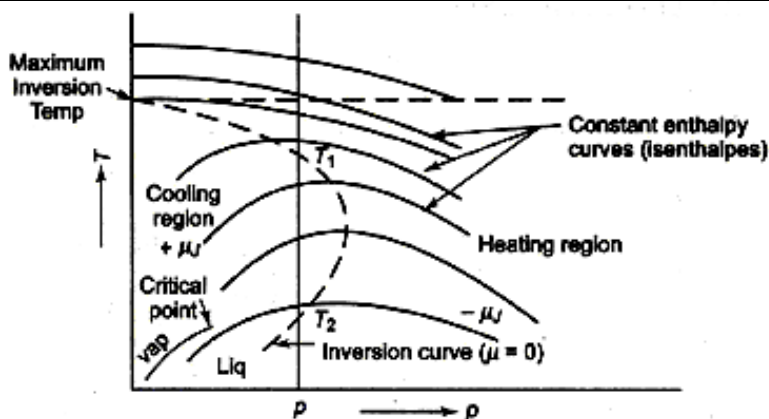
The partial derivative  $(\partial h / \partial p)_T$ , called the **constant-temperature coefficient**, can be eliminated. The following expression results:

$$c_p = \frac{1}{\mu_J} \left[ T \left( \frac{\partial v}{\partial T} \right)_p - v \right]$$

allows the value of  $c_p$  at a state to be determined using  $p$ - $v$ - $T$  data and the value of the Joule-Thomson coefficient at that state. Let us consider next how the Joule-Thomson coefficient can be found experimentally.

The numerical value of the slope of an isenthalpic on a  $T$ - $p$  diagram at any point is called the **Joule-Kelvin coefficient** and is denoted by  $\mu_J$ . Thus the locus of all points at which  $\mu_J$  is zero is the inversion curve. The region inside the inversion curve where  $\mu_J$  is positive is called the **cooling region** and the region outside where  $\mu_J$  is negative is called the **heating region**. So,

$$\mu_J = \left( \frac{\partial T}{\partial p} \right)_h$$



*Isenthalpic Curves and the Inversion Curve*

### Energy Equation

For a system undergoing an infinitesimal reversible process between two equilibrium states, the change of internal energy is

$$dU = TdS - pdV$$

Substituting the first TdS equation

$$dU = C_v dT + T \left( \frac{\partial p}{\partial T} \right)_V dV - p dV$$

$$= C_v dT + \left[ T \left( \frac{\partial p}{\partial T} \right)_V - p \right] dV$$

if  $U = (T, V)$

$$dU = \left( \frac{\partial U}{\partial T} \right)_V dT + \left( \frac{\partial U}{\partial V} \right)_T dV$$

$$\left( \frac{\partial U}{\partial V} \right)_T = T \left( \frac{\partial p}{\partial T} \right)_V - p$$

This is known as energy equation. Two application of the equation are given below-

(a) For an ideal gas,  $p = \frac{n\bar{R}T}{V}$

$$\therefore \left( \frac{\partial p}{\partial T} \right)_V = \frac{n\bar{R}}{V} = \frac{p}{T}$$

$$\therefore \left( \frac{\partial U}{\partial V} \right)_T = T \cdot \frac{p}{T} - p = 0$$

U does not change when V changes at  $T = C$ .

$$\left( \frac{\partial U}{\partial p} \right)_T \left( \frac{\partial p}{\partial V} \right)_T \left( \frac{\partial V}{\partial U} \right)_T = 1$$

$$\left( \frac{\partial U}{\partial p} \right)_T \left( \frac{\partial p}{\partial V} \right)_T = \left( \frac{\partial U}{\partial V} \right)_T = 0$$

$$\text{since } \left( \frac{\partial p}{\partial V} \right)_T \neq 0, \left( \frac{\partial U}{\partial p} \right)_T = 0$$

U does not change either when p changes at  $T = C$ . So the internal energy of an ideal gas is a function of temperature only.

Another important point to note is that for an ideal gas

$$pV = n\bar{R}T \text{ and } T \left( \frac{\partial p}{\partial T} \right)_V - p = 0$$

Therefore

$$dU = C_v dT$$

holds good for an ideal gas in any process (even when the volume changes). But for any other substance

$$dU = C_v dT$$

is true only when the volume is constant and  $dV = 0$

Similarly

$$dH = TdS + Vdp$$

$$\text{and} \quad TdS = C_p dT - T \left( \frac{\partial V}{\partial T} \right)_p dp$$

$$\therefore dH = C_p dT + \left[ V - T \left( \frac{\partial V}{\partial T} \right)_p \right] dp$$

$$\therefore \left( \frac{\partial H}{\partial p} \right)_T = V - T \left( \frac{\partial V}{\partial T} \right)_p$$

As shown for internal energy, it can be similarly proved from Eq. shown in above that the enthalpy of an ideal gas is not a function of either volume or pressure.

$$\left[ i.e. \left( \frac{\partial H}{\partial p} \right)_T = 0 \text{ and } \left( \frac{\partial H}{\partial V} \right)_T = 0 \right]$$

but a function of temperature alone.

Since for an ideal gas,  $pV = n\bar{R}T$

$$\text{and} \quad V - T \left( \frac{\partial V}{\partial T} \right)_p = 0$$

the relation  $dH = C_p dT$  is true for any process (even when the pressure changes.)

However, for any other substance the relation  $dH = C_p dT$  holds good only when the pressure remains constant or  $dp = 0$ .

(b) Thermal radiation in equilibrium with the enclosing walls processes an energy that depends only on the volume and temperature. The energy density ( $u$ ), defined as the ratio of energy to volume, is a function of temperature only, or

$$u = \frac{U}{V} = f(T) \text{ only.}$$

The electromagnetic theory of radiation states that radiation is equivalent to a photon gas and it exerts a pressure, and that the pressure exerted by the black body radiation in an enclosure is given by

$$p = \frac{u}{3}$$

Black body radiation is thus specified by the pressure, volume and temperature of the radiation.

since.

$$U = uV \text{ and } p = \frac{u}{3}$$

$$\left( \frac{\partial U}{\partial V} \right)_T = u \text{ and } \left( \frac{\partial p}{\partial T} \right)_V = \frac{1}{3} \frac{du}{dT}$$

By substituting in the energy Eq.

$$u = \frac{T}{3} \frac{du}{dT} - \frac{u}{3}$$

$$\therefore \frac{du}{u} = 4 \frac{dT}{T}$$

or  $\ln u = \ln T^4 + \ln b$

or  $u = bT^4$

where  $b$  is a constant. This is known as the **Stefan - Boltzmann Law**.

Since  $U = uV = VbT^4$

$$\left(\frac{\partial U}{\partial T}\right)_V = C_v = 4VbT^3$$

and  $\left(\frac{\partial p}{\partial T}\right)_V = \frac{1}{3} \frac{du}{dT} = \frac{4}{3} bT^3$

From the first TdS equation

$$\begin{aligned} TdS &= C_v dT + T \left(\frac{\partial p}{\partial T}\right)_V dV \\ &= 4VbT^3 dT + \frac{4}{3} bT^4 dV \end{aligned}$$

For a reversible isothermal change of volume, the heat to be supplied reversibly to keep temperature constant.

$$Q = \frac{4}{3} bT^4 \Delta V$$

For a reversible adiabatic change of volume

$$\frac{4}{3} bT^4 dV = -4VbT^3 dT$$

$$\text{or } \frac{dV}{V} = -3 \frac{dT}{T}$$

$$\text{or } VT^3 = \text{const}$$

If the temperature is one-half the original temperature. The volume of black body radiation is to be increased adiabatically eight times its original volume so that the radiation remains in equilibrium with matter at that temperature.

## Gibbs Phase Rule

Gibbs Phase Rule determines what is expected to define the state of a system

$$\mathbf{F = C - P + 2}$$

$F$  = Number of degrees of freedom (i.e., no. of properties required)

$C$  = Number of components

$P$  = Number of phases

e.g., Nitrogen gas  $C = 1$ ;  $P = 1$ . Therefore,  $F = 2$

- To determine the state of the nitrogen gas in a cylinder two properties are adequate.
- A closed vessel containing water and steam in equilibrium:  $P = 2$ ,  $C = 1$
- Therefore,  $F = 1$ . If any one property is specified it is sufficient.
- A vessel containing water, ice and steam in equilibrium
- $P = 3$ ,  $C = 1$  therefore  $F = 0$ . The triple point is uniquely defined.

**Question:** Which one of the following can be considered as property of a system?

(a)  $\int p dv$       (b)  $\int v dp$       (c)  $\int \left( \frac{dT}{T} + \frac{p \cdot dv}{v} \right)$       (d)  $\int \left( \frac{dT}{T} - \frac{v \cdot dp}{T} \right)$

**Given:**  $p$  = pressure,  $T$  = Temperature,  $v$  = specific volume

[IES-1993]

**Solution:** P is a function of v and both are connected by a line path on p and v coordinates.

Thus  $\int p dv$  and  $\int v dp$  are not exact differentials and thus not properties.

If X and Y are two properties of a system, then dx and dy are exact differentials. If the differential is of the form  $Mdx + Ndy$ , then the test for exactness is

$$\left[ \frac{\partial M}{\partial y} \right]_x = \left[ \frac{\partial N}{\partial x} \right]_y$$

Now applying above test for

$$\int \left( \frac{dT}{T} + \frac{p \cdot dv}{v} \right), \left[ \frac{\partial(1/T)}{\partial v} \right]_T = \left[ \frac{\partial(p/v)}{\partial T} \right]_v = \left[ \frac{\partial(RT/v^2)}{\partial T} \right]_v \text{ or } 0 = \frac{R}{v^2}$$

This differential is not exact and hence is not a point function and hence

$\int \left( \frac{dT}{T} + \frac{p \cdot dv}{v} \right)$  is not a point function and hence not a property.

$$\text{And for } \int \left( \frac{dT}{T} - \frac{v \cdot dp}{T} \right) \left[ \frac{\partial(1/T)}{\partial p} \right]_T = \left[ \frac{\partial(-v/T)}{\partial T} \right]_p = \left[ \frac{\partial(-R/P)}{\partial T} \right]_p \text{ or } 0 = 0$$

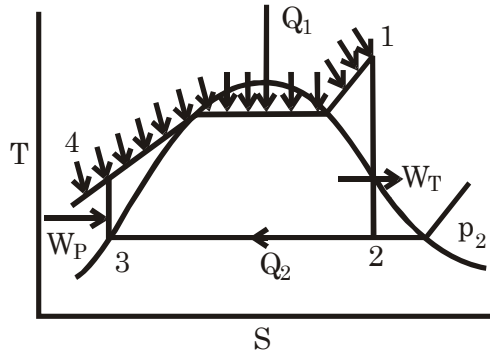
Thus  $\int \left( \frac{dT}{T} - \frac{v \cdot dp}{T} \right)$  is exact and may be written as ds, where s is a point function and hence a property



# 12. Vapour Power Cycles

## Some Important Notes

### A. Rankine Cycle



For 1 kg of fluid using S.F.E.E.

$$(i) \quad h_4 + Q_1 = h_1$$

$$\text{or} \quad \boxed{Q_1 = h_1 - h_4}$$

$$(ii) \quad h_1 = W_T + h_2$$

$$\text{or} \quad \boxed{W_T = h_1 - h_2}$$

$$(iii) \quad h_3 + W_P = h_4$$

$$\text{or} \quad \boxed{W_P = h_4 - h_3}$$

**About pump:** The pump handles liquid water which is incompressible.

For reversible Adiabatic Compression  $Tds = dh - vdp$  where  $ds = 0$

$$\therefore \quad dh = vdp \quad \text{as } v = \text{constant}$$

$$\Delta h = v\Delta p$$

$$\text{or} \quad h_4 - h_3 = v(p_1 - p_2) = W_P$$

$$(iv) \quad \boxed{W_P = h_4 - h_3 = v(p_1 - p_2) \text{ kJ/kg}} \quad \text{Where } v \text{ in m}^3/\text{kg} \text{ and } p \text{ in kPa}$$

### B. Rankine Cycle efficiency:

$$\eta = \frac{W_{\text{net}}}{Q_1} = \frac{W_T - W_P}{Q_1} = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)}$$

$$C. \quad \text{Steam rate} = \frac{3600}{W_T - W_P} \frac{\text{kg}}{\text{kWh}}$$

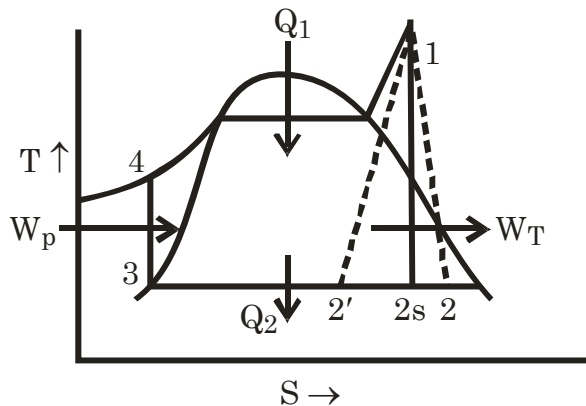
$$D. \quad \text{Heat Rate} = \text{Steam rate} \times Q_1 = \frac{3600 Q_1}{W_T - W_P} \frac{\text{kJ}}{\text{kWh}} = \frac{3600}{\eta} \frac{\text{kJ}}{\text{kWh}}$$

- E. About Turbine Losses:** If there is heat loss to the surroundings,  $h_2$  will decrease, accompanied by a decrease in entropy. If the heat loss is large, the end state of steam from the turbine may be  $2'$ . (figure in below).

It may so happen that the entropy increase due to frictional effects just balances the entropy decrease due to heat loss, with the result that the initial and final entropies of steam in the expansion process are equal, but the expansion is neither adiabatic nor reversible.

- F. Isentropic Efficiency:**

$$\eta_{\text{isen}} = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{\text{Actual Enthalpy drop}}{\text{isentropic enthalpy drop}}$$

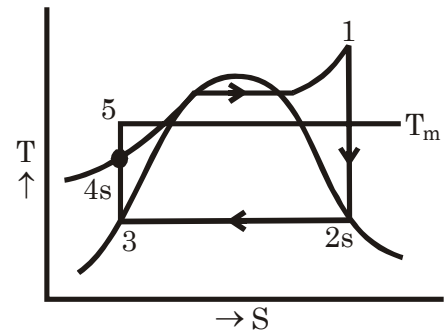


- G. Mean temperature of heat addition:**

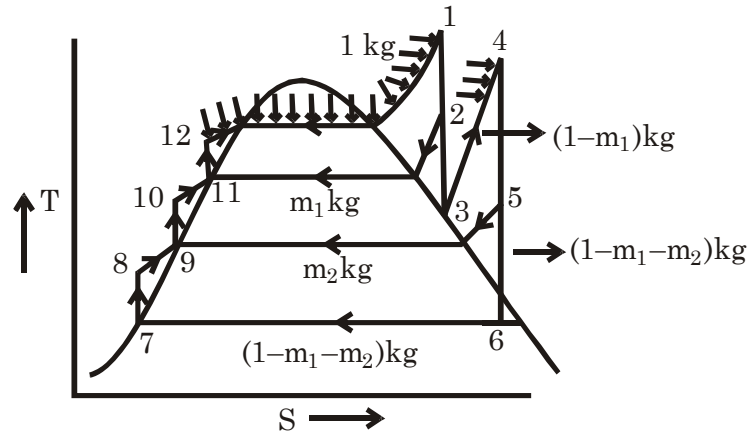
$$Q_1 = h_1 - h_{4s} = T_m (s_1 - s_{4s})$$

$\therefore$

$$T_m = \frac{h_1 - h_{4s}}{s_1 - s_{4s}}$$



### H. For Reheat – Regenerative Cycle:



$$W_T = (h_1 - h_2) + (1 - m_1) (h_2 - h_3) + (1 - m_1) (h_4 - h_5) + (1 - m_1 - m_2) (h_5 - h_6) \text{ kJ/kg}$$

$$W_P = (1 - m_1 - m_2) (h_8 - h_7) + (1 - m_1) (h_{10} - h_9) + 1(h_{12} - h_{11}) \text{ kJ/kg}$$

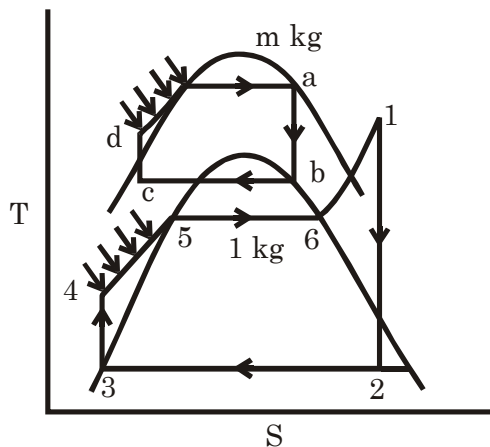
$$Q_1 = (h_1 - h_{12}) + (1 - m_1) (h_4 - h_3) \text{ kJ/kg}$$

Energy balance of heater 1 and 2

$$m_1 h_2 + (1 - m_1) h_{10} = 1 \times h_{11} \dots\dots\dots \text{For calculation of } m_1$$

$$\text{And } m_2 h_5 + (1 - m_1 - m_2) h_8 = (1 - m_1) h_9 \dots\dots\dots \text{For calculation of } m_2.$$

### I. For Binary vapour Cycles:



$$W_T = m (h_a - h_b) + (h_1 - h_2) \text{ kJ/kg of steam}$$

$$W_P = m (h_d - h_c) + (h_4 - h_3) \text{ kJ /kg of steam}$$

$$Q_1 = m (h_a - h_d) + (h_1 - h_6) + (h_5 - h_4) \text{ kJ /kg of steam.}$$

Energy balance in mercury condenser-steam boiler

$$m (h_b - h_c) = (h_6 - h_5)$$

$$\therefore m = \frac{h_6 - h_5}{h_b - h_c} \text{ kg of Hg/kg of H}_2\text{O i.e. } \approx 8 \text{ kg}$$

**J. Efficiency of Binary vapour cycle:**

$$1 - \eta = (1 - \eta_1)(1 - \eta_2) \dots (1 - \eta_n)$$

∴ For two cycles

$$\eta = \eta_1 + \eta_2 - \eta_1 \eta_2$$

**K. Overall efficiency of a power plant**

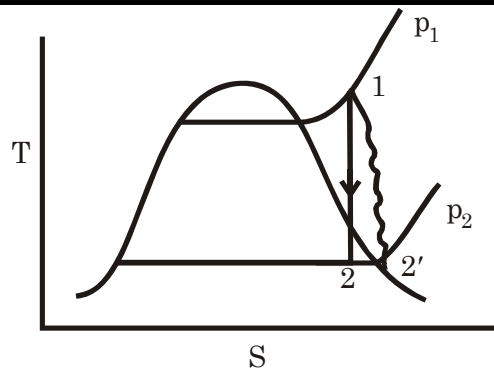
$$\eta_{\text{overall}} = \eta_{\text{boiler}} \times \eta_{\text{cycle}} \times \eta_{\text{turbine (mean)}} \times \eta_{\text{generator}}$$

## Questions with Solution P. K. Nag

- Q. 12.1** for the following steam cycles find
- (a)  $W_T$  in kJ/kg
  - (b)  $W_p$  in kJ/kg,
  - (c)  $Q_1$  in kJ/kg,
  - (d) cycle efficiency,
  - (e) steam rate in kg/kW h, and
  - (f) moisture at the end of the turbine process. Show the results in tabular form with your comments.

Boiler Outlet	Condenser Pressure	Type of Cycle
10 bar, saturated	1 bar	Ideal Rankine Cycle
-do-	-do-	Neglect $W_p$
-do-	-do-	Assume 75% pump and Turbine efficiency
-do-	0.1 bar	Ideal Rankine Cycle
10 bar, 300°C	-do-	-do-
150 bar, 600°C	-do-	-do-
-do-	-do-	Reheat to 600°C maximum intermediate pressure to limit end moisture to 15%
-do-	-do-	-do- but with 85% turbine efficiencies
10 bar, saturated	0.1 bar	Isentropic pump process ends on saturated
Boiler Outlet	Condenser Pressure	Type of Cycle
10 bar, saturated	0.1 bar	-do- but with 80% machine efficiencies
-do-	-do-	Ideal regenerative cycle
-do-	-do-	Single open heater at 110°C
-do-	-do-	Two open heaters at 90°C and 135°C
-do-	-do-	-do- but the heaters are closed heaters





At 4 bar		at 5 bar	
150°C	200°C	152°C	200°C
$h = 2752.8$	$h = 2860.5$	$h = 2748.7$	$h = 2855.4$
$s = 6.9299$	$s = 7.1706$	$s = 6.8213$	$s = 7.0592$

∴ at 4 bar 175°C at 5 bar, 175°C

$$h = 2752.8 + \frac{1}{2} (2860.5 - 2752.8)$$

$$h = 2748.7 + \left( \frac{175 - 152}{200 - 152} \right) (2855.4 - 2748.7)$$

$$= 2806.7 \text{ kJ/kg} \quad = 2800 \text{ kJ/kg}$$

$$s = 6.9299 + \frac{1}{2} (7.1706 - 6.9299) \quad s = 6.8213 + \frac{23}{48} (7.0592 - 6.8213)$$

$$= 7.0503 \text{ kJ/kg - K} \quad = 6.9353$$

∴ at 4.5 bar 175°C

$$h_1 = \frac{2806.7 + 2800}{2} = 2803.4 \text{ kJ/kg}$$

$$s_1 = \frac{7.0503 + 6.9353}{2} = 6.9928 \text{ kJ/kg - K}$$

Pressure 100 mm Hg

$$= \frac{100}{1000} \text{ m} \times (13.6 \times 10^3) \text{ kg/m}^3 \times 9.81 \text{ m/s}^2$$

$$= 0.13342 \text{ bar} = 13.342 \text{ kPa}$$

Here also entropy 6.9928 kJ/kg - K

So from S. T.

	At 10 kPa	at 15 kPa
$h_f = 191.83$	$s_f = 0.6493$	$s_f = 0.7549$
$h_{fg} = 2392.8$	$s_g = 8.1502$	$s_g = 8.0085$
		$h_{fg} = 2373.1$

∴ at 13.342 kPa [Interpolation]

$$s_f = 0.6493 + \left( \frac{15 - 13.342}{15 - 10} \right) (0.7549 - 0.6493) = 0.68432 \text{ kJ/kg - K}$$

$$s_g = 8.1502 + \left( \frac{15 - 13.342}{15 - 10} \right) (8.0085 - 8.1502) = 8.1032 \text{ kJ/kg - K}$$

∴ If dryness fraction is x then

$$6.9928 = 0.68432 + x (8.1032 - 0.68432)$$

∴  $x = 0.85033$

∴ At 13.342 kPa

$$h_f = 191.83 + \left( \frac{15 - 13.342}{15 - 10} \right) (225.94 - 191.83) = 203.14 \text{ kJ/kg}$$

$$h_{fg} = 2392.8 + \left( \frac{15 - 13.342}{15 - 10} \right) (2373.1 - 2392.8) = 2386.3 \text{ kJ/kg}$$

$$h_{2s} = h_f + x h_{fg} = 203.14 + 0.85033 \times 2386.3 = 2232.3 \text{ kJ/kg}$$

$$\eta_{\text{isentropic}} = \frac{h_1 - h_{2'}}{h_1 - h_{2s}}$$

$$\therefore h_1 - h_{2'} = \eta_{\text{isentropic}} \times (h_1 - h_{2s})$$

$$\therefore h_{2'} = h_1 - \eta_{\text{isentropic}} (h_1 - h_{2s})$$

$$= 2803.4 - 0.75 (2803.4 - 2232.3) = 2375 \text{ kJ/kg.}$$

$$\therefore \text{Turbine work } (W_T) = h_1 - h_{2'} = (2803.4 - 2375) \%$$

$$= 428.36 \text{ kJ/kg}$$

$$\therefore \text{Efficiency of the plant} = \frac{W_T}{h_1} = \frac{428.36}{2803.4} \approx 0.1528 = 25.28\%$$

If mass flow rate is  $\dot{m}$  kg/s

$$\dot{m} \cdot W_T = 12.5 \times 10^3$$

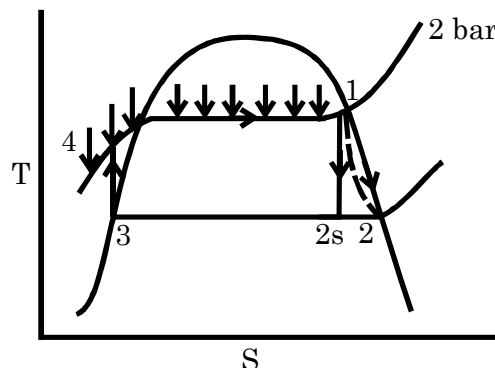
$$\text{or } \dot{m} = \frac{12.5 \times 10^3}{428.36} = 29.18 \text{ kg/s}$$

### Q.12.3

A simple steam power cycle uses solar energy for the heat input. Water in the cycle enters the pump as a saturated liquid at 40°C, and is pumped to 2 bar. It then evaporates in the boiler at this pressure, and enters the turbine as saturated vapour. At the turbine exhaust the conditions are 40°C and 10% moisture. The flow rate is 150 kg/h. Determine (a) the turbine isentropic efficiency, (b) the net work output (c) the cycle efficiency, and (d) the area of solar collector needed if the collectors pick up 0.58 kW/m<sup>2</sup>.

(Ans. (c) 2.78%, (d) 18.2 m<sup>2</sup>)

**Solution:** From Steam Table  $T_1 = 120.23^\circ\text{C} = 393.23 \text{ K}$   
 $h_1 = 2706.7 \text{ kJ/kg}$   
 $s_1 = 7.1271 \text{ kJ/kg} - \text{K}$



At 40°C saturated pressure 7.384 kPa

$$h_f = 167.57$$

$$h_{fg} = 2406.7$$

$$s_f = 0.5725$$

$$s_g = 8.2570$$

$$\therefore h_2 = h_f + 0.9 \times 2406.7 = 2333.6 \text{ kJ/kg}$$

For  $h_{2s}$  if there is dryness fraction  $x$

$$7.1271 = 0.5725 + x \times (8.2570 - 0.5725)$$

$$\therefore x = 0.853$$

$$\therefore h_{2s} = 167.57 + 0.853 \times 2406.7 = 2220.4 \text{ kJ/kg}$$

$$(a) \quad \therefore \text{Isentropic efficiency, } \eta_{\text{isentropic}} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

$$= \frac{2706.7 - 2333.6}{2706.7 - 2220.4} = 76.72\%$$

$$(b) \quad \text{Net work output } W_T = h_1 - h_2 = 373.1 \text{ kJ/kg}$$

$$\therefore \text{Power} = 15.55 \text{ kW i.e. } (W_T - W_P) \times \dot{m}$$

$$\text{Pump work, } W_P = v(p_1 - p_2)$$

$$= 1.008 \times 10^{-3} (200 - 7.384) \text{ kJ/kg} = 0.1942 \text{ kJ/kg}$$

$$\therefore h_3 = 167.57 \text{ kJ/kg, } h_a = 167.76 \text{ kJ/kg}$$

$$\therefore Q_1 = (h_1 - h_4) = (2706.7 - 167.76) \text{ kJ/kg} = 2539 \text{ kJ/kg}$$

$$\therefore \eta_{\text{cycle}} = \frac{W_T - W_P}{Q_1} = \frac{373.1 - 0.1942}{2539} = 14.69 \%$$

$$\begin{aligned} \text{Required area } A &= \frac{Q_1 \times \dot{m}}{\text{collection picup}} \\ &= \frac{2539 \times 150}{0.58 \times 3600} = 182.4 \text{ m}^2 \end{aligned}$$

**Q.12.4** In a reheat cycle, the initial steam pressure and the maximum temperature are 150 bar and 550°C respectively. If the condenser pressure is 0.1 bar and the moisture at the condenser inlet is 5%, and assuming ideal processes, determine (a) the reheat pressure, (b) the cycle efficiency, and (c) the steam rate.

(Ans. 13.5 bar, 43.6%, 2.05 kg/kW h)

**Solution :** From Steam Table at 150 bar 550°C

$$h_1 = 3448.6 \text{ kJ/kg}$$

$$s_1 = 6.520 \text{ kJ/kg - K}$$

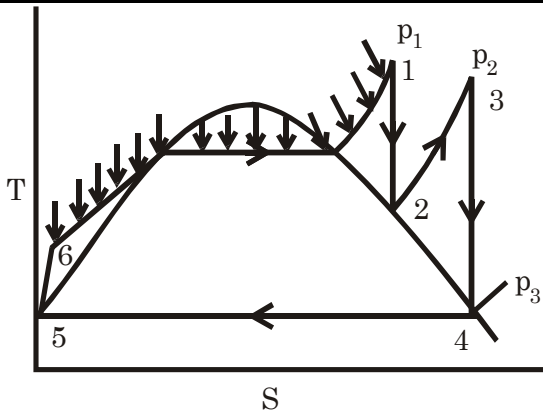
$$\text{At } p_3 = 0.1 \text{ bar}$$

$$T = 45.8^\circ\text{C}$$

$$h_f = 191.8 \text{ kJ/kg}$$

$$h_{fg} = 2392.8 \text{ kJ/kg}$$





$$\begin{aligned}\therefore h_4 &= h_f + x h_{fg} = 191.8 + 0.95 \times 2392.8 = 2465 \text{ kJ/kg} \\ s_f &= 0.649; s_{fg} = 7.501 \\ s_4 &= s_f + x s_{fg} = 0.649 + 0.95 \times 7.501 = 7.775 \text{ kJ/kg} - \text{K}\end{aligned}$$

From Molier Diagram at 550°C and 7.775 entropy, 13.25 bar

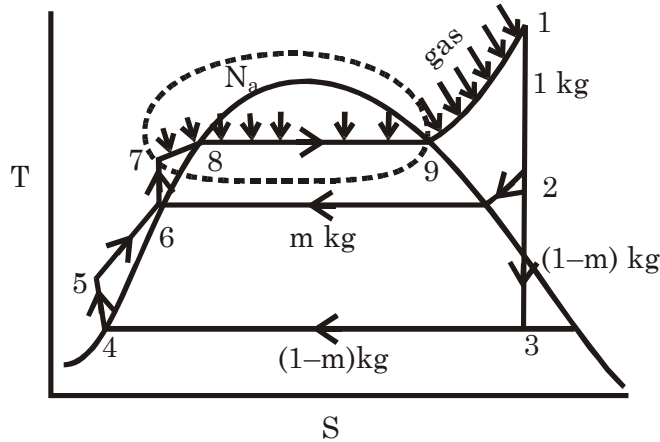
$$\begin{aligned}\text{From S.T. at 10 bar } 550^\circ\text{C} & \quad s = 7.8955 \\ \therefore \quad 15 \text{ bar } 550^\circ\text{C} & \quad s = 7.7045 \\ \therefore \quad 7.775 &= 7.8955 + \left( \frac{p - 10}{15 - 10} \right) (7.7045 - 7.8955) \\ -0.1205 &= (p - 10) (-0.0382) \\ \therefore \quad p - 10 &= 3.1544 \Rightarrow p = 13.15 \text{ bar} \\ \therefore \quad \text{from Molier Dia. At 13 bar } 550^\circ\text{C} & \\ h_3 &= 3580 \text{ kJ/kg} \\ h_2 &= 2795 \text{ kJ/kg} \quad t_2 = 195^\circ\text{C} \\ h_5 &= 191.8 \text{ kJ/kg} \\ W_P &= v_5 (p_1 - p_3) = 0.001010 (15000 - 10) \text{ kJ/kg} \\ &= 1.505 \text{ kJ/kg} \\ \therefore \quad h_6 &= h_5 + W_P = 193.3 \text{ kJ/kg} \\ \therefore \quad W_T &= (h_1 - h_2) + (h_3 - h_4) = 1768.6 \text{ kJ/kg} \\ W_P &= 1.50 \text{ kJ/kg} \\ \therefore \quad W_{\text{net}} &= 1767.5 \text{ kJ/kg} \\ Q &= (h_1 - h_6) + (h_3 - h_2) = 4040.3 \text{ kJ/kg} \\ \therefore \quad \eta_{\text{cycle}} &= \frac{W_{\text{net}}}{Q} = \frac{1767.5}{4040.3} \times 100 \% = 43.75\%\end{aligned}$$

$$\text{Steam rate} = \frac{3600}{W_T - W_P} = \frac{3600}{1767.5} \text{ kg/kWh} = 2.0368 \text{ kg/kWh}$$

### Q.12.5

In a nuclear power-plant heat is transferred in the reactor to liquid sodium. The liquid sodium is then pumped to a heat exchanger where heat is transferred to steam. The steam leaves this heat exchanger as saturated vapour at 55 bar, and is then superheated in an external gas-fired super heater to 650°C. The steam then enters the turbine, which has one extraction point at 4 bar, where steam flows to an open feed water heater. The turbine efficiency is 75% and the condenser temperature is 40°C. Determine the heat transfer in the reactor and in the super heater to produce a power output of 80 MW.

**Solution:** From Steam Table at 55 bar saturated state  
 $h_9 = 2789.9 \text{ kJ/kg}$



From super heated S.T. at 55 bar  $650^\circ\text{C}$   
 at 50 bar  $600^\circ\text{C}$ ,  $700^\circ\text{C}$   $\therefore$  By calculation at  $650^\circ\text{C}$   
 $h = 3666.5$   $h = 3900.1$   $h = 3783.3$   
 $s = 7.2589$   $s = 7.5122$   $s = 7.3856$

At 60 bar  $600^\circ$   $C = 700^\circ\text{C}$   $\therefore$  by calculation  
 $h = 3658.4$   $h = 3894.2$   $h = 3776.3$   
 $s = 7.1677$   $s = 7.4234$   $s = 7.2956$

$\therefore$  at 55 bar  $650^\circ\text{C}$  (by interpolation)  
 $h_1 = 3770.8 \text{ kJ/kg}$   
 $s_1 = 7.3406 \text{ kJ/kg}$

For  $h_2$ , at 4 bar where  $S = 7.3406$   
 At  $200^\circ\text{C}$  at  $250^\circ\text{C}$  if temp is  $t$   
 $s = 7.172$ ,  $s = 7.379$   
 $h = 2860.5$   $h = 2964.2$

$$\text{Then } h_2 = 2860.5 + \left( \frac{7.3406 - 7.171}{7.379 - 7.171} \right) \times (2964.2 - 2860.5) = 2945 \text{ kJ/kg}$$

For  $h_3$ , at point 3 at  $40^\circ\text{C}$   
 $h_f = 167.6$ ,  $h_{fg} = 2406.7$   $s_f = 0.573$   $s_{fg} = 7.685$

If dryness fraction is  $x$  then

$$\begin{aligned} 0.573 + x \times 7.685 &= 7.3406 \\ \therefore x &= 0.8806 \\ \therefore h_3 &= 167.6 + 0.8806 \times 2406.7 = 2287 \text{ kJ/kg} \\ h_4 &= h_f = 167.6 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} W_{p_{4-5}} &= v_4 (p_2 - p_3) = 0.001010 \times (400 - 7.38) \text{ kJ/kg} \\ &= 0.397 \text{ kJ/kg} \approx 0.4 \text{ kJ/kg} \end{aligned}$$

$$\therefore h_5 = h_4 + W_{p_{4-5}} = 168 \text{ kJ/kg}$$

$$h_6 = 604.7 \text{ kJ/kg}$$

$$W_{p_{6-7}} = v_6 (p_1 - p_2) = 0.001084 (5500 - 400) = 5.53 \text{ kJ/kg}$$

$$\therefore h_7 = h_6 + W_{p_{6-7}} = 610.23 \text{ kJ/kg}$$

From heater energy balance

$$(1 - m) h_5 + m h_2 = h_5 \quad \Rightarrow m = 0.1622 \text{ kg}$$

$$\therefore W_T = [(h_1 - h_2) + (1 - m) (h_2 - h_3)] \times 0.75 = 1049.8 \text{ kJ/kg ;}$$

$$W_{\text{net}} = W_T - W_{p_{4-5}} - W_{p_{6-7}} = 1043.9 \text{ kJ/kg}$$

$$\therefore \text{Steam flow rate } (\dot{m}) = \frac{80 \times 10^3}{1049.8} \text{ kg/s} = 76.638 \text{ kg/s}$$

$$\therefore \text{Heat transfer in heater} = \dot{m}(h_9 - h_7)$$

$$= 76.638(2789.9 - 610.23) = 167.046 \text{ MW}$$

$$\text{Heat transfer in super heater} = \dot{m}(h_1 - h_9)$$

$$= 76.638(3779.8 - 2789.9) = 75.864 \text{ MW}$$

**Q.12.6** In a reheat cycle, steam at 500°C expands in a h.p. turbine till it is saturated vapour. It is reheated at constant pressure to 400°C and then expands in a l.p. turbine to 40°C. If the maximum moisture content at the turbine exhaust is limited to 15%, find (a) the reheat pressure, (b) the pressure of steam at the inlet to the h.p. turbine, (c) the net specific work output, (d) the cycle efficiency, and (e) the steam rate. Assume all ideal processes.

What would have been the quality, the work output, and the cycle efficiency without the reheating of steam? Assume that the other conditions remain the same.

**Solution:** Try please.

**Q.12.7** A regenerative cycle operates with steam supplied at 30 bar and 300°C and -condenser pressure of 0.08 bar. The extraction points for two heaters (one Closed and one open) are at 3.5 bar and 0.7 bar respectively. Calculate the thermal efficiency of the plant, neglecting pump work.

**Solution:** Try please.

**Q.12.8** The net power output of the turbine in an ideal reheat-regenerative cycle is 100 MW. Steam enters the high-pressure (H.P.) turbine at 90 bar, 550°C. After expansion to 7 bar, some of the steam goes to an open heater and the balance is reheated to 400°C, after which it expands to 0.07 bar. (a) What is the steam flow rate to the H.P. turbine? (b) What is the total pump work? (c) Calculate the cycle efficiency. (d) If there is a 10°C rise in the temperature of the cooling water, what is the rate of flow of the cooling water in the condenser? (e) If the velocity of the steam flowing from the turbine to the condenser is limited to a maximum of 130 m/s, find the diameter of the connecting pipe.

**Solution:** Try please.

**Q.12.9** A mercury cycle is superposed on the steam cycle operating between the boiler outlet condition of 40 bar, 400°C and the condenser temperature

of 40°C. The heat released by mercury condensing at 0.2 bar is used to impart the latent heat of vaporization to the water in the steam cycle. Mercury enters the mercury turbine as saturated vapour at 10 bar. Compute (a) kg of mercury circulated per kg of water, and (b) the efficiency of the combined cycle.

The property values of saturated mercury are given below

p (bar)	T(°C)	$h_f$ (kJ/kg) $h_g$	$s_f$ (kJ/kg K) $s_g$	$v_f$ (m <sup>3</sup> /kg) $v_g$
10	515.5	72.23 363.0	0.1478 0.5167	80.9 x10 <sup>-6</sup> 0.0333
0.2	277.3	38.35 336.55	0.0967 0.6385	77.4 x10 <sup>-6</sup> 1.163

**Solution:** Try please.

**Q.12.10** In an electric generating station, using a binary vapour cycle with mercury in the upper cycle and steam in the lower, the ratio of mercury flow to steam flow is 10 : 1 on a mass basis. At an evaporation rate of 1,000,000 kg/h for the mercury, its specific enthalpy rises by 356 kJ/kg in passing through the boiler. Superheating the steam in the boiler furnace adds 586 kJ to the steam specific enthalpy. The mercury gives up 251.2 kJ/kg during condensation, and the steam gives up 2003 kJ/kg in its condenser. The overall boiler efficiency is 85%. The combined turbine metrical and generator efficiencies are each 95% for the mercury and steam units. The steam auxiliaries require 5% of the energy generated by the units. Find the overall efficiency of the plant.

**Solution:** Try please

**Q.12.11** A sodium-mercury-steam cycle operates between 1000°C and 40°C. Sodium rejects heat at 670°C to mercury. Mercury boils at 24.6 bar and rejects heat at 0.141 bar. Both the sodium and mercury cycles are saturated. Steam is formed at 30 bar and is superheated in the sodium boiler to 350°C. It rejects heat at 0.08 bar. Assume isentropic expansions, no heat losses, and no generation and neglect pumping work. Find (a) the amounts of sodium and mercury used per kg of steam, (b) the heat added and rejected in the composite cycle per kg steam, (c) the total work done per kg steam. (d) the efficiency of the composite cycle, (e) the efficiency of the corresponding Carnot cycle, and (f) the work, heat added, and efficiency of a supercritical pressure steam (single fluid) cycle operating at 250 bar and between the same temperature limits.

For mercury, at 24.6 bar,  $h_g = 366.78$  kJ/kg

$$s_g = 0.48 \text{ kJ/kg K} \quad \text{and at } 0.141 \text{ bar, } s_g = 0.09$$

And  $s_g = 0.64 \text{ kJ/kg K}$ ,  $h_f = 36.01$  and  $h_g = 330.77$  kJ/kg

For sodium, at 1000°C,  $h_g = 4982.53$  kJ/kg

At turbine exhaust = 3914.85 kJ/kg

$$\text{At } 670^\circ\text{C, } h_f = 745.29 \text{ kJ/kg}$$

For a supercritical steam cycle, the specific enthalpy and entropy at the turbine inlet may be computed by extrapolation from the steam tables.

**Solution:** Try please.

**Q.12.12** A textile factory requires 10,000 kg/h of steam for process heating at 3 bar saturated and 1000 kW of power, for which a back pressure turbine of 70% internal efficiency is to be used. Find the steam condition required at the inlet to the turbine.

**Solution:** Try please.

**Q.12.13** A 10,000 kW steam turbine operates with steam at the inlet at 40 bar, 400°C and exhausts at 0.1 bar. Ten thousand kg/h of steam at 3 bar are to be extracted for process work. The turbine has 75% isentropic efficiency throughout. Find the boiler capacity required.

**Solution:** Try please.

**Q.12.14** A 50 MW steam plant built in 1935 operates with steam at the inlet at 60 bar, 450°C and exhausts at 0.1 bar, with 80% turbine efficiency. It is proposed to scrap the old boiler and put in a new boiler and a topping turbine of efficiency 85% operating with inlet steam at 180 bar, 500°C. The exhaust from the topping turbine at 60 bar is reheated to 450°C and admitted to the old turbine. The flow rate is just sufficient to produce the rated output from the old turbine. Find the improvement in efficiency with the new set up. What is the additional power developed?

**Solution:** Try please.

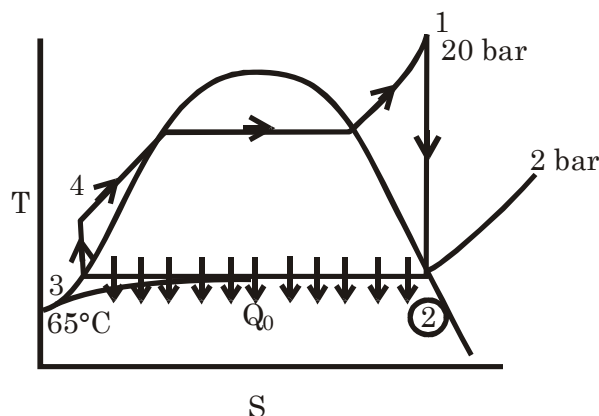
**Q.12.15** A steam plant operates with an initial pressure at 20 bar and temperature 400°C, and exhausts to a heating system at 2 bar. The condensate from the heating system is returned to the boiler plant at 65°C, and the heating system utilizes for its intended purpose 90% of the energy transferred from the steam it receives. The turbine efficiency is 70%. (a) What fraction of the energy supplied to the steam plant serves a useful purpose? (b) If two separate steam plants had been set up to produce the same useful energy, one to generate heating steam at 2 bar, and the other to generate power through a cycle working between 20 bar, 400°C and 0.07 bar, what fraction of the energy supplied would have served a useful purpose?

(Ans. 91.2%, 64.5%)

**Solution:** From S.T. at 20 bar 400°C

$$h_1 = 3247.6 \text{ kJ/kg}$$

$$s_1 = 7.127 \text{ kJ/kg} - \text{K}$$



At 2 bar

$$s_f = 1.5301, s_{fg} = 5.5967$$

$$s_g = 7.127 \text{ kJ/kg} - K \text{ so at point (2)}$$

Steam is saturated vapour

$$\text{So } h_2 = 2706.3 \text{ kJ/kg}$$

At 2 bar saturated temperature is 120.2°C but 65°C liquid

$$\text{So } h_3 = h_2 - C_P \Delta T$$

$$= 504.7 - 4.187 \times (120.2 - 65) = 273.6 \text{ kJ/kg}$$

$$W_{P_{3-4}} = v_3 (p_1 - p_2) = 0.001 \times (2000 - 200) = 1.8 \text{ kJ/kg}$$

$$\therefore h_4 = 275.4 \text{ kJ/kg}$$

$$\therefore \text{Heat input (Q)} = h_1 - h_4 = (3247.6 - 275.4) = 2972.2 \text{ kJ/kg}$$

$$\begin{aligned} \text{Turbine work} &= (h_1 - h_2) \eta = (3247.6 - 2706.3) \times 0.7 \text{ kJ/kg} \\ &= 378.9 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} \text{Heat rejection that utilized (Q}_0) &= (h_2 - h_3) \eta \\ &= (2706.3 - 273.6) \times 0.9 = 2189.4 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} \therefore \text{Net work output (W}_{\text{net}}) &= W_T - W_P = 378.9 - 1.8 \\ &= 377.1 \text{ kJ/kg} \end{aligned}$$

$\therefore$  Fraction at energy supplied utilized

$$\begin{aligned} &= \frac{W_{\text{net}} + Q_0}{Q_1} = \frac{377.1 + 2189.4}{2972.2} \times 100\% \\ &= 86.35\% \end{aligned}$$

(b) At 0.07 bar

$$s_f = 0.559, s_{fg} = 7.717$$

$$\therefore \text{Dryness fraction } x, 0.559 + x \times 7.717 = 7.127$$

$$\therefore x = 0.85137$$

$$\therefore h_2 = 163.4 + 0.85137 \times 2409.1 = 2214.4 \text{ kJ/kg}$$

$$h_3 = 163.4 \text{ kJ/kg}$$

$$W_P = 0.001007 \times (2000 - 7) = 2 \text{ kJ/kg}$$

$$\therefore h_4 = 165.4 \text{ kJ/g.}$$

$$\therefore W_T = (h_1 - h_2) \times 0.7 = 723.24 \text{ kJ/kg}$$

$$W_{\text{net}} = W_T - W_P = 721.24 \text{ kJ/kg}$$

Here heat input for power =  $(h_1 - h_4) = 3082.2 \text{ kJ/kg}$

For same 377.1 kg power we need 0.52285 kg of water

So heat input = 1611.5 kJ for power

$$\text{Heat input for heating} = \frac{2189.4}{0.9} \text{ kJ} = 2432.7 \text{ kJ}$$

$$\begin{aligned} \therefore \text{Fraction of energy used} &= \frac{377.1 + 2189.4}{1611.5 + 2432.7} \times 100\% \\ &= 63.46\% \end{aligned}$$

**Q.12.16**

**In a nuclear power plant saturated steam at 30 bar enters a h.p. turbine and expands isentropically to a pressure at which its quality is 0.841. At this pressure the steam is passed through a moisture separator which removes all the liquid. Saturated vapour leaves the separator and is**

expanded isentropically to 0.04 bar in I.p. turbine, while the saturated liquid leaving the separator is returned via a feed pump to the boiler. The condensate leaving the condenser at 0.04 bar is also returned to the boiler via a second feed pump. Calculate the cycle efficiency and turbine outlet quality taking into account the feed pump term. Recalculate the same quantities for a cycle with the same boiler and condenser pressures but without moisture separation.

(Ans. 35.5%, 0.824; 35%; 0.716)

**Solution:** Form Steam Table at 30 bar saturated

$$h_1 = 2802.3 \text{ kJ/kg}$$

$$s_1 = 6.1837$$

From Molier diagram

$$h_2 = 2300 \text{ kJ/kg}$$

$$p_r = 2.8 \text{ bar}$$

From S.T.  $h_g = 2721.5 \text{ kJ/kg}$ ,  $s_g = 7.014 \text{ kJ/kg K}$

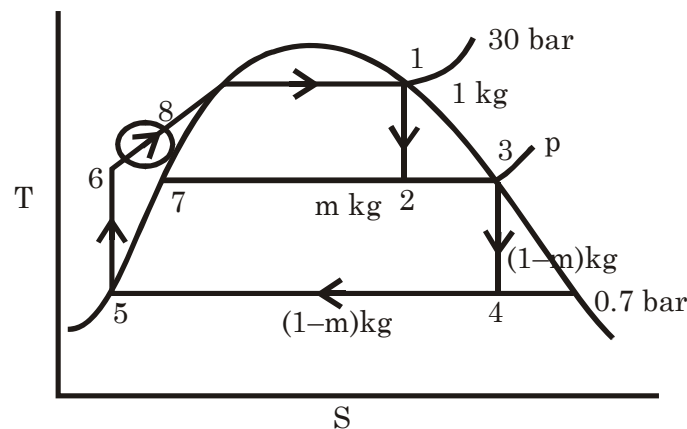
$$t = 131.2^\circ\text{C}$$

$$h_f = 551.4 \text{ kJ/kg}$$

From 0.04 bar S.T

$$s_f = 0.423 \text{ kJ/kg}, s_{fg} = 8.052 \text{ kJ/kg}$$

$$h_f = 121.5 \text{ kJ/kg}, h_{fg} = 2432.9 \text{ kJ/kg}$$



∴ If dryness fraction is  $x$  the

$$7.014 = 0.423 + x \times 8.052 \Rightarrow x = 0.8186$$

$$\therefore h_4 = h_f + x h_{fg} = 2113 \text{ kJ/kg}$$

$$W_{P_{5-6}} = 0.001(3000 - 4) = 3 \text{ kJ/kg}$$

$$W_{P_{7-8}} = 0.001071(3000 - 280) = 2.9 \text{ kJ/kg}$$

$$\text{So } h_1 = 2802.3 \text{ kJ/kg} \quad h_5 = 121.5 \text{ kJ/kg}$$

$$h_2 = 2380 \text{ kJ/kg} \quad h_6 = 124.5 \text{ kJ/kg}$$

$$h_3 = 2721.5 \text{ kJ/kg} \quad h_7 = 551.4 \text{ kJ/kg}$$

$$h_4 = 2113 \text{ kJ/kg} \quad h_8 = 554.3 \text{ kJ/kg}$$

$$\therefore m = 1 - 0.841 = 0.159 \text{ kg of sub/ kg of steam}$$

$$\therefore W_T = (h_1 - h_2) + (1 - m)(h_3 - h_4) = 934 \text{ kJ/kg}$$

$$W_P = m \times W_{P_{7-8}} + (1 - m) W_{P_{5-6}} = 2.98 \text{ kJ/kg} \approx 3 \text{ kJ/kg}$$

$$\therefore W_{\text{net}} = 931 \text{ kJ/kg}$$

$$\text{Heat supplied (Q)} = m(h_1 - h_8) + (1 - m)(h_1 - h_6) = 2609.5 \text{ kJ/kg}$$

$$\therefore \eta = \frac{931}{2609.5} \times 100\% = 35.68\% \text{ with turbine exhaust quality } 0.8186$$

If No separation is taking place, Then is quality of exhaust is x

$$\text{Then } 6.1837 = 0.423 + x \times 8.052 \Rightarrow x = 0.715$$

$$\therefore h_4 = h_f + x \times h_{fg} = 1862 \text{ kJ/kg}$$

$$\therefore W_T = h_1 - h_4 = 941.28 \text{ kJ/kg}$$

$$W_P = W_{P_{3-6}} = 3 \text{ kJ/kg}$$

$$\therefore W_{\text{net}} = 938.28 \text{ kJ/kg}$$

$\therefore$

$$\text{Heat input, } Q = h_1 - h_6 = 2677.8 \text{ kJ/kg}$$

$$\therefore \eta = \frac{938.28}{2677.8} \times 100\% = 35\%$$

**Q.12.17** The net power output of an ideal regenerative-reheat steam cycle is 80MW. Steam enters the h.p. turbine at 80 bar, 500°C and expands till it becomes saturated vapour. Some of the steam then goes to an open feedwater heater and the balance is reheated to 400°C, after which it expands in the I.p. turbine to 0.07 bar. Compute (a) the reheat pressure, (b) the steam flow rate to the h.p. turbine, and (c) the cycle efficiency. Neglect pump work.

(Ans. 6.5 bar, 58.4 kg/s, 43.7%)

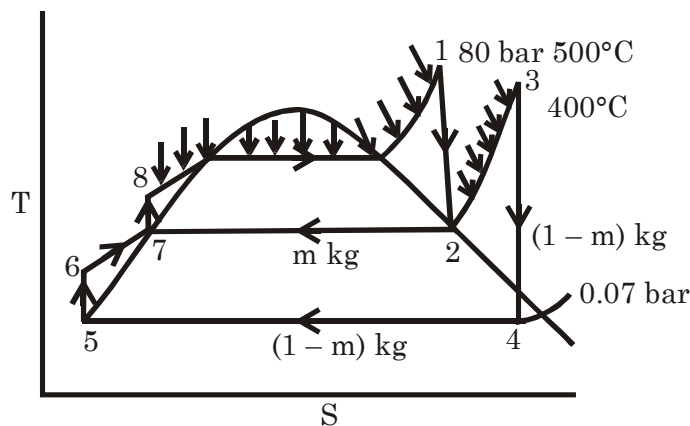
**Solution:** From S.T of 80 bar 500°C

$$h_1 = 3398.3 \text{ kJ/kg}$$

$$s_1 = 6.724 \text{ kJ/kg - K}$$

$$s_2 = 6.725 \text{ at 6.6 bar so}$$

Reheat pr. 6.6 bar



$$\therefore h_2 = 2759.5 \text{ kJ/kg}$$

$$h_3 = 3270.3 + 0.6(3268.7 - 3270.3) = 3269.3 \text{ kJ/kg}$$

$$s_3 = 7.708 + 0.6(7.635 - 7.708)$$

$$= 7.6643 \text{ kJ/kg - K}$$

At 0.07 bar

$$h_f = 163.4, \quad h_{fg} = 2409.1$$



$$h_f = 0.559, \quad s_{fg} = 7.717$$

∴ If quality is  $x$  then

$$7.6642 = 0.559 + x \times 7.717 \Rightarrow x = 0.9207$$

$$\therefore h_4 = 163.4 + 0.9207 \times 2409.1 = 2381.5 \text{ kJ/kg}$$

$$h_5 = 163.4 \text{ kJ/kg} \approx h_6, \quad h_7 = 686.8 \text{ kJ/kg} \approx h_8$$

∴ From Heat balance of heater

$$m \times h_2 + (1 - m) h_6 = h_7 \quad \therefore m = 0.2016 \text{ kg/kg of steam at H.P.}$$

$$\therefore (1 - m) = 0.7984$$

$$W_T = h_1 - h_2 + (1 - m) (h_3 - h_4) = 1347.6 \text{ kJ/kg}$$

$W_P$  neglected

$$Q = (h_1 - h_8) + (1 - m) (h_3 - h_2) = 3118.5 \text{ kJ/kg at H.P.}$$

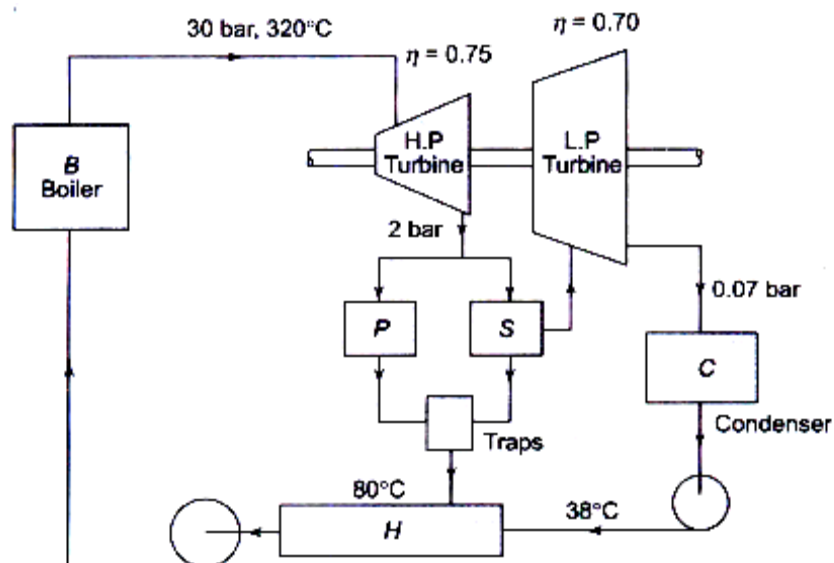
∴ (a) Reheat pr. 6.6 bar

$$(b) \text{ Steam flow rate at H.P.} = \frac{80 \times 10^3}{1347.6} \text{ kg/s} = 59.36 \text{ kg/s}$$

$$(c) \text{ Cycle efficiency } (\eta) = \frac{W}{Q} = \frac{1347.6}{3118.5} \times 100\% = 43.21\%$$

**Q.12.18**

Figure shows the arrangement of a steam plant in which steam is also required for an industrial heating process. The steam leaves boiler  $B$  at 30 bar, 320°C and expands in the H.P. turbine to 2 bar, the efficiency of the H.P. turbine being 75%. At this point one half of the steam passes to the process heater  $P$  and the remainder enters separator  $S$  which removes all the moisture. The dry steam enters the L.P. turbine at 2 bar and expands to the condenser pressure 0.07 bar, the efficiency of the L.P. turbine being 70%. The drainage from the



**Fig. 12.51**

Separator mixes with the condensate from the process heater and the combined flow enters the hotwell H at 50°C. Traps are provided at the exist from P and S. A pump extracts the condensate from condenser C and this enters the hot well at 38°C. Neglecting the feed pump work and radiation loss, estimate the temperature of water leaving the hotwell which is at atmospheric pressure. Also calculate, as percentage of heat transferred in the boiler, (a) the heat transferred in the process heater, and (b) the work done in the turbines.

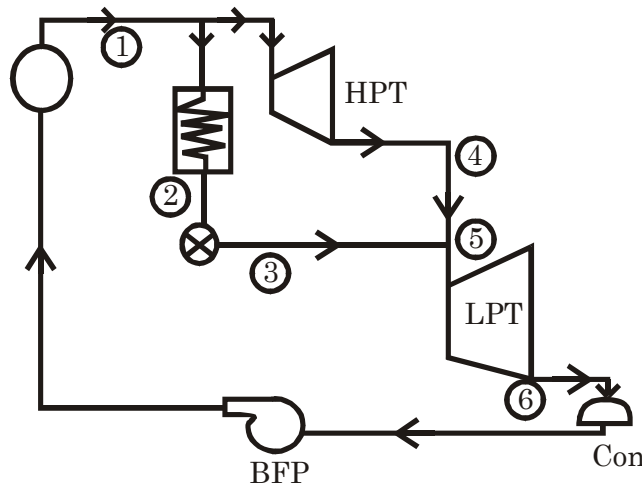
**Solution:** Try please.

**Q.12.19** In a combined power and process plant the boiler generates 21,000 kg/h of steam at a pressure of 17 bar, and temperature 230°C. A part of the steam goes to a process heater which consumes 132.56 kW, the steam leaving the process heater 0.957 dry at 17 bar being throttled to 3.5 bar. The remaining steam flows through a H.P. turbine which exhausts at a pressure of 3.5 bar. The exhaust steam mixes with the process steam before entering the L.P. turbine which develops 1337.5 kW. At the exhaust the pressure is 0.3 bar, and the steam is 0.912 dry. Draw a line diagram of the plant and determine (a) the steam quality at the exhaust from the H.P. turbine, (b) the power developed by the H.P. turbine, and (c) the isentropic efficiency of the H.P. turbine.

(Ans. (a) 0.96, (b) 1125 kW, (c) 77%)

**Solution:** Given steam flow rate

$$\dot{m} = 21000 \text{ kg/h} = \frac{35}{6} \text{ kg/s}$$



From Steam Table at 17 bar 230°C

15 bar 200°C	250°C
$h = 2796.8$	2923.3
$s = 6.455$	6.709

∴ at 230°C

$$h = 2796.8 + \frac{30}{50} (2623.3 - 2796.8) = 2872.7 \text{ kJ/kg}$$

$$s = 6.455 + \frac{30}{50} (6.709 - 6.455) = 6.6074 \text{ kJ/kg}$$

20 bar	212.4°C	250°C
--------	---------	-------

$$\begin{aligned}
 h &= 2797.2 & h &= 2902.5 \\
 s &= 6.3366 & s &= 6.545 \\
 \therefore & \text{ at } 230^\circ\text{C} \\
 h &= 2797.2 + \frac{17.6}{37.6} (2902.5 - 2797.2) = 2846.4 \text{ kJ/kg} \\
 s &= 6.3366 + \frac{17.6}{37.6} (6.545 - 6.3366) = 6.434 \text{ kJ/kg} \\
 \therefore & \text{ at 17 bar } 230^\circ\text{C} \\
 h_1 &= 2872.7 + \frac{2}{5} (2846.5 - 2872.7) = 2862.2 \text{ kJ/kg} \\
 s_1 &= 6.6074 + \frac{2}{5} (6.434 - 6.6074) = 6.5381 \text{ kJ/kg} \\
 h_2 &= 871.8 + 0.957 \times 1921.5 = 2710.7 \text{ kJ/kg} \approx h_3 \\
 h_4 &= ? \\
 \therefore & \text{ Mass flow through process heater} \\
 &= (\dot{m}_1) = \frac{132.56}{h_1 - h_2} = 0.97597 \text{ kg/s} = 3513.5 \text{ kg/h} \\
 \therefore & \text{ Mass flow through HPT} \\
 &= 17486.5 \text{ kJ/kg} = 4.8574 \text{ kg/s} \\
 \therefore & 21000 h_5 = 17486.5 h_4 + 3513.5 h_3 \quad \dots (i) \\
 h_6 &= 289.3 = 0.912 \times 2336.1 = 2419.8 \text{ kJ/kg} \\
 W_T &= \dot{m}(h_5 - h_6) \\
 \therefore & h_5 = \frac{W_T}{\dot{m}} + h_6 = \left( \frac{1337.5 \times 3600}{21000} + 2419.8 \right) = 2649.1 \text{ kJ/kg} \\
 \therefore & \text{ From (i) } h_4 = 2636.7 \text{ kJ/kg} \\
 \text{At 3-5 bar } h_g &= 2731.6 \text{ kJ/kg so it is wet is quality } x \\
 (a) \quad \therefore & 2636.7 = 584.3 + x \times 2147.4 \Rightarrow x = 0.956 \\
 (b) \quad W_{\text{HPT}} &= \dot{m}_2 (h_1 - h_4) \\
 &= \frac{17486.5}{3600} (2862.2 - 2636.7) \text{ kJ/kg} = 1095 \text{ kW} \\
 (c) \quad \text{At 3.5 bar, } s_f &= 1.7273, s_{fg} = 5.2119 \text{ quality is isentropic} \\
 6.5381 &= 1.7273 + x \times 5.2119 \quad x = 0.923 \\
 \therefore h_{4s} &= 584.3 + 0.923 \times 2147.4 = 2566.4 \text{ kJ/kg} \\
 \therefore \eta_{\text{isen.}} &= \frac{h_1 - h_4}{h_1 - h_{4s}} = \frac{2862.2 - 2636.7}{2862.2 - 2566.4} \times 100\% = 76.24\%
 \end{aligned}$$

**Q.12.20**

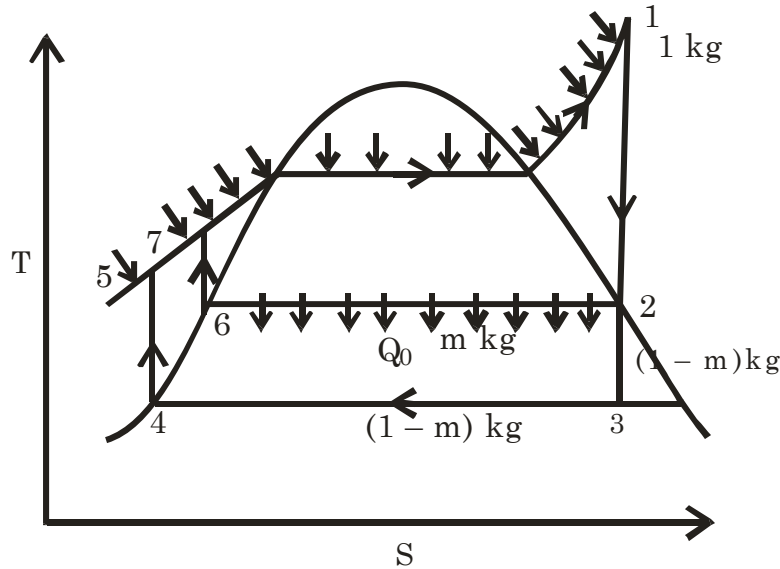
**In a cogeneration plant, the power load is 5.6 MW and the heating load is 1.163 MW. Steam is generated at 40 bar and 500°C and is expanded isentropically through a turbine to a condenser at 0.06 bar. The heating load is supplied by extracting steam from the turbine at 2 bar which condensed in the process heater to saturated liquid at 2 bar and then pumped back to the boiler. Compute (a) the steam generation capacity of the boiler in tonnes/h, (b) the heat input to the boiler in MW, and (c) the heat rejected to the condenser in MW.**

(Ans. (a) 19.07 t/h, (b) 71.57 MW, and (c) 9.607 MW)

**Solution:** From steam table at 40 bar 500°C

$$h_1 = 3445.3 \text{ kJ/kg}$$

$$s_1 = 7.090 \text{ kJ/kg}$$



→ at 2 bar

$$s_f = 1.5301, s_{fg} = 5.5967$$

$$\therefore 7.090 = 1.5301 + x \times 5.5967$$

$$\therefore x = 0.9934$$

$$h_2 = 504.7 + 0.9934 \times 2201.6$$

$$= 2691.8 \text{ kJ/kg}$$

→ at 0.06 bar

$$s_f = 0.521, s_{fg} = 7.809$$

$$\therefore 7090 = 0521 + x \times 7.809 \Rightarrow x = 0.8412$$

$$\therefore h_3 = 151.5 + 0.8412 \times 2415.9 = 2183.8 \text{ kJ/kg}$$

$$h_4 = 151.5 \text{ kJ/kg}$$

$$h_6 = 504.7 \text{ kJ/kg}$$

$$W_{P_{4-5}} = 0.001006 \times (4000 - 6) = 4 \text{ kJ/kg}$$

$$\text{so } h_5 = h_4 + W_P = 155.5 \text{ kJ/kg}$$

$$W_{P_{6-7}} = 0.001061 \times (4000 - 100) = 4 \text{ kJ/kg}$$

$$\text{so } h_7 = h_6 + W_P = 508.7 \text{ kJ/kg}$$

$$\text{For heating load } Q_0 = h_2 - h_6 = (2691.8 - 504.7) \text{ kJ/kg}$$

$$= 2187.1 \text{ kJ/kg}$$

$$\text{For } W_T = (h_1 - h_2) + (1 - m)(h_2 - h_3)$$

$$= 753.5 + (1 - m) 508$$

$$= 1261.5 - 508 m$$

$$\therefore W_{\text{net}} = W_T - W_{P_{4-5}}(1 - m) - m W_{P_{6-7}}$$

$$= (1257.5 - 508 m) \text{ kJ/kg}$$

If mass flow rate at '1' of steam is  $w$  kg/s then

$$w(1257.5 - 508m) = 5600 \quad \dots(i)$$

$$w_m \times 2187.1 = 1163 \quad \dots(ii)$$

From (i) & (ii)  $w = 4.668 \text{ kg/s} = 16.805 \text{ Ton/h}$

$$\therefore m = 0.11391 \text{ kg/kg of the generation}$$

(a) Steam generation capacity of boiler = 16.805 t/h

(b) Heat input to the boiler  
 $= W [(1 - m) (h_1 - h_5) + m (h_1 - h_7)] = 15.169 \text{ MW}$

(c) Heat rejection to the condenser  
 $= (1 - m) (h_3 - h_4) = 8.406 \text{ MW}$

**Q.12.21** Steam is supplied to a pass-out turbine at 35 bar, 350°C and dry saturated process steam is required at 3.5 bar. The low pressure stage exhausts at 0.07 bar and the condition line may be assumed to be straight (the condition line is the locus passing through the states of steam leaving the various stages of the turbine). If the power required is 1 MW and the maximum process load is 1.4 kW, estimate the maximum steam flow through the high and low pressure stages. Assume that the steam just condenses in the process plant.

(Ans. 1.543 and 1.182 kg/s)

**Solution:** Form Steam Table 35 bar 350°C

$$s_1 = \frac{6.743 + 6.582}{2} = 6.6625 \text{ kJ/kg}$$

$$h_1 = \frac{3115.3 + 3092.5}{2} = 3103.9 \text{ kJ/kg}$$

at 3.5 bar  $s_f = 1.7273$   $s_{fg} = 5.2119$

∴ if condition of steam is  $x_1$

$$6.6625 = 1.7273 + x_1 \times 5.2119$$

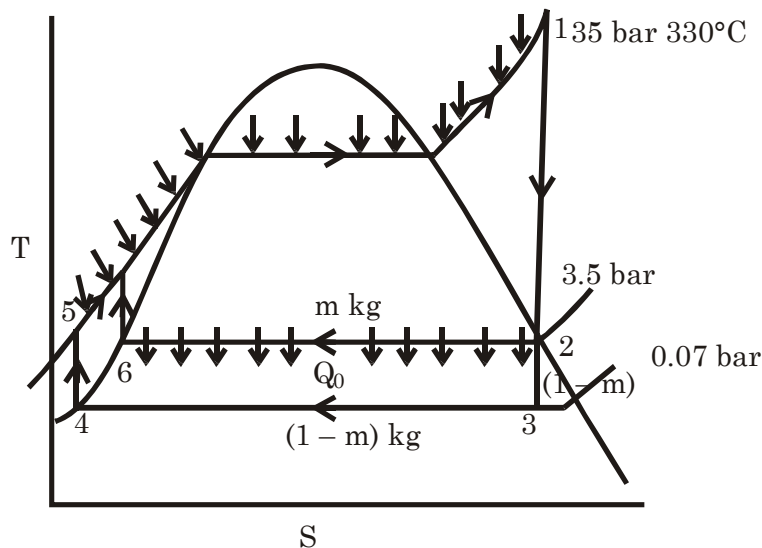
$$x_1 = 0.9469$$

$$\therefore h_2 = 584.3 + 0.9469 \times 2147.4 = 2617.7 \text{ kJ/kg}$$

At 0.07 bar

$$s_f = 0.559 \quad s_{fg} = 7.717$$

$$\therefore 6.6625 = 0.559 + x \times 7.717 \Rightarrow x_2 = 0.7909$$



$$\therefore h_3 = 163.4 + 0.7909 \times 2409.1 = 2068.8 \text{ kJ/kg}$$

$$h_4 = 163.4 \text{ kJ/kg} \quad \therefore h_6 = 584.3 \text{ kJ/kg}$$

$$W_{p_{4-5}} = 0.001007 (3500 - 7) = 3.5 \text{ kJ/kg}$$

$$\therefore h_5 = h_4 + W_{P_{4-5}} = 166.9 \text{ kJ/kg}$$

$$W_{P_{6-7}} = 0.001079 (3500 - 350) = 3.4 \text{ kJ/kg}$$

$$\therefore h_7 = h_6 + W_{P_{6-7}} = 587.7 \text{ kJ/kg}$$

Let boiler steam generation rate =  $w$  kg/s

$$\therefore W_T = w [(h_1 - h_2) + (1 - m) (h_2 - h_3)]$$

$$\begin{aligned} W_{\text{net}} &= w [(h_1 - h_2) + (1 - m) (h_2 - h_3) - (1 - m) 3.5 - m \times 3.4] \text{ kW} \\ &= w [486.2 + 545.4 - 542 \text{ m}] \\ &= w [1031.6 - 542 \text{ m}] \text{ kW} \end{aligned}$$

$$Q = mw [h_2 - h_6] = mw (2033.4) \text{ kW}$$

$$\text{Here } w [1031.6 - 542m] = 1000 \quad \dots (i)$$

$$mw \times 2033.4 = 1400 \quad \dots (ii)$$

$$\therefore w = 1.3311 \text{ kg/s} \quad ;$$

$$m = 0.51724 \text{ kg/kg of steam at H.P}$$

$$\therefore \text{AT H.P flow } 1.3311 \text{ kg/s}$$

$$\text{At L.P flow} = (1 - m) w = 0.643 \text{ kg/s}$$

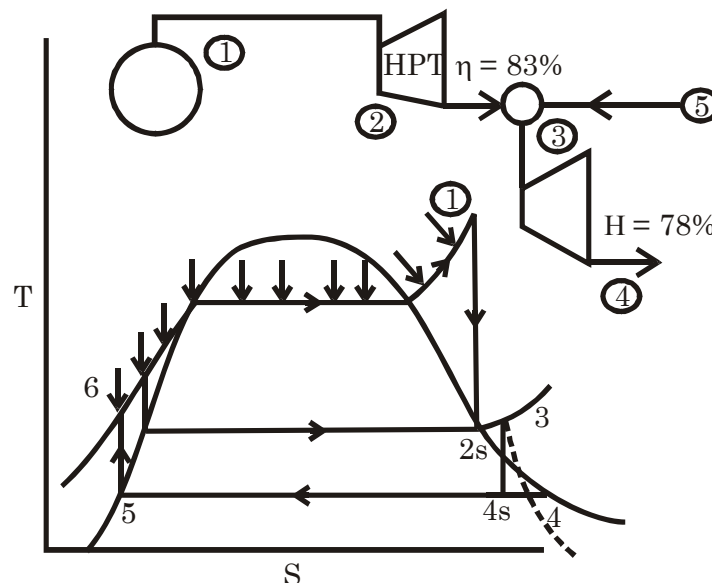
**Q.12.22** Geothermal energy from a natural geyser can be obtained as a continuous supply of steam 0.87 dry at 2 bar and at a flow rate of 2700 kg/h. This is utilized in a mixed-pressure cycle to augment the superheated exhaust from a high pressure turbine of 83% internal efficiency, which is supplied with 5500 kg/h of steam at 40 bar and  $500^\circ\text{C}$ . The mixing process is adiabatic and the mixture is expanded to a condenser pressure of 0.10 bar in a low pressure turbine of 78% internal efficiency. Determine the power output and the thermal efficiency of the plant.

(Ans. 1745 kW, 35%)

**Solution :** From Steam Table 40 bar  $500^\circ\text{C}$

$$h_1 = 3445.3 \text{ kJ/kg}$$

$$s_1 = 7.090 \text{ kJ/kg} - \text{K}$$



At 2 bar

$$h_f = 504.7 \text{ kJ/kg}, h_{fg} = 2201.6 \text{ kJ/kg}$$

$$s_f = 1.5301 \text{ kJ/kg}, s_{fg} = 5.5967 \text{ kJ/kg}$$

$$7.090 = 1.4301 + x_1 \times 4.5967$$

$$\therefore x_1 = 0.99342$$

$$\therefore h_{2s} = 504.7 + 0.99342 \times 2201.6 \text{ kJ/kg} = 2691.8 \text{ kJ/kg}$$

$$\eta_{isen.} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

$$\therefore h_2 - h_1 - \eta_{in} (h_1 - h_{2s}) = 3445.3 - 0.83 (3445.3 - 2691.8) = 2819.9 \text{ kJ/kg}$$

From molier diagram  $s_2 = 7.31 \text{ kJ/kg} - K$

Adiabatic mixing  $h_5 = 504.7 + 0.87 \times 2201.6 = 2420 \text{ kJ/kg}$

$$\therefore h_2 \times 5500 + h_5 \times 2700 = h_3 \times (5500 + 2700)$$

$$\therefore h_3 = 2688.3 \text{ kJ/kg from molier dia at 2 bar } 2688.3 \text{ kJ/kg}$$

quality of steam  $x_3$

$$\text{Then } 504.7 + x_2 \times 2201.6 = 2688.3 \Rightarrow x_3 = 0.9912$$

$$\therefore s_3 = 1.5301 + 0.9918 \times 5.5967 = 7.081 \text{ kJ/kg} - K$$

at 0.1 bar

$$s_f = 0.649 + s_{fg} = 7.501$$

$$\therefore x_4 \times 7.501 + 0.649 = 7.081 \Rightarrow x_4 = 0.8575$$

$$\therefore h_{4s} = 191.8 + 0.8575 \times 2392.8 = 2243.6 \text{ kJ/kg}$$

$$\therefore W_{T.H.P} = \frac{5500}{3600} (h_1 - h_2) = 955.47 \text{ kW}$$

$$W_{T.L.P} = \frac{8200}{3600} (h_3 - h_{4s}) \times 0.78 = 790 \text{ kW } 790.08 \text{ kW}$$

$$\therefore W_T = 1745.6 \text{ kW}$$

$$W_P = \frac{5500}{3600} \times 0.001010 \times (4000 - 10) = 6.16 \text{ kW}$$

$$\therefore W_{net} = 1739.44 \text{ kW}$$

$$h_5 = 191.8 \text{ kJ/kg}, h_6 = h_5 + W_P = 195.8 \text{ kJ/kg}$$

$$\therefore \text{Heat input} = \frac{5500}{3600} (h_1 - h_6) = 4964.5 \text{ kW}$$

$$\therefore \eta = \frac{1739.44}{4964.5} \times 100\% = 35.04\%$$

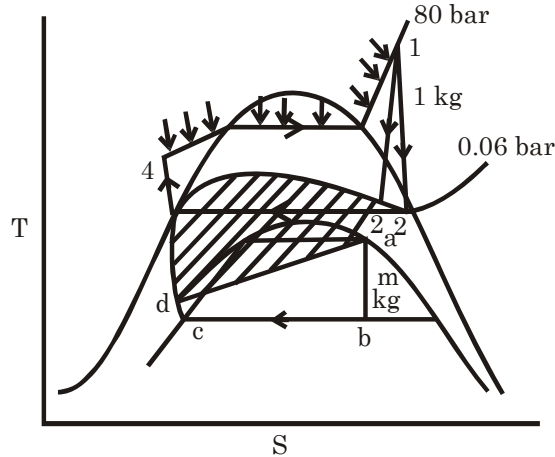
### Q.12.23

In a study for a space projects it is thought that the condensation of a working fluid might be possible at  $-40^\circ\text{C}$ . A binary cycle is proposed, using Refrigerant 12 as the low temperature fluid, and water as the high temperature fluid. Steam is generated at 80 bar,  $500^\circ\text{C}$  and expands in a turbine of 81% isentropic efficiency to 0.06 bar, at which pressure it is condensed by the generation of dry saturated refrigerant vapour at  $30^\circ\text{C}$  from saturated liquid at  $-40^\circ\text{C}$ . The isentropic efficiency of the R-12 turbine is 83%. Determine the mass ratio of R-12 to water and the efficiency of the cycle. Neglect all losses.

**Solution :** at 80 bar 500°C

$$h_1 = 3398.3 \text{ kJ/kg}$$

$$s_1 = 6.724 \text{ kJ/kg} - \text{K}$$



at 0.06 bar if quality is  $x$ , then

$$6.724 = 0.521 + x_2 \times 7.809$$

$$\therefore x_2 = 0.79434$$

$$\therefore h_{2s} = 151.5 + 0.79434 \times 2415.9 = 2070.5 \text{ kJ/kg}$$

$$h_3 = 151.5$$

$$W_P = 0.001006 (8000 - 6) = 8 \text{ kJ/kg}$$

$$\therefore h_4 = 159.5 \text{ kJ/kg}$$

$$\therefore W_T = (h_1 - h_{2s}) \times \eta = (3398.3 - 2070.5) \times 0.81 = 1075.5 \text{ kJ/kg}$$

$$\therefore W_{\text{net}} = W_T - W_P = 1067.5 \text{ kJ/kg}$$

$$Q_1 = h_1 - h_4 = 3238.8 \text{ kJ/kg}$$

$$Q_2 = h_2 - h_3 = h_1 - \eta (h_1 - h_{2s}) - h_3 = 2171.3 \text{ kJ/kg}$$

For R-12

at 30°C saturated vapour

$$h_a = 199.6 \text{ kJ/kg}, p = 7.45 \text{ bar } s_g = 0.6854 \text{ kJ/kg} - \text{K}$$

at 40°C

$$s_f = 0, \quad s_{fg} = 0.7274, \quad \therefore \text{if dryness } x_b \text{ then}$$

$$x_b \times 0.7274 = 0.6854 \quad \Rightarrow x_b = 0.94226$$

$$\therefore s_f = 0, h_{fg} = 169.0 \text{ kJ/kg}$$

$$\therefore h_b = 0.94226 \times 169 = 159.24 \text{ kJ/kg}$$

$$h_c = 0$$

$$W_P = V_c (p_a - p_c) = \left( \frac{0.66 + 0.77}{2} \right) \times 10^{-3} (745 - 64.77) x = 0.4868 \text{ kJ/kg}$$

$$\therefore h_b = h_c + W_P = 0.4868 \text{ kJ/kg}$$

$$\begin{aligned} \therefore \text{Heat input} &= m (h_a - h_b) \\ &= m (199.6 - 0.4868) = 199.11 m = 2171.3 \end{aligned}$$

$$\therefore m = 10.905 \text{ kg of R-12/kg of water}$$

$$\text{Power output } W_{T_r} = m (h_a - h_b) \times \eta$$

$$= 10.905 \times (199.6 - 159.24) \times 0.83 = 365.3 \text{ K}$$



$$\begin{aligned}
 \therefore W_{\text{net}} &= 364.8 \text{ kJ/kg of steam} \\
 \therefore W_{\text{output}} &= W_{\text{net H}_2\text{O}} + W_{\text{net R12}} \\
 &= (1067.5 + 364.8) = 1432.32 \text{ kJ/kg} \\
 \therefore \eta &= \frac{W_{\text{output}}}{\text{Heat input}} = \frac{1432.32}{3238.8} \times 100\% = 44.22\%
 \end{aligned}$$

**Q.12.24** Steam is generated at 70 bar, 500°C and expands in a turbine to 30 bar with an isentropic efficiency of 77%. At this condition it is mixed with twice its mass of steam at 30 bar, 400°C. The mixture then expands with an isentropic efficiency of 80% to 0.06 bar. At a point in the expansion where the pressure is 5 bar, steam is bled for feedwater heating in a direct contact heater, which raises the feed water to the saturation temperature of the bled steam. Calculate the mass of steam bled per kg of high pressure steam and the cycle efficiency. Assume that the L.P. expansion condition line is straight.

(Ans. 0.53 kg; 31.9%)

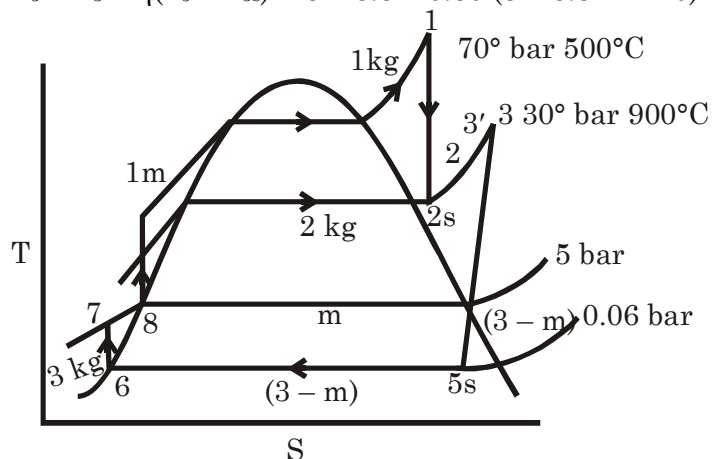
**Solution :** From Steam Table 70 bar 500°C  
 $h_1 = 3410.3 \text{ kJ/kg}$   
 $s_1 = 6.798 \text{ kJ/kg - K}$

$s_1$  at 30 bar 400°C  
 $h'_3 = 3230.9 \text{ kJ/kg}$   
 $s'_3 = 6.921 \text{ kJ/kg}$

From Molier diagram  $h_{2s} = 3130 \text{ kJ/kg}$   
 $\therefore h_2 = h_1 - \eta_{\text{isentropic}} \times (h_1 - h_{2s})$   
 $= 3410.3 - 0.77 (3410.3 - 3130) = 3194.5 \text{ kJ/kg}$

For adiabatic mixing

$$\begin{aligned}
 1 \times h_2 + 2 \times h'_3 &= 3 \times h_3 \\
 h_3 &= 3218.8 \text{ kJ/kg} \\
 s_3 &= 6.875 \text{ kJ/kg} \quad (\text{From Molier diagram}) \\
 h'_4 &= 2785 \text{ kJ/kg} \\
 \therefore h_{5s} &= 2140 \text{ kJ/kg} \\
 \therefore h_5 &= h_3 - \eta (h_3 - h_{5s}) = 3218.8 - 0.80 (3218.8 - 2140) \text{ kJ/kg}
 \end{aligned}$$



From S.L in H.P.  $h_4 = 2920 \text{ kJ/kg}$

From Heat balance & heater

$$m \times h_4 + (3 - m)h_7 = 3h_g \quad h_7 = h_6 + W_P$$

$$= 151.5 + 0.001006 \times (500 - 6) \approx 0.5 \text{ kJ/kg}$$

$$\therefore m \times 2920 + (3 - m) \times 152 = 3 \times 640.1 = 152 \text{ kJ/kg}$$

$$m = 0.529 \text{ kg} \quad h_g = 640.1$$

$$W_{T_{H.P}} = 1 \times (h_1 - h_2) = (3410.3 - 3194.5) \text{ kJ/kg} = 215.8 \text{ kJ/kg}$$

$$W_{T_{L.P}} = 3 (h_3 - h_4) + (3 - m) (h_4 - h_5)$$

$$= 3 (3218.8 - 2920) + (3 - 0.529) (2920 - 2355.8)$$

$$= 2290.5 \text{ kJ/kg of steam H.P}$$

$$W_P = (3 - m) (h_7 - h_6) + 2 \times 0.001(3000 - 500)$$

$$+ 1 \times 0.001 (7000 - 500) = 12.74 \text{ kJ/kg of H.P}$$

$$\therefore W_{\text{net}} = (215.8 + 2290.5 - 12.74) \text{ kJ/kg \& H.P steam}$$

$$= 2493.6 \text{ kJ/kg of H.P steam}$$

$$\text{Heat input } Q_1 = (h_1 - h_{10}) + 2 (h_3' - h_9)$$

$$\therefore h_{10} + h_8 + W_{P_{8-10}} = 646.6 \text{ kJ/kg}$$

$$= (3410.3 - 646.6) + 2 (3230.9 - 642.6) \quad h_9 = h_8 + W_{P_{8-9}}$$

$$= 7940.3 \text{ kJ/kg of H.P steam} \quad = 642.6 \text{ kJ/kg}$$

$$\therefore \eta_{\text{cycle}} = \frac{2493.6}{7940.3} \times 100\% = 31.4\%$$

**Q.12.25** An ideal steam power plant operates between 70 bar, 550°C and 0.075 bar. It has seven feed water heaters. Find the optimum pressure and temperature at which each of the heaters operate.

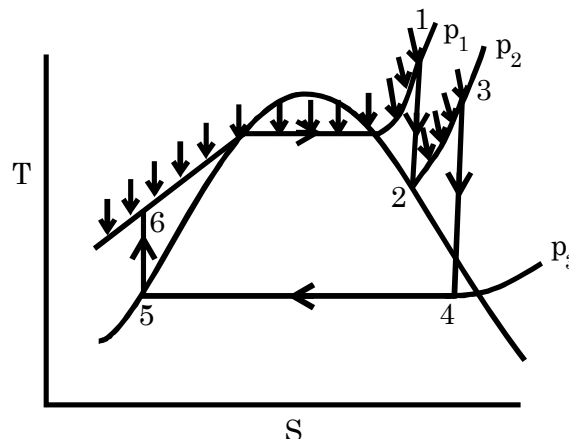
**Solution:** Try please.

**Q.12.26** In a reheat cycle steam at 550°C expands in an h.p. turbine till it is saturated vapour. It is reheated at constant pressure to 400°C and then expands in a l.p. turbine to 40°C. If the moisture content at turbine exhaust is given to be 14.67%, find (a) the reheat pressure, (b) the pressure of steam at inlet to the h.p. turbine, (c) the net work output per kg, and (d) the cycle efficiency. Assume all processes to be ideal.

(Ans. (a) 20 bar, (b) 200 bar, (c) 1604 kJ/kg, (d) 43.8%)

**Solution:** From S.T. at 40°C, 14.67% moisture

$$\therefore x = 0.8533$$



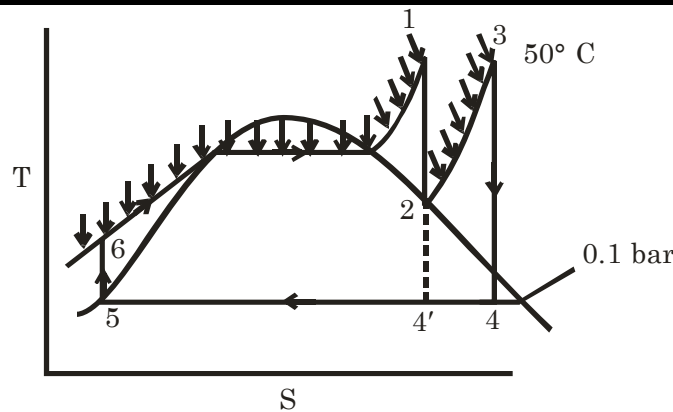
$$\begin{aligned}
 p_3 &= 0.0738 \text{ bar} \\
 h_f &= 167.6 \text{ kJ/kg}; h_{fg} = 2406.7 \text{ kJ/kg} \\
 \therefore h_4 &= 167.6 + 0.85322 \times 2406.7 \\
 &= 2221.2 \text{ kJ/kg} \\
 s_f &= 0.573 \text{ kJ/kg} - \text{K} & s_{fg} &= 7.685 \text{ kJ/kg} - \text{K} \\
 s_4 &= 0.573 + 0.8533 \times 7.685 = 7.1306 \text{ kJ/kg} \\
 \therefore & \text{ at } 400^\circ\text{C} \text{ and } s_4 = 7.1306 \text{ From Steam Table} \\
 \text{(a)} \quad p_r &= 20 \text{ bar}, & h_3 &= 3247.6 \text{ kJ/kg} \\
 & \text{At 20 bar saturation} \\
 h_2 &= 2797.2 \text{ kJ/kg} & \therefore S_2 &= 6.3366 \text{ kJ/kg} - \text{K} \\
 \\
 \text{(b)} \quad & \text{at } 550^\circ\text{C} \text{ and } 6.3366 \text{ kJ/kg} - \text{K} \\
 & \text{From Steam Table} \\
 p_r &= 200 \text{ bar} \\
 \therefore h_1 &= 3393.5 \text{ kJ/kg} \\
 \therefore h_5 &= 167.6 \text{ kJ/kg} & W_P &= 0.001 \times (20000 - 7.38) \text{ kJ/kg} \\
 \therefore h_6 &= h_5 + W = 187.6 \text{ kJ/kg} & &= 20 \text{ kJ/kg} \\
 \therefore W_T &= (h_1 - h_2) + (h_3 - h_4) = 1622.7 \text{ kJ/kg} \\
 \text{(c)} \quad \therefore W_{\text{net}} &= W_T - W_P = 1602.7 \text{ kJ/kg} \\
 \\
 & \text{Heat input } Q_1 = (h_1 - h_6) + (h_3 - h_2) \\
 &= (3393.5 - 187.6) + (3247.6 - 2797.2) \text{ kJ/kg} \\
 &= 3656.3 \text{ kJ/kg} \\
 \\
 \text{(d)} \quad \therefore \eta &= \frac{1602.7}{3656.3} \times 100\% = 43.83\%
 \end{aligned}$$

**Q.12.27** In a reheat steam cycle, the maximum steam temperature is limited to  $500^\circ\text{C}$ . The condenser pressure is 0.1 bar and the quality at turbine exhaust is 0.8778. Had there been no reheat, the exhaust quality would have been 0.7592. Assuming ideal processes, determine (a) reheat pressure, (b) the boiler pressure, (c) the cycle efficiency, and (d) the steam rate.

(Ans. (a) 30 bar, (b) 150 bar, (c) 50.51%, (d) 1.9412 kg/kWh)

**Solution:** From 0.1 bar (saturated S.T.)  
 $s_f = 0.649 \text{ kJ/kg} - \text{K}$   
 $s_{fg} = 7.501 \text{ kJ/kg} - \text{K}$

$$\begin{aligned}
 \therefore s_4 &= s_3 = 0.649 + 0.8778 \times 7.501 = 7.233 \text{ kJ/kg} - \text{K} \\
 s_4'' &= s_1 = 0.649 + 0.7592 \times 7.501 = 6.344 \text{ kJ/kg} - \text{K} \\
 h_4 &= 191.8 + 0.8778 \times 2392.8 = 2292.2 \text{ kJ/kg} \\
 & \text{From super heated steam turbine at } 500^\circ\text{C } 7.233 \text{ kJ/kg} \\
 p_3 &= 30 \text{ bar} \\
 \therefore h_3 &= 3456.5 \text{ kJ/kg}, h_2 = 2892.3 \text{ kJ/kg} \\
 & \text{From Molier diagram}
 \end{aligned}$$



At 500°C and 6.344 kJ/kg – K

$$p_1 = 150 \text{ bar}, \quad h_2 = 3308.6 \text{ kJ/kg – K}$$

$$h_5 = 191.8 \text{ kJ/kg}, \quad W_P = 0.001010 (15000 - 10) = 15.14 \text{ kJ/kg}$$

$$\therefore h_6 = 206.94 \text{ kJ/kg}$$

$$\therefore W_T = (h_1 - h_2) + (h_3 - h_4) = 1580.6 \text{ kJ/kg}$$

$$W_{\text{net}} = W_T - W_P = 1565.46 \text{ kJ/kg}$$

$$Q_1 = (h_1 - h_6) + (h_3 - h_2) = 3665.86 \text{ kJ/kg – K}$$

$$\therefore \eta = \frac{1565.45}{3665.86} \approx 42.7\%$$

**Q.12.28**

In a cogeneration plant, steam enters the h.p. stage of a two-stage turbine at 1 MPa, 200°C and leaves it at 0.3 MPa. At this point some of the steam is bled off and passed through a heat exchanger which it leaves as saturated liquid at 0.3 MPa. The remaining steam expands in the l.p. stage of the turbine to 40 kPa. The turbine is required to produce a total power of 1 MW and the heat exchanger to provide a heating rate of 500 kW. Calculate the required mass flow rate of steam into the h.p. stage of the turbine. Assume (a) steady condition throughout the plant, (b) velocity and gravity terms to be negligible, (c) both turbine stages are adiabatic with isentropic efficiencies of 0.80.

(Ans. 2.457 kg/s)

**Solution:** From S.T at 1 MPa 200°C

i.e 10 bar 200°C

$$h_1 = 2827.9 \text{ kJ/kg}$$

$$s_1 = 6.694 \text{ kJ/kg-K}$$

At 3 bar

$$s_f = 1.6716 \quad s_{fg} = 5.3193$$

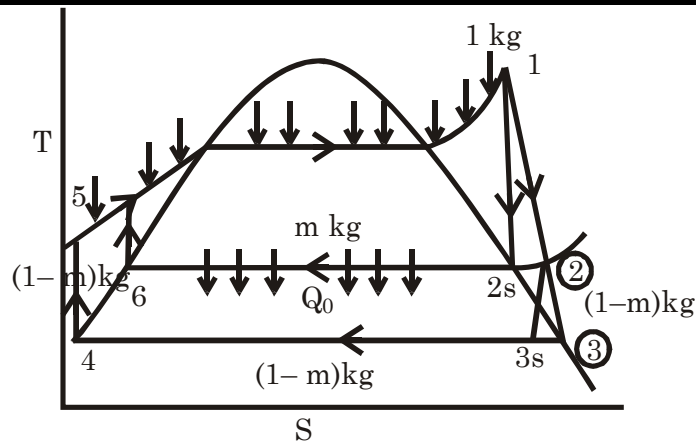
$$\therefore 6.694 = 1.6716 + x' \times 5.3193$$

$$\therefore x' = 0.9442$$

$$\therefore h_{2s} = 561.4 + 0.9242 \times 2163 = 2603.9 \text{ kJ/kg}$$

$$\therefore h_2 = h_1 - \eta(h_1 - h_{2s}) = 2827.9 - 0.8(2827.9 - 2603.0) \text{ kJ/kg}$$

$$= 2648.7 \text{ kJ/kg}$$



This is also wet so

$$s_2 = 1.6716 + x_2' \times 5.319.3 \quad [2648.7 = 561.4 + x_2 \times 2163.2]$$

$$= 6.8042 \text{ kJ/kg} - \text{K}$$

If at 3s condition of steam is  $x_3$  then 40 kPa = 0.4 bar

$$6.8042 = 1.0261 + x_3 \times 6.6440 \quad \Rightarrow x_3 = .8697$$

$$\therefore h_{3s} = 317.7 + 0.8697 \times 2319.2 = 2334.4 \text{ kJ/kg}$$

$$\therefore h_3 = h_2 - \eta (h_2 - h_{3s}) = 2397.3 \text{ kJ/kg}$$

$$h_4 = 317.7 \text{ kJ/kg}$$

$$W_{P_{4s}} = 0.001 \times (1000 - 40) \approx 1 \text{ kJ/kg}$$

$$\therefore h_5 = 318.7 \text{ kJ/kg}$$

$$h_6 = 562.1 \text{ kJ/kg}, W_{P_{6-7}} = 0.001 \times (1000 - 300)$$

$$= 0.07 \text{ kJ/kg}$$

$$h_7 = 562.1 \text{ kJ/kg}$$

$$\therefore W_T = (h_1 - h_2) + (1 - m) (h_2 - h_3)$$

$$\therefore W_{\text{net}} = (h_1 - h_2) + (1 - m) (h_2 - h_3) - (1 - m) 1 - m \times 0.7$$

$$= (429.6 - 251.1 \text{ m}) \text{ kJ/kg of steam of H.P}$$

$$\text{Process Heat} = m \times (h_2 - h_6) = m \times 2087.3$$

$\therefore$  If mass flow rate of w then

$$w (429.6 - 251.1 \text{ m}) = 1000$$

$$mw \times 2087.3 = 500$$

$$\therefore w = 2.4678 \text{ kg/s}$$

$$\therefore \text{required mass flow rate in H.P} = 2.4678 \text{ kg/s}$$



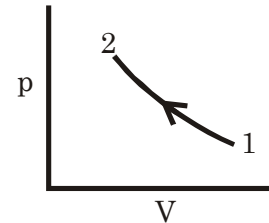
# 13. Gas Power Cycles

## Some Important Notes

### 1. Compression ratio, ( $r_c$ )

$$r_c = \frac{\text{Volume at the beginning of compression } (V_1)}{\text{Volume at the end of compression } (V_2)}$$

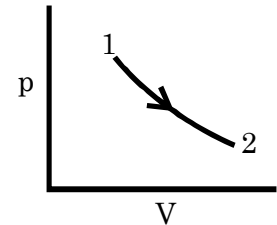
$$\therefore r_c = \frac{V_1}{V_2} = \frac{\text{bigger term}}{\text{smaller term}}$$



### 2. Expansion ratio, ( $r_e$ )

$$r_e = \frac{\text{Volume at the end of expansion } (V_2)}{\text{Volume at the beginning of expansion } (V_1)}$$

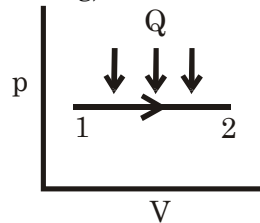
$$\therefore r_e = \frac{V_2}{V_1} = \frac{\text{bigger term}}{\text{smaller term}}$$



### 3. Cut-off ratio, $\rho = \frac{\text{volume after heat addition } (v_2)}{\text{volume before heat addition } (v_1)}$ (For constant Pressure heating)

$$\therefore \rho = \frac{V_2}{V_1} = \frac{\text{bigger term}}{\text{smaller term}}$$

**Relation**  $r_c = r_e \cdot \rho$

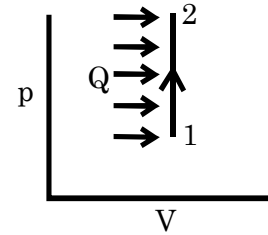


### 4. Constant volume pressure ratio,

$$\alpha = \frac{\text{Pressure after heat addition}}{\text{Pressure before heat addition}}$$

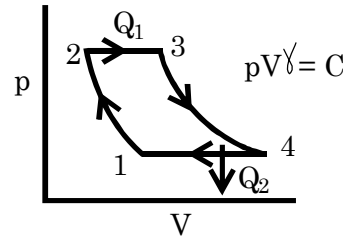
[For constant volume heating]

$$\therefore \alpha = \frac{p_2}{p_1} = \frac{\text{bigger term}}{\text{smaller term}}$$



**5. Pressure ratio,  $r_P$**  =  $\frac{\text{Pressure after compression or before expansion}}{\text{Pressure before compression or after expansion}} \left( \frac{p_2}{p_1} \right)$

$$\therefore r_P = \frac{p_2}{p_1}$$



**6. Carnot cycle:** The large back work (i.e compressor work) is a big draw back for the Carnot gas cycle, as in the case of the Carnot Vapour cycle.

**7. Stirling Cycle:** comparable with Otto.

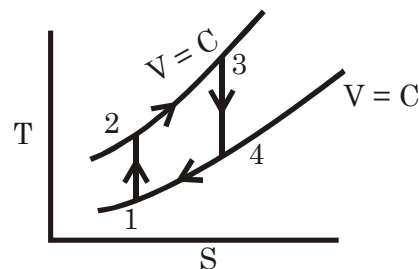
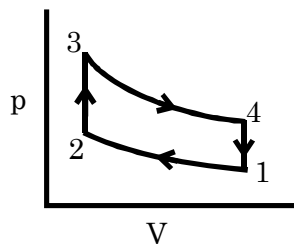
**8. Ericsson Cycle:** comparable with Brayton cycle.

**9. The regenerative, stirling and Ericsson cycles** have the same efficiency as the carnot cycle, but much less back work.

**10. Air standards cycles**

**a. Otto cycle (1876)**

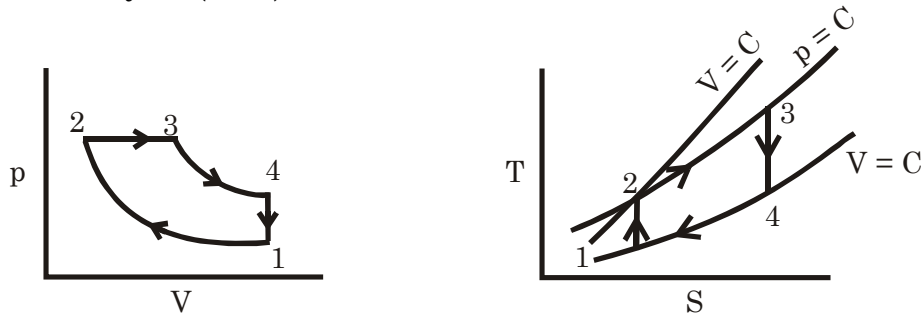
$$\eta = 1 - \frac{1}{r_c^{\gamma-1}}$$



**For  $W_{\max}$ ;  $r_c = \left( \frac{T_{\max}}{T_{\min}} \right)^{\frac{1}{2(\gamma-1)}}$**

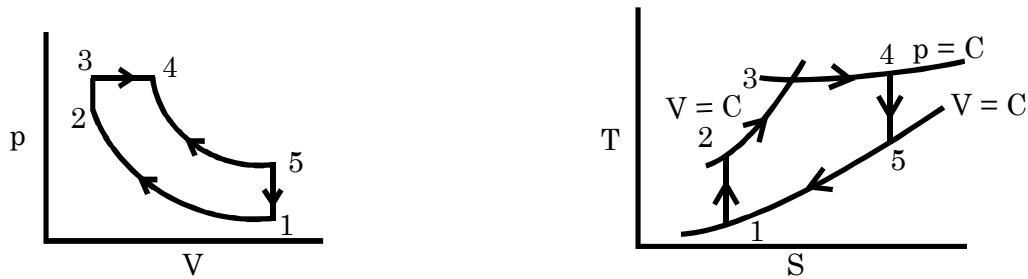


b. Diesel cycle (1892)



$$\eta = 1 - \frac{(\rho^\gamma - 1)}{r_c^{\gamma-1} \cdot \gamma(\rho - 1)}$$

C. Dual or Limited pressure or mixed cycle



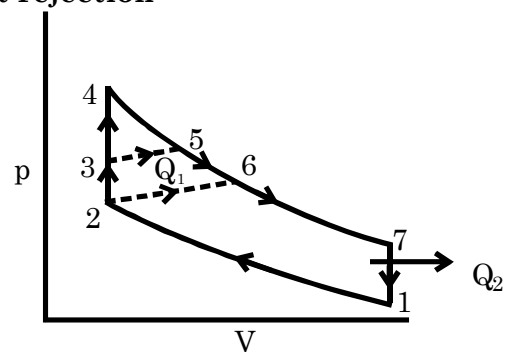
$$\eta = 1 - \frac{(\alpha \rho^\gamma - 1)}{r_c^{\gamma-1} [(\alpha - 1) + \alpha \gamma(\rho - 1)]}$$

Where  $\alpha = \frac{p_3}{p_2}$

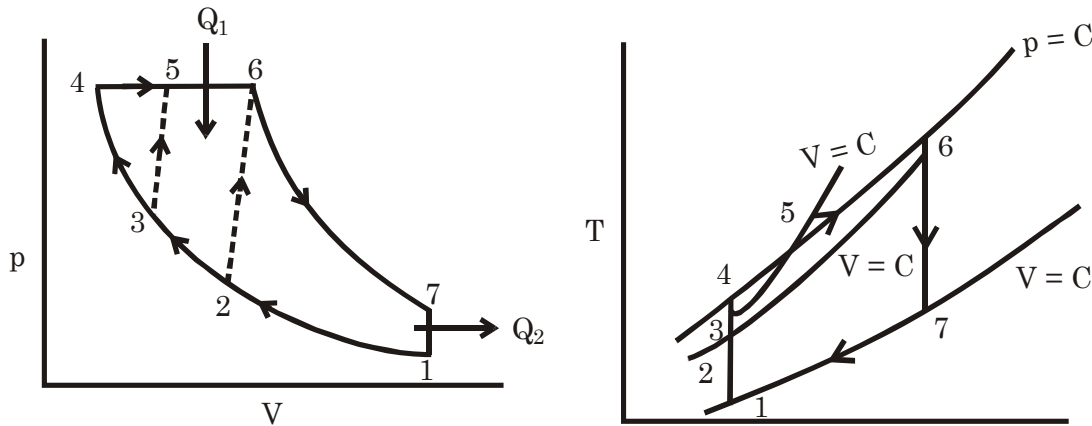
Comparison of Otto, Diesel and Dual cycle

a. With same compression ratio and heat rejection

$$\therefore \eta_{\text{otto}} > \eta_{\text{Dual}} > \eta_{\text{Diesel}}$$



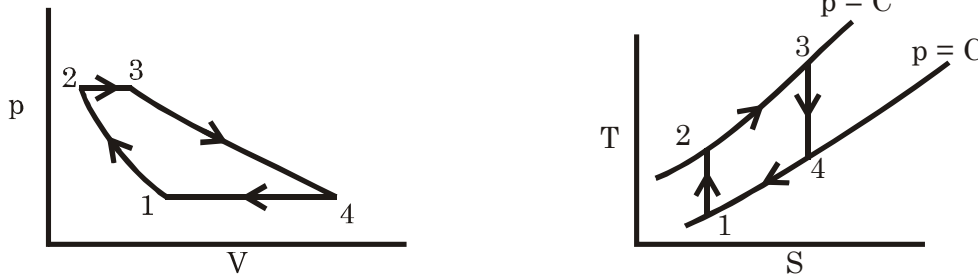
b. For the Same maximum Pressure and Temperature (also heat rejection same)



$$\eta_{\text{Diesel}} > \eta_{\text{Dual}} > \eta_{\text{Otto}}$$

### 11. Brayton cycle

$$\eta = 1 - \frac{1}{r_c^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}}$$



∴ Brayton cycle efficiency depends on either compression ratio ( $r_c$ ) or Pressure ratio  $r_p$

\* For same compression ratio  $[\eta_{\text{Otto}} = \eta_{\text{Brayton}}]$

a. For Maximum efficiency  $(r_p)_{\text{max}} = \left( \frac{T_{\text{max}}}{T_{\text{min}}} \right)^{\frac{\gamma}{\gamma-1}}$

$$\therefore \eta_{\text{max}} = \eta_{\text{Carnot}} = 1 - \frac{T_{\text{min}}}{T_{\text{max}}}$$

b. For Maximum work

$$(i) \quad (r_p)_{\text{opt.}} = \left( \frac{T_{\text{max}}}{T_{\text{min}}} \right)^{\frac{\gamma}{2(\gamma-1)}}$$

$$\therefore \eta_{\text{cycle}} = 1 - \sqrt{\frac{T_{\min}}{T_{\max}}} \text{ and } W_{\text{net, max}} = C_p [\sqrt{T_{\max}} - \sqrt{T_{\min}}]^2$$

(ii) If isentropic efficiency of Turbine is  $\eta_T$  and compressor is  $\eta_c$  then

$$(r_p)_{\text{opt.}} = \left( \eta_T \eta_c \frac{T_{\max}}{T_{\min}} \right)^{\frac{\gamma}{2(\gamma-1)}}$$

## Question and Solution (P K Nag)

**Q13.1** In a Stirling cycle the volume varies between  $0.03$  and  $0.06 \text{ m}^3$ , the maximum pressure is  $0.2 \text{ MPa}$ , and the temperature varies between  $540^\circ\text{C}$  and  $270^\circ\text{C}$ . The working fluid is air (an ideal gas). (a) Find the efficiency and the work done per cycle for the simple cycle. (b) Find the efficiency and the work done per cycle for the cycle with an ideal regenerator, and compare with the Carnot cycle having the same isothermal heat supply process and the same temperature range.

(Ans. (a) 27.7%, 53.7 kJ/kg, (b) 32.2%)

**Solution:**

Given  $V_1 = 0.06 \text{ m}^3 = V_4$

$V_2 = 0.03 \text{ m}^3 = V_3$

$p_3 = 200 \text{ kPa}$

$T_1 = T_2 = 270^\circ\text{C} = 543 \text{ K}$

$T_3 = T_4 = 540^\circ\text{C} = 813 \text{ K}$

$\therefore$  Heat addition  $Q_1 = Q_{2-3} = m c_v (T_3 - T_2)$

$$\text{Here } m = \frac{p_3 V_3}{R T_3} = \frac{200 \times 0.03}{0.287 \times 813} = 0.025715 \text{ kg}$$

$$\therefore Q_1 = 0.025715 \times 0.718 (813 - 543) \text{ kJ} = 4.985 \text{ kJ}$$

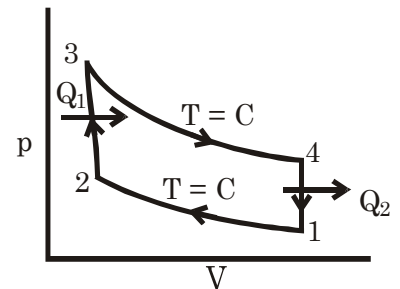
$$W_{3-4} = \int_3^4 p dV = m R T_3 \ln \left( \frac{V_4}{V_3} \right)$$

$$pV = mRT = C$$

$$W_{1-2} = \int_1^2 p dV = m R T_1 \ln \left( \frac{V_1}{V_2} \right)$$

$$\therefore p = \frac{mRT}{V}$$

$$\therefore \eta = \frac{m(RT_3 - RT_1) \ln \left( \frac{V_1}{V_2} \right)}{4.985} =$$



$$\frac{0.025715 \times 0.287 (813 - 543) \ln 2}{4.985} \times 100\% = 27.7\%$$

$$\text{Work done} = 1.3812 \text{ kJ} = 53.71 \text{ kJ/kg}$$

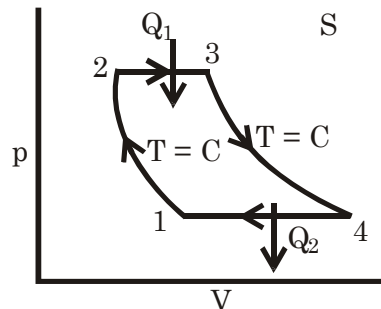
For ideal regeneration

$$\eta = 1 - \frac{543}{813} = 33.21\%$$

**Q13.2** An Ericsson cycle operating with an ideal regenerator works between 1100 K and 288 K. The pressure at the beginning of isothermal compression is 1.013 bar. Determine (a) the compressor and turbine work per kg of air, and (b) the cycle efficiency.

(Ans. (a)  $w_T = 465 \text{ kJ/kg}$ ,  $w_C = 121.8 \text{ kJ/kg}$  (b) 0.738)

**Solution:** Given  $T_1 = T_2 = 288 \text{ K}$   
 $T_3 = T_4 = 1100 \text{ K}$   
 $p_1 = 1.013 \text{ bar} = 101.3 \text{ kPa}$   
 $\therefore V_1 = \frac{RT_1}{p_1} = 0.81595 \text{ m}^3/\text{kg}$



$$W_C = RT_1 \ln \left( \frac{V_1}{V_2} \right)$$

$$W_T = R T_3 \ln \left( \frac{V_4}{V_3} \right)$$

$$p_3 = p_2 ; p_1 = p_4$$

$$\therefore \eta = 1 - \frac{288}{1100} = 73.82\%$$

$$\therefore \frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\therefore W = \eta Q_1 = \eta C_P (T_1 - T_2)$$

$$\therefore \frac{V_1}{V_2} = \frac{T_1}{T_2} \times \left( \frac{p_2}{p_1} \right) = \left( \frac{p_2}{p_1} \right)$$

$$= 0.7382 \times 1.005 (1100 - 288) \text{ kJ/kg} = 602.4 \text{ kJ/kg}$$

$$\therefore Q_{2-3} = C_P (T_3 - T_2) = C_v (T_3 - T_2) + p_2 (V_3 - V_2)$$

**Q13.5** An engine equipped with a cylinder having a bore of 15 cm and a stroke of 45 cm operates on an Otto cycle. If the clearance volume is  $2000 \text{ cm}^3$ , compute the air standard efficiency.

(Ans. 47.4%)

**Solution:**

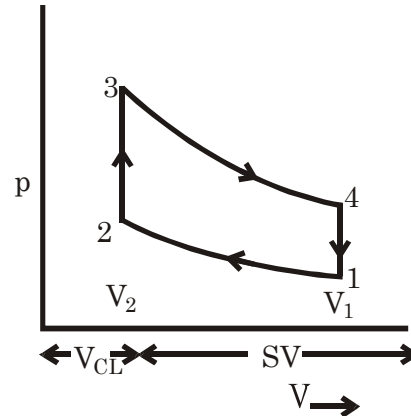
$$V_2 = 2000 \text{ cm}^3 = 0.002 \text{ m}^3$$

$$V_1 = V_2 + S.V.$$

$$= 0.002 + \frac{\pi \times 0.15^2}{4} \times 0.45 = 0.009952 \text{ m}^3$$

$$\therefore r_c = \frac{V_1}{V_2} = 4.9761$$

$$\eta_{\text{air std}} = 1 - \frac{1}{r_c^{\gamma-1}} = 47.4\%$$



**Q13.10**

Two engines are to operate on Otto and Diesel cycles with the following data: Maximum temperature 1400 K, exhaust temperature 700 K. State of air at the beginning of compression 0.1 MPa, 300 K.

Estimate the compression ratios, the maximum pressures, efficiencies, and rate of work outputs (for 1 kg/min of air) of the respective cycles.

(Ans. Otto--  $r_k = 5.656$ ,  $p_{\max} = 2.64 \text{ MPa}$ ,  $W = 2872 \text{ kJ/kg}$ ,  $\eta = 50\%$ )

Diesel-  $r_k = 7.456$ ,  $p_{\max} = 1.665 \text{ MPa}$ ,  $W = 446.45 \text{ kJ/kg}$ ,  $\eta = 60.8\%$ )

**Solution:**

$$T_3 = 1400 \text{ K}$$

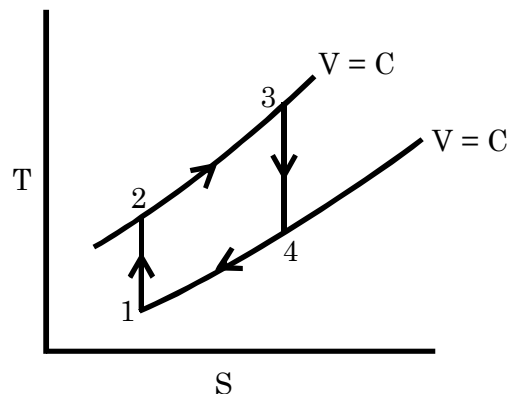
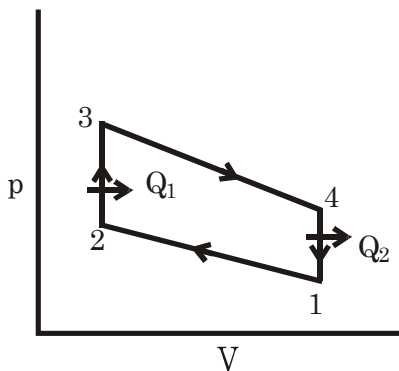
$$T_4 = 700 \text{ K}$$

$$p_1 = 100 \text{ kPa}$$

$$T_1 = 300 \text{ K}$$

$$\therefore v_1 = \frac{RT_1}{p_1} = 0.861 \text{ m}^3/\text{kg}$$

$$\therefore \frac{T_3}{T_4} = \left( \frac{p_3}{p_4} \right)^{\frac{\gamma-1}{\gamma}} = \left( \frac{v_4}{v_3} \right)^{\gamma-1}$$



$$\therefore \left( \frac{1400}{700} \right) = \left( \frac{v_1}{v_2} \right)^{\gamma-1}$$

$$\therefore v_2 = \frac{v_1}{2^{\frac{1}{\gamma-1}}} = \frac{0.861}{2^{0.9}} = 0.1522 \text{ m}^3/\text{kg}$$

$$\therefore \frac{T_2}{T_1} = \left( \frac{v_1}{v_2} \right)^{\gamma-1}$$

$$= (5.657)^{0.4} \times 300 = 600 \text{ K}$$

$$\therefore r_c = \frac{v_1}{v_2} = 2^{\frac{1}{\gamma-1}} = 5.657 \quad \frac{p_2}{p_1} = \left( \frac{v_1}{v_2} \right)^\gamma \Rightarrow P_2 = 1131.5 \text{ kPa}$$

$$\frac{p_3}{T_3} = \frac{p_2}{T_2} \Rightarrow p_3 = \frac{T_3}{T_2} \times p_2 = \frac{1400}{600} \times 1131.5 \text{ kPa} = 2.64 \text{ MPa}$$

$$\begin{aligned} \therefore W &= Q_1 - Q_2 = C_v (T_3 - T_2) - C_v (T_4 - T_1) \\ &= 0.718 [(1400 - 600) - (700 - 300)] \text{ kJ/kg} = 287.2 \text{ kJ/kg.} \end{aligned}$$

$$\therefore \eta = \frac{Q_1 - Q_2}{Q_1} = \frac{287.2}{0.718(1400 - 600)} = 0.5 \approx 50\%$$

Diesel  $T_3 = 1400 \text{ K}$   
 $T_4 = 700 \text{ K}$   
 $T_1 = 300 \text{ K}$   
 $p_1 = 100 \text{ kPa}$

$$\therefore v_1 = 0.861 \text{ m}^3/\text{kg}$$

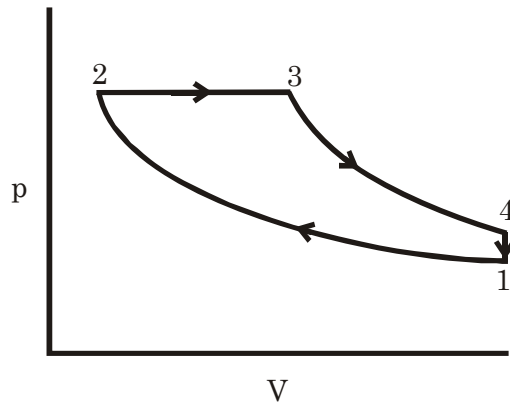
$$\frac{T_3}{T_4} = \left( \frac{v_4}{v_3} \right)^{\gamma-1}$$

$$\therefore \frac{1400}{700} = \left( \frac{v_1}{v_2} \right)^{0.4}$$

$$\therefore \frac{v_1}{v_3} = 2^{\frac{1}{0.4}} = 2^{2.5}$$

$$\therefore v_3 = \frac{v_1}{2^{2.5}} = 0.1522 \text{ m}^3/\text{kg}$$

$$\therefore p_3 = \frac{RT_3}{V_3} = \frac{0.287 \times 1400}{0.1522} = 2639.9 \text{ kPa}$$



$$\therefore p_2 = p_3$$

$$\therefore \frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} = \left( \frac{v_1}{v_2} \right)^{\gamma-1} \quad \therefore T_2 = 764 \text{ K}$$

$$r_c = \frac{v_1}{v_2} = \left( \frac{p_2}{p_1} \right)^{\frac{1}{\gamma}} = \left( \frac{2639.9}{100} \right)^{\frac{1}{1.4}} = 10.36$$

$$p_1 = p_3 = 2.64 \text{ MPa}$$

$$\begin{aligned} Q_1 &= Q_{2-3} = C_p (T_3 - T_2) = 1.005 (1400 - 764) \text{ kJ/kg} \\ &= 638.84 \text{ kJ/kg} \end{aligned}$$

$$Q_2 = Q_{4-1} = C_v(T_4 - T_1) = 0.718(700 - 300) = 287.2 \text{ kJ/kg}$$

$$\therefore W = Q_1 - Q_2 = 351.64 \text{ kJ/kg}$$

$$\eta = \frac{W}{Q_1} = \frac{351.64}{638.84} = 55\%$$

**Q13.11** An air standard limited pressure cycle has a compression ratio of 15 and compression begins at 0.1 MPa, 40°C. The maximum pressure is limited to 6 MPa and the heat added is 1.675 MJ/kg. Compute (a) the heat supplied at constant volume per kg of air, (b) the heat supplied at constant pressure per kg of air, (c) the work done per kg of air, (d) the cycle efficiency, (e) the temperature at the end of the constant volume heating process, (f) the cut-off ratio, and (g) the m.e.p. of the cycle.

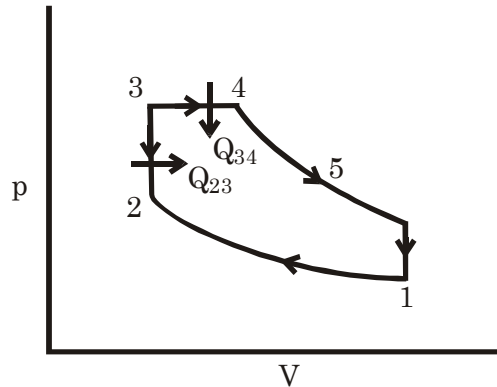
(Ans. (a) 235 kJ/kg, (b) 1440 kJ/kg, (c) 1014 kJ/kg,  
(d) 60.5%, (e) 1252 K, (f) 2.144 (g) 1.21 MPa)

**Solution:**  $r_c = \frac{v_1}{v_2} = 15$

$$p_1 = 100 \text{ kPa}$$

$$T_1 = 40^\circ\text{C} = 313 \text{ K}$$

$$\therefore v_1 = \frac{RT_1}{p_1} = 0.89831 \text{ m}^3/\text{kg}$$



$$p_3 = p_4 = 6000 \text{ kPa}$$

$$Q_{2-4} = 1675 \text{ kJ/kg}$$

$$\therefore \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = (15)^{1.4-1} \Rightarrow T_2 = 924.7 \text{ K}$$

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} = 15^{1.4} \Rightarrow p_2 = 4431 \text{ kPa}$$

$$\therefore \frac{p_2 V_2}{T_2} = \frac{p_3 V_3}{T_3} \Rightarrow T_3 = \frac{p_3}{p_2} \times T_2 = \frac{6000}{4431} \times 924.7 = 1252 \text{ K}$$

$$\therefore Q_{2-4} = C_v(T_3 - T_2) + C_p(T_4 - T_3) = 1675$$

$$\therefore T_4 = T_3 + 1432.8 \text{ K} = 2684.8 \text{ K}$$

∴

$$\therefore v_4 = \frac{RT_4}{p_4} = 0.12842 \text{ m}^3/\text{kg}.$$

$$\therefore \frac{T_4}{T_5} = \left( \frac{v_5}{v_4} \right)^{\gamma-1} = \left( \frac{v_1}{v_4} \right)^{\gamma-1} \Rightarrow \frac{T_4}{T_5} = 2.1773 \quad \therefore T_5 = 1233 \text{ K}$$

(a) Heat supplied at constant volume =  $C_v (T_3 - T_2) = 235 \text{ kJ/kg}$

(b) Heat supplied at constant Pressure =  $(1675 - 235) = 1440 \text{ kJ/kg}$

(c) Work done =  $Q_1 - Q_2 = 1675 - C_v (T_5 - T_1) = 1014.44 \text{ kJ/kg}$

(d) Efficiency  $\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{1014.44}{1675} \times 100\% = 60.56\%$

(e) Temperature at the end of the heating ( $T_3$ ) = 1252 K

(f) Cut-off ratio ( $\rho$ ) =  $\frac{v_4}{v_3} = \frac{0.12842}{0.05988} = 2.1444$

$$[\therefore v_3 = \frac{RT_3}{p_3} = 0.059887]$$

(g) m. e. p.  $\therefore p_m (V_1 - V_2) = W$

$$\therefore p_m = \frac{1014.44}{v_1 - \frac{v_1}{15}} = 1209.9 \text{ kPa} = 1.2099 \text{ MPa}$$

**Q13.13**

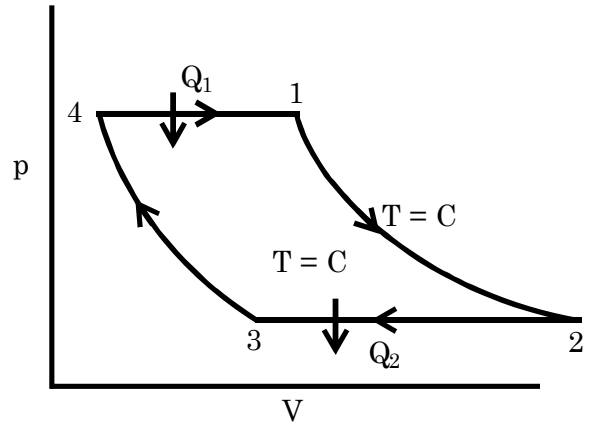
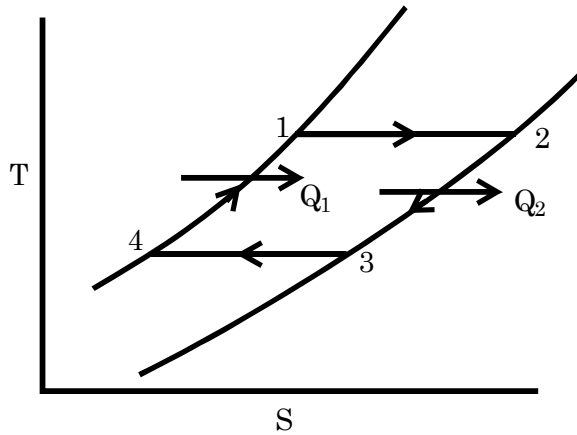
**Show that the air standard efficiency for a cycle comprising two constant pressure processes and two isothermal processes (all reversible) is given by**

$$\eta = \frac{(T_1 - T_2) \ln(r_p)^{(\gamma-1)/\gamma}}{T_1 \left[ 1 + \ln(r_p)^{(\gamma-1)/\gamma} - T_2 \right]}$$

Where  $T_1$  and  $T_2$  are the maximum and minimum temperatures of the cycle, and  $r_p$  is the pressure ratio.



Solution:



$$r_p = \frac{p_4}{p_3} = \frac{p_1}{p_2} \quad \therefore W_{1-2} = \int p dV = RT_1 \int_1^2 \frac{dV}{V} = RT_1 \ln \frac{V_2}{V_1} = RT_1 \ln r_p$$

$$Q_{1-2} = 0 + W_{1-2}$$

$$W_{3-4} = \int_3^4 p dV = RT_3 \ln \left( \frac{V_4}{V_3} \right) = -RT_3 \ln \left( \frac{V_3}{V_4} \right) = -RT_3 \ln r_p.$$

$$\therefore W_{\text{net}} = W_{1-2} + W_{3-4} = R(T_1 - T_3) \ln \left( \frac{V_2}{V_1} \right) \\ = R(T_1 - T_2) \ln r_p.$$

$$\text{Constant pressure heat addition} = C_p(T_1 - T_4)$$

$$= \frac{\gamma R}{\gamma - 2}(T_1 - T_4) \quad T_1 = T_{\text{max}}$$

$$= \frac{\gamma R}{\gamma - 1}(T_1 - T_2) \quad T_2 = T_{\text{min.}}$$

$$\text{Total heat addition (Q}_1\text{)} = Q_{12} + \text{const Pr.} \\ = RT_1 \ln r_p + \frac{\gamma R}{\gamma - 1}(T_1 - T_2)$$

$$\eta = \frac{R(T_1 - T_2) \ln r_p}{R \left[ (T_1 \ln r_p + \frac{\gamma}{\gamma - 1}(T_1 - T_2)) \right]}$$

$$\text{Multiply } \frac{\gamma - 1}{\gamma}, D^\gamma, N^\gamma$$

$$= \frac{\left( \frac{\gamma - 1}{\gamma} \right) (T_1 - T_2) \ln r_p}{(T_1 - T_2) + \frac{\gamma - 1}{\gamma} T_1 \ln r_p}$$

$$\eta = \frac{(T_1 - T_2) \ln r_p^{\left( \frac{\gamma - 1}{\gamma} \right)}}{T_1 [1 + \ln r_p^{\left( \frac{\gamma - 1}{\gamma} \right)}] - T_2}$$

**Q13.14** Obtain an expression for the specific work done by an engine working on the Otto cycle in terms of the maximum and minimum

Temperatures of the cycle, the compression ratio  $r_k$ , and constants of the working fluid (assumed to be an ideal gas).

Hence show that the compression ratio for maximum specific work output is given by

$$r_k = \left( \frac{T_{\min}}{T_{\max}} \right)^{1/2(1-\gamma)}$$

**Solution:**

$$T_{\min} = T_1$$

$$T_{\max} = T_3$$

$$Q_1 = C_v (T_3 - T_2)$$

$$Q_2 = C_v (T_4 - T_1)$$

$$\begin{aligned} \therefore W &= Q_1 - Q_2 \\ &= C_v [(T_3 - T_2) - (T_4 - T_1)] \end{aligned}$$

$$\text{Hence } \frac{T_2}{T_1} = \left( \frac{v_1}{v_2} \right)^{\gamma-1} = r_c^{\gamma-1}$$

$$\therefore T_2 = T_1 r_c^{\gamma-1}$$

$$\text{And } \frac{T_4}{T_3} = \left( \frac{v_3}{v_4} \right)^{\gamma-1} = \left( \frac{v_2}{v_1} \right)^{\gamma-1} = r_c^{-(\gamma-1)} \quad \text{Let } r_c^{\gamma-1} = x$$

$$= x$$

$$\therefore T_4 = T_3 \cdot r_c^{-(\gamma-1)} = \frac{T_3}{x}$$

Then

$$W = C_v \left[ T_3 - T_1 x - \frac{T_3}{x} + T_1 \right]$$

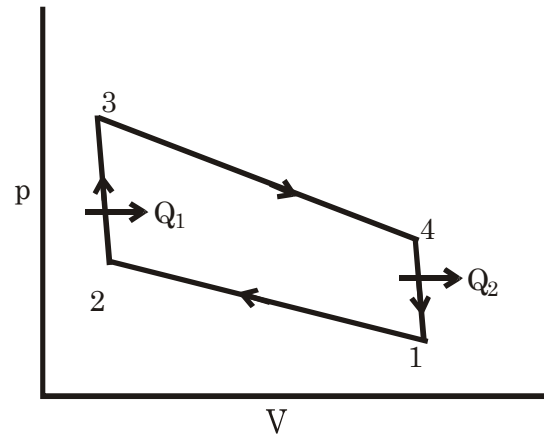
$$\text{For maximum } W, \frac{dW}{dx} = 0$$

$$\therefore C_v \left[ 0 - T_1 + \frac{T_3}{x^2} + 0 \right] = 0$$

$$\therefore x^2 = \frac{T_3}{T_1}$$

$$\therefore r_c^{\gamma-1} = \sqrt{\frac{T_3}{T_1}} = \sqrt{\frac{T_{\max}}{T_{\min}}}$$

$$\therefore r_c = \left( \frac{T_{\max}}{T_{\min}} \right)^{\frac{1}{2(\gamma-1)}} = \left( \frac{T_{\min}}{T_{\max}} \right)^{\frac{1}{2(1-\gamma)}} \quad \text{Proved.}$$



**Q13.15** A dual combustion cycle operates with a volumetric compression ratio  $r_k = 12$ , and with a cut-off ratio 1.615. The maximum pressure is given by  $p_{\max} = 54 p_1$  where  $p_1$  is the pressure before compression. Assuming

indices of compression and expansion of 1.35, show that the m.e.p. of the cycle

$$p_m = 10 p_1$$

Hence evaluate (a) temperatures at cardinal points with  $T_1 = 335$  K, and (b) Cycle efficiency.

(Ans. (a)  $T_2 = 805$  K,  $p_2 = 29.2$   $p_1$ ,  $T_3 = 1490$  K,  $T_4 = 2410$  K,  $T_5 = 1200$  K, (b)  $\eta = 0.67$ )

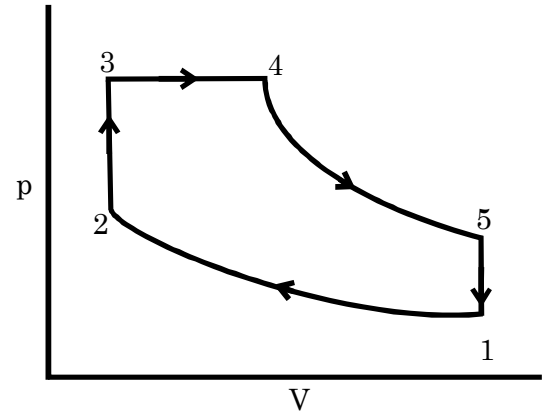
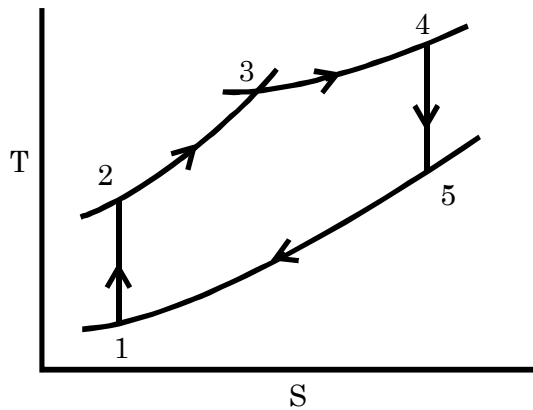
**Solution:**

Here  $\frac{v_1}{v_2} = r_c = 12$

$$\frac{v_4}{v_3} = \rho = 1.615$$

$$pv^{1.35} = C, n = 1.35$$

$$p_{\max} = p_3 = p_4 = 54 p_1$$



$$\therefore \frac{T_2}{T_1} = \left( \frac{v_1}{v_2} \right)^{n-1} \therefore T_2 = T_1 \times (12)^{(1.35-1)} = 2.3862 T_1$$

And  $\frac{p_2}{p_1} = \left( \frac{v_1}{v_2} \right)^n \therefore p_2 = p_1 \times (12)^{1.35} = 28.635 p_1$

$$\frac{p_2}{T_2} = \frac{p_3}{T_3} \therefore T_3 = \frac{p_3}{T_2} \times T_2 = \frac{54 p_1}{28.635 p_1} \times 2.3862 T_1 = 4.5 T_1$$

$$v_3 = v_2 = \left( \frac{v_1}{12} \right)$$

$$\therefore v_4 = \rho v_3 = \frac{1.615}{12} v_1 = 0.13458 v_1$$

$$\therefore \frac{p_4 v_4}{T_4} = \frac{p_3 v_3}{T_3} \quad p_3 = p_4$$

$$T_4 = T_3 \times \frac{v_4}{v_3} = 1.615 T_3 = 1.615 \times 4.5 T_1 = 7.2675 T_1$$

$$\therefore \frac{T_5}{T_4} = \left( \frac{v_4}{v_5} \right)^{n-1} = \left( \frac{v_4}{v_1} \right)^{n-1}$$

$$\therefore T_5 = 3.6019 T_1$$

$$\therefore W = [C_v(T_3 - T_2) + C_p(T_4 - T_3) - C_v(T_5 - T_1)] = 2.4308 T_1 \text{ kJ/kg.}$$

$$p_m (v_1 - v_2) = W$$

$$\therefore p_m = \frac{2.4308 T_1}{v_1 - \frac{v_1}{12}} = \frac{2.4308 p_1}{\frac{11}{12} \times R} = 9.25 p_1$$

$$(b) \therefore \eta = \frac{2.4308 T_1}{4.299 T_1} \times 100\% = 56.54\%$$

$$(a) \quad T_1 = 335 \text{ K}, T_2 = 799.4 \text{ K}, T_3 = 1507.5 \text{ K}, T_4 = 2434.6 \text{ K}, \\ T_5 = 1206.6 \text{ K}.$$

**Q13.16** Recalculate (a) the temperatures at the cardinal points, (b) the m.e.p., and (c) the cycle efficiency when the cycle of Problem 13.15 is a Diesel cycle with the same compression ratio and a cut-off ratio such as to give an expansion curve coincident with the lower part of that of the dual cycle of Problem 13.15.

(Ans. (a)  $T_2 = 805 \text{ K}$ ,  $T_3 = 1970 \text{ K}$ ,  $T_4 = 1142 \text{ K}$

(b)  $6.82 p_1$ , (c)  $\eta = 0.513$ )

**Solution:**

Given  $\frac{v_1}{v_2} = 12 = r_c$

$$\frac{v_3}{v_2} = \rho = 1.615$$

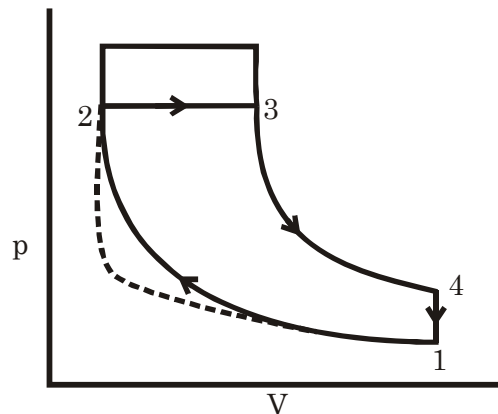
$$\therefore T_3 = \frac{v_3}{v_2} \times T_2 = 1.615 \times 799.4 = 1291 \text{ K}$$

$$\text{Then } \frac{T_2}{T_1} = \left( \frac{v_1}{v_2} \right)^{n-1}$$

$$\therefore T_2 = T_1 (12)^{1.35-1} = 799.4 \text{ K}$$

$$\text{But } \frac{p_2}{p_1} = \left( \frac{v_1}{v_2} \right)^n$$

Continue to try.....



**Q13.19** In a gas turbine plant working on the Brayton cycle the air at the inlet is at  $27^\circ\text{C}$ ,  $0.1 \text{ MPa}$ . The pressure ratio is  $6.25$  and the maximum temperature is  $800^\circ\text{C}$ . The turbine and compressor efficiencies are each  $80\%$ . Find (a) the compressor work per kg of air, (b) the turbine work per kg of air, (c) the heat supplied per kg of air, (d) the cycle efficiency, and (e) the turbine exhaust temperature.

(Ans. (a)  $259.4 \text{ kJ/kg}$ , (b)  $351.68 \text{ kJ/kg}$ , (c)  $569.43 \text{ kJ/kg}$ ,  
(d)  $16.2\%$ , (e)  $723 \text{ K}$ )

**Solution:** Maximum Temperature

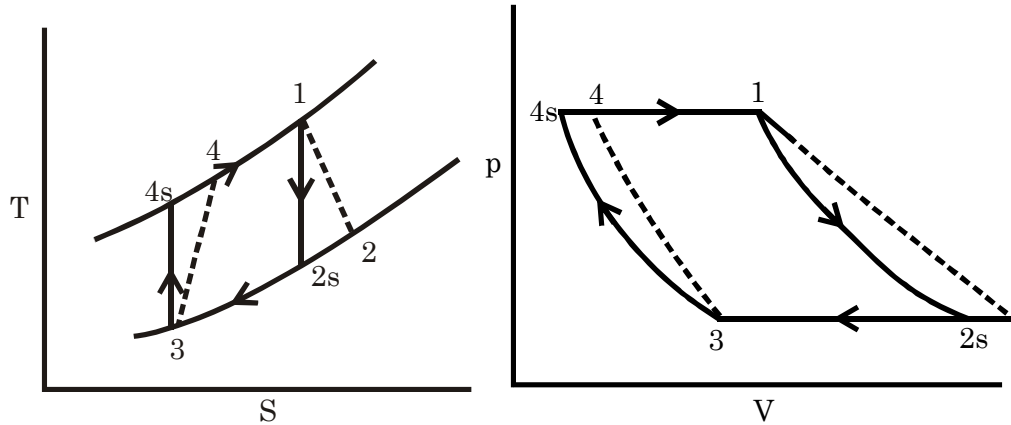
$$T_1 = 800^\circ \text{C} = 1073 \text{ K}$$

$$p_3 = 100 \text{ kPa}$$

$$T_3 = 300 \text{ K}$$

$$r_p = 6.25$$

$$\frac{p_4}{p_3} = 6.25$$



$$\therefore p_4 = 625 \text{ kPa}$$

$$p_2 = 100 \text{ kPa}$$

$$p_1 = p_4$$

$$\therefore v_3 = \frac{RT_3}{p_3} = 0.861$$

$$v_3 = 0.861$$

$$\frac{p_4}{p_3} = \left( \frac{v_3}{v_4} \right)^\gamma \therefore \frac{v_4}{v_3} = \left( \frac{p_3}{p_4} \right)^{\frac{1}{\gamma}}$$

$$T_3 = 300 \text{ K}$$

$$v_4 = v_3 \times \left( \frac{p_3}{p_4} \right)^{\frac{1}{\gamma}}$$

$$p_2 = p_3$$

$$\frac{T_4}{T_3} = \left( \frac{v_3}{v_4} \right)^{\gamma-1}$$

$$p_4 = 625 \text{ kPa}$$

$$\therefore T_4 = T_3 \times (3.70243)^{0.4}$$

$$v_{4s} = 0.23255$$

$$\therefore 0.8 = \frac{T_{4s} - T_3}{T_4 - T_3}$$

$$\therefore T_4 = 558$$

$$T_{4s} = 506.4 \text{ K}$$

$$T_{2s} = 635.6 \text{ K}$$

$$\frac{T_1}{T_{2s}} = \left( \frac{p_1}{p_2} \right)^{\frac{\gamma-1}{\gamma}} = \left( \frac{p_4}{p_3} \right)^{\frac{\gamma-1}{\gamma}} = 1.68808$$

$$T_4 = 558 \text{ K}$$

$$T_2 = 723 \text{ K}$$

$$\eta = \frac{T_1 - T_2}{T_1 - T_{2s}} \Rightarrow T_1 - T_2 = 350$$

$$\therefore T_2 = T_1 - 350 = 723 \text{ K}$$

$$(a) \text{ Compressor work } (W_c) = (h_4 - h_3) = C_p(T_4 - T_3) = 259.3 \text{ kJ/kg}$$

(b) Turbine work ( $W_T$ ) =  $(h_1 - h_2) = C_p(T_1 - T_2) = 351.75 \text{ kJ/kg}$

(c) Heat supplied ( $Q_1$ ) =  $C_p(T_1 - T_4) = 517.6 \text{ kJ/kg}$

(d) Cycle efficiency ( $\eta$ ) =  $\frac{W_T - W_C}{Q_1} = 17.86\%$

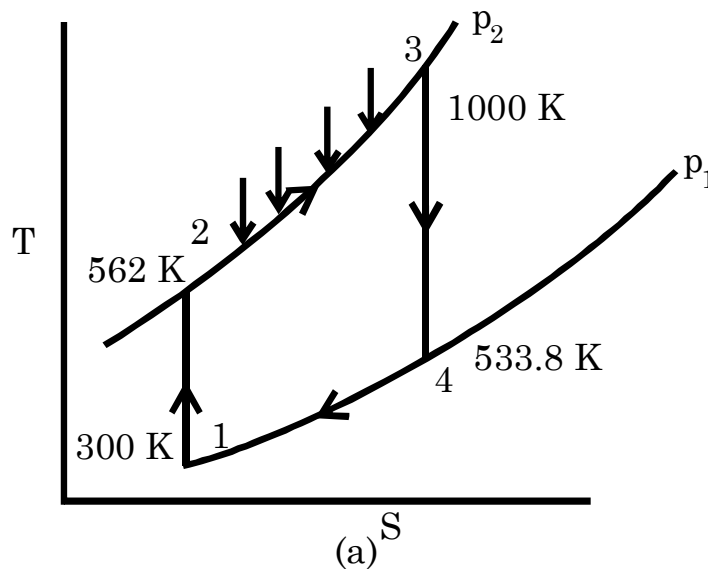
(e) Turbine exhaust temperature ( $T_2$ ) = 723 K

**Q13.27**

A simple gas turbine plant operating on the Brayton cycle has air inlet temperature  $27^\circ\text{C}$ , pressure ratio 9, and maximum cycle temperature  $727^\circ\text{C}$ . What will be the improvement in cycle efficiency and output if the turbine process is divided into two stages each of pressure ratio 3, with intermediate reheating to  $727^\circ\text{C}$ ?

(Ans. - 18.3%, 30.6%)

**Solution:**



For (a)

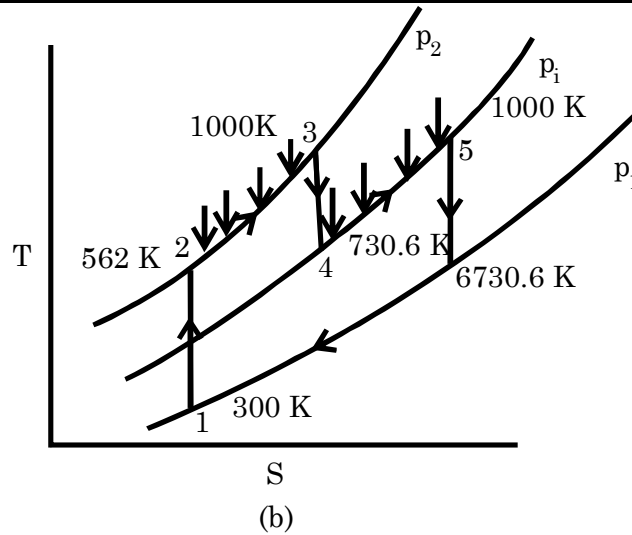
$$T_1 = 300 \text{ K}$$

$$\frac{p_2}{p_1} = 9$$

$$T_3 = 1000 \text{ K}$$

$$\therefore T_2 = \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \times T_1 = 562 \text{ K}$$

$$\frac{T_4}{T_3} = \left( \frac{p_4}{p_3} \right)^{\frac{\gamma-1}{\gamma}} = \left( \frac{p_1}{p_2} \right)^{\frac{\gamma-1}{\gamma}} = \left( \frac{1}{9} \right)^{\frac{\gamma-1}{\gamma}} \quad \therefore T_4 = \frac{T_3}{9^{\frac{\gamma-1}{\gamma}}} = 533.8 \text{ K}$$



$$\text{For (b)} \quad \frac{T_4}{T_3} = \left( \frac{p_1}{p_2} \right)^{\frac{\gamma-1}{\gamma}} \quad \therefore T_4 = T_3 \times \left( \frac{1}{3} \right)^{\frac{\gamma-1}{\gamma}} = 730.6 \text{ K}$$

$$\frac{T_6}{T_5} = \left( \frac{p_1}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \quad \therefore T_6 = T_5 \times \left( \frac{1}{3} \right)^{\frac{\gamma-1}{\gamma}} = 730.6 \text{ K}$$

$$\begin{aligned} \therefore \text{For (a)} \quad W &= (h_3 - h_4) - (h_2 - h_1) \\ &= C_p [T_3 - T_4] - (T_2 - T_1) = 205.22 \text{ kJ/kg} \\ Q &= h_3 - h_2 = C_p (T_3 - T_2) = 440.19 \text{ kJ/kg} \\ \therefore \eta &= 46.62 \% \end{aligned}$$

$$\begin{aligned} \text{For (b)} \quad W &= (h_3 - h_4) + (h_5 - h_6) - (h_2 - h_1) \\ &= C_p [(T_3 - T_4) + (T_5 - T_6) - (T_2 - T_1)] = 278.18 \text{ kJ/kg} \\ Q &= h_3 - h_2 + h_5 - h_4 = C_p [(T_3 - T_2) + (T_5 - T_4)] = 710.94 \text{ kJ/kg} \\ \therefore \eta &= 39.13 \% \end{aligned}$$

$$\therefore \text{Efficiency change} = \frac{39.13 - 46.62}{46.62} \times 100\% = -16.07 \%$$

$$\text{Work output change} = \frac{278.18 - 205.22}{205.22} \times 100 = 35.6\%$$

**Q13.28** Obtain an expression for the specific work output of a gas turbine unit in terms of pressure ratio, isentropic efficiencies of the compressor and turbine, and the maximum and minimum temperatures,  $T_3$  and  $T_1$ .

Hence show that the pressure ratio  $r_p$  for maximum power is given by

$$r_p = \left( \eta_T \eta_C \frac{T_3}{T_1} \right)^{\frac{1}{2(\gamma-1)}}$$

If  $T_3 = 1073 \text{ K}$ ,  $T_1 = 300 \text{ K}$ ,  $\eta_c = 0.8$ ,  $\eta_T = 0.8$  and  $\gamma = 1.4$  compute the optimum

Value of pressure ratio, the maximum net work output per kg of air, and corresponding cycle efficiency.

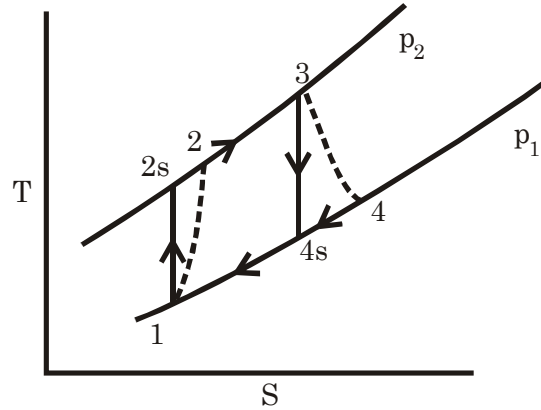
(Ans. 4.263, 100 kJ/kg, 17.2%)

**Solution:**

$$T_1 = T_{\min} \quad T_3 = T_{\max}$$

Hence

$$\begin{aligned} T_{2s} &= T_1 \times \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \\ &= T_1 \times r_p^{\frac{\gamma-1}{\gamma}} \end{aligned}$$



$$\text{Let } r_p^{\frac{\gamma-1}{\gamma}} = x$$

$$\therefore T_{2s} = x T_1$$

$\therefore$  If isentropic efficiency and compressor is  $\eta_c$

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1}$$

$$\therefore T_2 = T_1 + \frac{T_{2s} - T_1}{\eta_c} = T_1 \left[ 1 + \frac{x - 1}{\eta_c} \right]$$

$$\text{Similarly } T_{4s} = T_3 \left( \frac{p_4}{p_3} \right)^{\frac{\gamma-1}{\gamma}} = T_3 \left( \frac{p_1}{p_2} \right)^{\frac{\gamma-1}{\gamma}} = \frac{T_3}{x}$$

$\therefore$  If isentropic efficiency of turbine is  $\eta_T$

$$\text{Then } \eta_T = \frac{T_3 - T_4}{T_3 - T_{4s}} \Rightarrow -T_3 + T_4 = \eta_T (T_{4s} - T_3)$$

$$\begin{aligned} T_4 &= T_3 + \eta_T \left( \frac{T_3}{x} - T_3 \right) \\ &= T_3 \left[ 1 + \eta_T \left( \frac{1}{x} - 1 \right) \right] \end{aligned}$$



∴ Specific work output

$$\begin{aligned}
 W &= (h_3 - h_4) - (h_2 - h_1) \\
 &= C_P [(T_3 - T_4) - (T_2 - T_1)] \\
 &= C_P \left[ \eta_T \left( T_3 - \frac{T_3}{x} \right) - \frac{xT_1 - T_1}{\eta_c} \right] \text{kJ/kg} \\
 &= C_P \left[ \eta_T T_{\max} \left( 1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}} \right) - \frac{T_{\min}}{\eta_c} \left( r_p^{\frac{\gamma-1}{\gamma}} - 1 \right) \right] \text{kJ/kg}
 \end{aligned}$$

For maximum Sp. Work  $\frac{dW}{dx} = 0$

$$\therefore \frac{dW}{dx} = C_P \left[ \frac{\eta_T T_3}{x^2} - \frac{T_1}{\eta_c} \right] = 0$$

$$\therefore x^2 = \eta_T \eta_c \frac{T_3}{T_1}$$

$$\therefore x = \sqrt{\eta_T \eta_c \frac{T_{\max}}{T_{\min}}}$$

$$\therefore r_p = \left( \eta_T \eta_c \frac{T_{\max}}{T_{\min}} \right)^{\frac{\gamma}{2(\gamma-1)}} \text{ Proved.}$$

⇒ If  $T_3 = 1073 \text{ K}$ ,  $T_1 = 300 \text{ K}$ ,  $\eta_l = 0.8$ ,  $\eta_r = 0.8$ ,  $\gamma = 1.4$  then

$$(r_p)_{\text{opt.}} = \left( 0.8 \times 0.8 \times \frac{1073}{300} \right)^{\frac{1.4}{2(1.4-1)}} = 4.26$$

$$(r_p)_{\text{opt.}}^{\frac{\gamma-1}{\gamma}} = x = 1.513$$

$$\begin{aligned}
 \therefore W_{\max} &= C_p \left[ \eta_T T_3 \left( 1 - \frac{1}{x} \right) - \frac{T_1(x-1)}{\eta_c} \right] \\
 &= 1.005 \left[ 0.8 \times 1073 \left( 1 - \frac{1}{1.513} \right) - \frac{300}{0.08} (1.513 - 1) \right] \text{kJ/kg} \\
 &= 99.18 \text{ kJ/kg}
 \end{aligned}$$

$$\begin{aligned}
 \text{Heat input } Q_1 &= h_3 - h_2 = C_p (T_3 - T_2) & T_2 &= T_1 \left[ 1 + \frac{x-1}{\eta_c} \right] \\
 &= 1.005 (1073 - 492.4) & &= 492.4 \text{ K} \\
 &= 583.5 \text{ kJ/kg}
 \end{aligned}$$

$$\therefore \eta = \frac{99.18}{583.5} \times 100\% = 17\%$$

**Q13.29**

A gas turbine plant draws in air at 1.013 bar, 10°C and has a pressure ratio of 5.5. The maximum temperature in the cycle is limited to 750°C. Compression is conducted in an uncooled rotary compressor having an isentropic efficiency of 82%, and expansion takes place in a turbine with an isentropic efficiency of 85%. A heat exchanger with an efficiency of 70% is fitted between the compressor outlet and combustion chamber. For an air flow of 40 kg/s, find (a) the overall cycle efficiency, (b) the turbine output, and (c) the air-fuel ratio if the calorific value of the fuel used is 45.22 MJ/kg.

(Ans. (a) 30.4%, (b) 4272 kW, (c) 115)

**Solution:**

$$p_1 = 101.3 \text{ kPa}$$

$$T_1 = 283 \text{ K}$$

$$\frac{p_2}{p_1} = 5.5$$

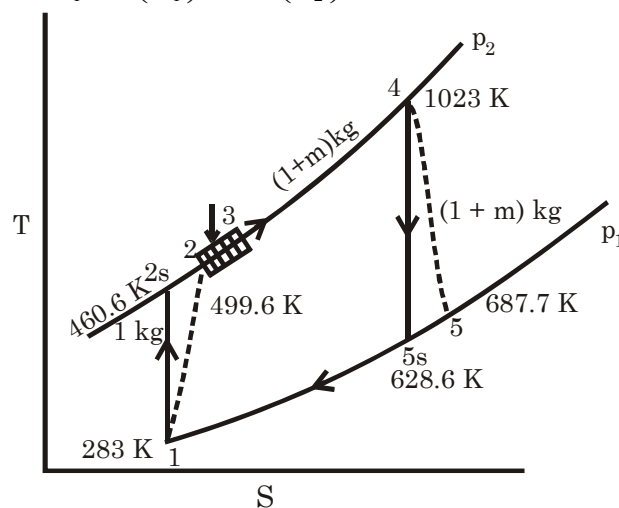
$$T_4 = 750^\circ\text{C} = 1023 \text{ K}$$

$$\therefore \frac{T_{2s}}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_{2s} = 460.6 \text{ K}$$

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1} \therefore T_2 = T_1 + \frac{T_{2s} - T_1}{\eta_c}$$

$$= 499.6 \text{ K}$$

$$\therefore \frac{T_{5s}}{T_4} = \left( \frac{p_5}{p_4} \right)^{\frac{\gamma-1}{\gamma}} = \left( \frac{p_1}{p_2} \right)^{\frac{\gamma-1}{\gamma}} = \left( \frac{1}{5.5} \right)^{\frac{\gamma-1}{\gamma}}$$



$$\therefore T_{5s} = T_4 \times \left( \frac{1}{5.5} \right)^{\frac{1.4-1}{1.4}} = 628.6 \text{ K}$$

$$\eta_T = \frac{T_4 - T_5}{T_4 - T_{5s}} \therefore T_4 - T_5 = \eta_T (T_4 - T_{5s}) = 335.3 \text{ K}$$

$$\therefore T_5 = 687. \text{K}$$

Maximum possible heat from heat exchanger =  $C_p (T_5 - T_2)$

$$\therefore \text{Actual heat from} = 0.7 C_p (T_5 - T_2) = 132.33 \text{ kJ/kg of air}$$

$$\therefore C_p (T_3 - T_2) = (1 + m) 132.33 \text{ and}$$

$$C_p T_3 = 132.33 + 132.33 m + C_p T_2 = 634.43 + 132.33 m$$

$$\text{Heat addition } (Q_1) = C_p (T_4 - T_3) = C_p T_4 - C_p T_3$$

$$= 393.7 - 132.33m = m \times 45.22 \times 10^3$$

$$\therefore m = 8.68 \times 10^{-3} \text{ kJ/kg of air}$$

$$\therefore Q_1 = 392.6 \text{ kJ/kg of air}$$

$$W_T = (1 + m) (h_4 - h_5) = (1 + m) C_p (T_4 - T_5)$$

$$= 1.00868 \times 1.005 \times (1023 - 687.7) \text{ kJ/kg of air} = 340 \text{ kJ/kg}$$

$$W_c = (h_2 - h_1) = C_p (T_2 - T_1) = 1.005 \times (499.6 - 283)$$

$$= 217.7 \text{ kJ/kg of air}$$

$$\therefore W_{\text{net}} = W_T - W_c = 122.32 \text{ kJ/kg}$$

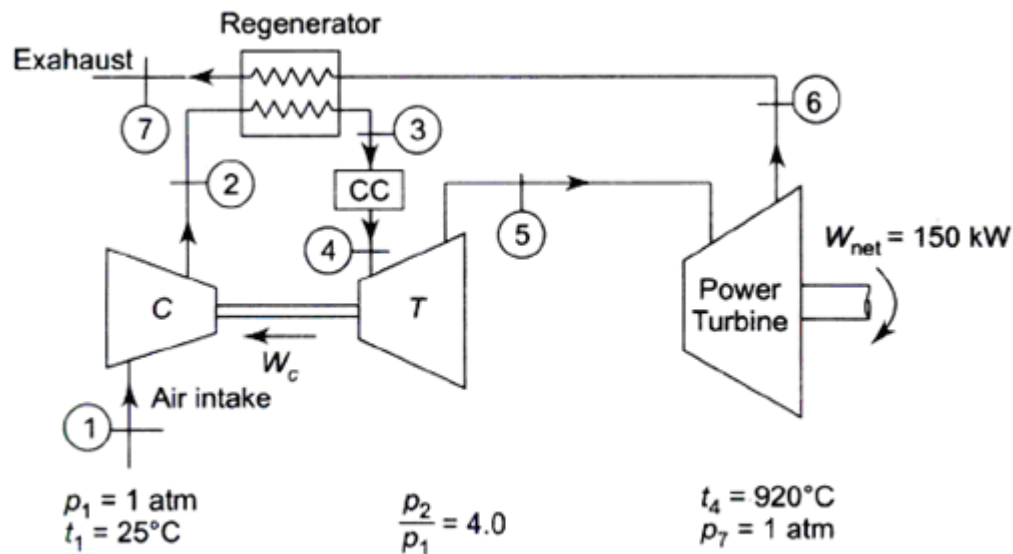
$$(a) \quad \eta = \frac{122.32}{392.6} \times 100\% = 31.16\%$$

$$(b) \text{ Turbine output} = (W_T) = 122.32 \text{ kJ/kg of air} \\ = 4893 \text{ kW}$$

$$(c) \text{ Air fuel ratio} = \frac{1 \text{ kg air}}{0.00868 \text{ kg of fuel}} = 115.2 \text{ kg of air/kg of fuel}$$

**Q13.30**

A gas turbine for use as an automotive engine is shown in Fig. 13.43. In the first turbine, the gas expands to just a low enough pressure  $p_5$ , for the turbine to drive the compressor. The gas is then expanded through a second turbine connected to the drive wheels. Consider air as the working fluid, and assume that all processes are ideal. Determine (a) pressure  $p_5$  (b) the net work per kg and mass flow rate, (c) temperature  $T_3$  and cycle thermal efficiency, and (d) the  $T - S$  diagram for the cycle.



**Fig. .**

**Solution :** Try please.

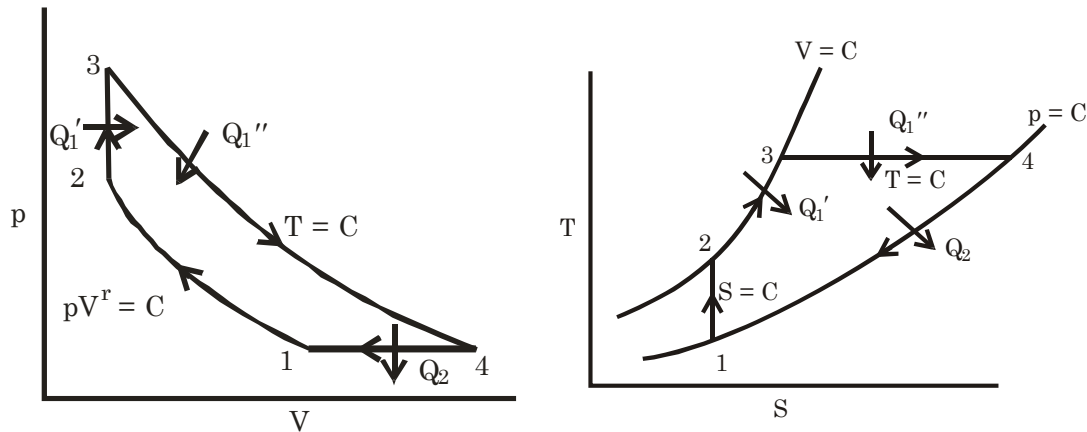
**Q13.31** Repeat Problem 13.30 assuming that the compressor has an efficiency of 80%, both the turbines have efficiencies of 85%, and the regenerator has an efficiency of 72%.

**Solution:** Try please.

**Q13.32** An ideal air cycle consists of isentropic compression, constant volume heat transfer, isothermal expansion to the original pressure, and constant pressure heat transfer to the original temperature. Deduce an expression for the cycle efficiency in terms of volumetric compression ratio  $r_c$ , and isothermal expansion ratio,  $r_k$ . In such a cycle, the pressure and temperature at the start of compression are 1 bar and 40°C, the compression ratio is 8, and the maximum pressure is 100 bar. Determine the cycle efficiency and the m.e.p.

(Ans. 51.5%, 3.45 bar)

**Solution:**



$$\therefore \text{Compression ratio, } r_c = \frac{V_1}{V_2}$$

$$\text{Expansion ratio, } r_e = \frac{V_4}{V_3} = \frac{V_4}{V_2}$$

$$\begin{aligned} \text{Heat addition } Q_1 &= Q_{1'} + Q_{1''} \\ &= \text{constant volume heat addition} \quad (Q_{1'} + \text{constant temperature heat addition } Q_{1''}) \end{aligned}$$

$$\text{Heat rejection, } Q_2 = C_p (T_4 - T_1)$$

$$\text{Hence } \frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} = \left( \frac{v_1}{v_2} \right)^{\gamma-1} = r_c^{\gamma-1}$$

$$\therefore T_2 = T_1 \cdot r_c^{\gamma-1} \text{ and } v_2 = \frac{v_1}{r_c} \text{ and } p_2 = p_1 r_c^\gamma$$

$$T_3 = T_4$$

$$\therefore \frac{p_1 v_3}{T_3} = \frac{p_4 v_4}{T_4} \quad \therefore \frac{v_4}{v_3} = \frac{p_3}{p_4} = \frac{p_3}{p_1} = r_e$$

$$\therefore p_3 = p_1 \cdot r_e$$

$$\begin{aligned} \therefore \frac{p_2}{T_2} &= \frac{p_3}{T_3} \quad \text{or} \quad T_3 = \frac{p_3}{p_2} \times T_2 = \frac{r_e}{r_c^\gamma} \times T_1 \cdot r_c^{\gamma-1} = T_1 \frac{r_e}{r_c} = T_4 \\ \therefore \eta &= 1 - \frac{Q_2}{Q_1} = 1 - \frac{C_p(T_4 - T_1)}{C_v(T_3 - T_2) + R T_3 \ln r_e} \\ &= 1 - \frac{C_p \left( T_1 \cdot \frac{r_e}{r_c} - T_1 \right)}{C_v \left( T_1 \cdot \frac{r_e}{r_c} - T_1 r_c^{\gamma-1} \right) + R \cdot T_1 \frac{r_e}{r_c} \ln r_e} \\ &= 1 - \frac{\gamma \left( \frac{r_e}{r_c} - 1 \right)}{\left( \frac{r_e}{r_c} - r_c^{\gamma-1} \right) + (\gamma - 1) \frac{r_e}{r_c} \ln r_e} \\ &= 1 - \frac{\gamma(r_e - r_c)}{(r_e - r_c^\gamma) + (\gamma - 1) r_e \ln r_e} \\ \therefore \eta &= 1 - \frac{\gamma[r_e - r_c]}{(r_e - r_c^\gamma) + (\gamma - 1) r_e \ln r_e} \end{aligned}$$

Given  $p_1 = 1 \text{ bar} = 100 \text{ kPa}$

$T_1 = 40^\circ\text{C} = 313 \text{ K}$

$r_c = 8$  and  $p_3 = 100 \text{ bar} = 10000 \text{ kPa}$

$$\therefore p_3 = p_1 \cdot r_e \quad \therefore r_e = \frac{p_3}{p_1} = 100$$

$$\begin{aligned} \therefore \eta &= 1 - \frac{1.4(100 - 8)}{(100 - 8^{1.4}) + (1.4 - 1) \times \ln 100} \\ &= 1 - \frac{128.8}{265.83} \\ &= 0.51548 = 51.548 \% \end{aligned}$$

$$\Rightarrow T_3 = T_1 \times \frac{r_e}{r_c} = \frac{313 \times 100}{8} = 3912.5 \text{ K}$$

$$T_2 = T_1 \times r_c^{\gamma-1} = 719 \text{ K}$$

$$\begin{aligned} \therefore \text{Heat addition, } Q &= C_v (T_3 - T_2) + R T_3 \ln r_e \\ &= 0.718 (3912.5 - 719) + 0.287 \times 3912.5 \times \ln 100 \\ &= 7464 \text{ kJ/kg} \end{aligned}$$

$$\therefore \text{Work, } W = Q \eta = 3847.5 \text{ kJ/kg}$$

$$\therefore p_m (V_4 - V_2) = W \quad \therefore v_4 = 100 v_2 \quad v_2 = \frac{v_1}{r_c}$$

$$\therefore p_m (100 - 1) v_2 = W$$

$$\therefore p_m (99) \times \frac{v_1}{8} = W$$

$$\therefore p_m = \frac{8 \text{ W}}{99 \times v_1} = 346.1 \text{ kPa} \quad v_1 = \frac{RT_1}{p_1} = 0.89831 \text{ kJ/kg}$$

$$= 3.461 \text{ bar}$$

$$\therefore p_m (v_4 - v_3) = 4058$$

$$\therefore p_m = \frac{4058}{v_4 - \frac{v_4}{100}} = 365 \text{ bar}$$

**Q13.37** Show that the mean effective pressure,  $p_m'$  for the Otto cycle is  
Given by

$$p_m = \frac{(p_3 - p_1 r_k^\gamma) \left(1 - \frac{1}{r_k^{\gamma-1}}\right)}{(\gamma - 1)(r_k - 1)}$$

Where  $p_3 = p_{\max}$ ,  $p_1 = p_{\min}$  and  $r_k$  is the compression ratio.

**Solution:**

Intake  $p_1, v_1, T_1$

$$\therefore \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = r_c^{\gamma-1}$$

$$\therefore T_2 = T_1 \cdot r_c^{\gamma-1}$$

$$p_2 = p_1 \times r_c^\gamma$$

$$v_2 = \frac{v_1}{r_c}$$

$$\frac{p_3}{T_3} = \frac{p_2}{T_2}$$

$$\therefore T_3 = T_2 \times \frac{p_3}{p_2} = T_2 \times \frac{p_3}{p_1 r_c^\gamma} = T_1 \frac{r_c^{\gamma-1} \times p_3}{p_1 r_c^\gamma} = \frac{T_1 \times p_3}{r_c p_1}$$

$$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma-1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = r_c^{\gamma-1}$$

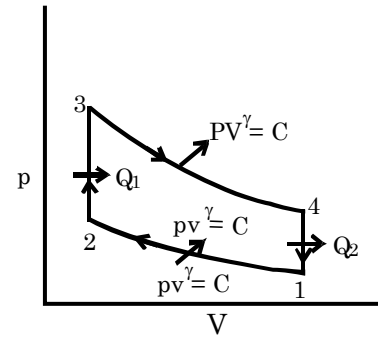
$$\therefore T_4 = \frac{T_3}{r_c^{\gamma-1}} = \frac{T_1 p_3}{r_c p_1 \times r_c^{\gamma-1}} = \frac{T_1 p_3}{r_c^\gamma p_1}$$

$$W = Q_1 - Q_2$$

$$= C_v (T_3 - T_2) - C_v (T_4 - T_1)$$

$$\therefore p_m (V_1 - V_2) = W$$

$$\therefore p_m = \frac{C_v [(T_3 - T_2) - (T_4 - T_1)]}{V_1 - V_2}$$



$$= \frac{c_v \left[ \frac{T_1 p_3}{r_c p_1} - T_1 r_c^{\gamma-1} - \frac{T_1 p_3}{r_c^{\gamma} p_1} + T_1 \right]}{v_1 - \frac{v_1}{r_c}}$$

$$= \frac{c_v T_1}{V_1 p_1} \left[ \frac{p_3 - p_1 r_c^{\gamma} - \frac{p_3}{r_c^{\gamma-1}} + p_1 r_c}{(r_c - 1)} \right]$$

$$\left[ \begin{aligned} c_v &= \frac{R}{\gamma - 1} \\ \because p_1 V_1 &= RT_1 \end{aligned} \right]$$

$$= \frac{RT_1}{V_1 p_1} \frac{[(p_3 - p_1 r_c^{\gamma}) - \frac{p_3}{r_c^{\gamma-1}} + (p_3 - p_1 r_c^{\gamma})]}{(\gamma - 1)(r_c - 1)}$$

$$= \frac{(p_3 - p_1 r_c^{\gamma}) \left( 1 - \frac{1}{r_c^{\gamma-1}} \right)}{(\gamma - 1)(r_c - 1)} \text{ Proved}$$

**Q13.38**

**A gas turbine plant operates on the Brayton cycle using an optimum pressure ratio for maximum net work output and a regenerator of 100% effectiveness. Derive expressions for net work output per kg of air and corresponding efficiency of the cycle in terms of the maximum and the minimum temperatures.**

**If the maximum and minimum temperatures are 800°C and 30°C respectively, compute the optimum value of pressure ratio, the maximum net work output per kg and the corresponding cycle efficiency.**

$$(\text{Ans. } (W_{net})_{\max} = C_p (\sqrt{T_{\max}} - \sqrt{T_{\min}})^2 (\eta_{cycle})_{\max} = 1 - \sqrt{\frac{T_{\min}}{T_{\max}}}, (r_p)_{opt} = 9.14$$

$$(W_{net})_{\max} = 236.97 \text{ kJ/kg}; \eta_{cycle} = 0.469)$$

**Solution:**

$$\begin{aligned} T_1 &= T_{\min} \\ T_4 &= T_{\max} \\ \therefore \frac{T_2}{T_1} &= \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} = r_p^{\frac{\gamma-1}{\gamma}} = x \text{ (say)} \\ \therefore T_2 &= T_1 x \\ \frac{T_5}{T_4} &= \left( \frac{p_5}{p_4} \right)^{\frac{\gamma-1}{\gamma}} = \left( \frac{p_1}{p_2} \right)^{\frac{\gamma-1}{\gamma}} = \frac{1}{x} \\ \therefore T_5 &= \frac{T_4}{x} \end{aligned}$$

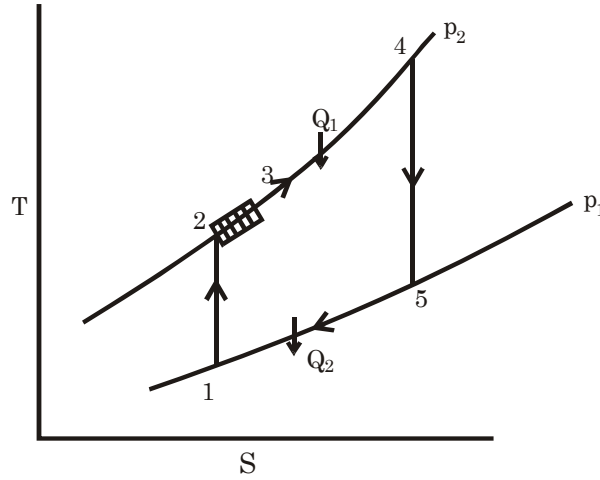
For regeneration 100% effective number

$$C_p (T_5 - T_2) = C_p (T_3 - T_2)$$

$$\therefore T_3 = T_5 = \frac{T_4}{x}$$

$$W_T = h_4 - h_5 = C_p (T_4 - T_5)$$

$$= C_p \left( T_4 - \frac{T_4}{x} \right)$$



$$\begin{aligned} \text{And } W_c &= h_2 - h_1 = C_p T_4 \left( 1 - \frac{1}{x} \right) \\ &= C_p (T_2 - T_1) \\ &= C_p T_1 (x - 1) \end{aligned}$$

$$\therefore W_{\text{net}} = W_T - W_C = C_p \left[ T_4 \left( 1 - \frac{1}{x} \right) - T_1 (x - 1) \right]$$

For Maximum Net work done

$$\frac{\partial W_{\text{net}}}{\partial x} = 0 \therefore T_4 \times \frac{1}{x^2} - T_1 = 0$$

$$\therefore x^2 = \frac{T_4}{T_1} = \frac{T_{\text{max}}}{T_{\text{min}}}$$

$$\therefore x = \sqrt{\frac{T_{\text{max}}}{T_{\text{min}}}}$$

$$\text{Heat addition } \therefore (r_p)_{\text{opt.}} = \left( \frac{T_{\text{max}}}{T_{\text{min}}} \right)^{\frac{\gamma}{2(\gamma-1)}}$$

$$Q_1 = h_4 - h_3 = C_p (T_4 - T_3) = C_p \left( T_4 - \frac{T_4}{x} \right)$$

$$= C_p T_4 \left( 1 - \frac{1}{x} \right)$$

$$= C_p T_4 \left[ 1 - \sqrt{\frac{T_1}{T_4}} \right]$$

$$\therefore \eta_{\text{opt.}} = \frac{W_{\text{net}}}{Q_1} = \frac{T_4 \left( 1 - \frac{1}{x} \right) - T_1 (x - 1)}{T_4 \left( 1 - \frac{1}{x} \right)}$$

$$= 1 - \frac{T_1}{T_4} \times x = 1 - \frac{T_1}{T_4} \times \sqrt{\frac{T_4}{T_1}} = 1 - \sqrt{\frac{T_{\text{min}}}{T_{\text{max}}}}$$



$$W_{\text{opt.}} = C_p [T_4 - \sqrt{T_1 T_4} - \sqrt{T_1 T_4} + T_1]$$

$$= C_p [\sqrt{T_4} - \sqrt{T_1}]^2 = C_p [\sqrt{T_{\text{max}}} - \sqrt{T_{\text{min}}}]^2$$

If  $T_{\text{max}} = 800^\circ\text{C} = 1073 \text{ K}$ ;  $T_{\text{min}} = 30^\circ\text{C} = 303 \text{ K}$

$$\therefore r_{\text{p,opt}} = \left( \frac{1073}{303} \right)^{\frac{1.4}{2(1.4-1)}} = 9.14$$

$$\eta_{\text{opt.}} = 1 - \sqrt{\frac{T_{\text{min}}}{T_{\text{max}}}} = 46.9\%$$

$$W_{\text{opt.}} = 1.005 (\sqrt{1073} - \sqrt{303})^2 = 236.8 \text{ kJ/kg}$$

**Q13.40**

**Show that for the Sterling cycle with all the processes occurring reversibly but where the heat rejected is not used for regenerative heating, the efficiency is given: by**

$$\eta = 1 - \frac{\left( \frac{T_1}{T_2} - 1 \right) + (\gamma - 1) \ln r}{\left( \frac{T_1}{T_2} - 1 \right) + (\gamma - 1) \frac{T_1}{T_2} \ln r}$$

Where  $r$  is the compression ratio and  $T_1/T_2$  the maximum to minimum temperature ratio.

Determine the efficiency of this cycle using hydrogen ( $R = 4.307 \text{ kJ/kg K}$ ,  $c_p = 14.50 \text{ kJ/kg K}$ ) with a pressure and temperature prior to isothermal.

Compression of 1 bar and 300 K respectively, a maximum pressure of 2.55 MPa and heat supplied during the constant volume heating of 9300 kJ/kg. If the heat rejected during the constant volume cooling can be utilized to provide the constant volume heating, what will be the cycle efficiency? Without altering the temperature ratio, can the efficiency be further improved in the cycle?

**Solution:**

Minimum temperature

$$(T_2) = T_{\text{min}}$$

Maximum temperature

$$(T_1) = T_{\text{max}}$$

$\therefore$  Compression ratio

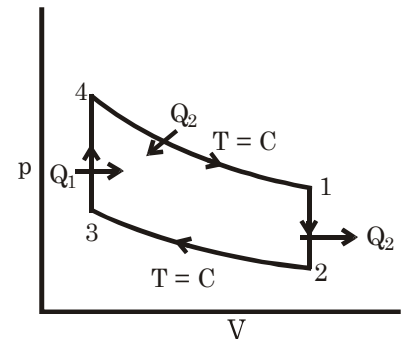
$$(r_c) = \frac{v_2}{v_3} = \frac{v_1}{v_4}$$

$\therefore T_1 - T_4 \quad \text{and} \quad T_3 = T_2$

$$W_T = RT_1 \ln \frac{v_1}{v_4} = RT_1 \ln r_c$$

$$W_C = RT_2 \ln \left( \frac{v_2}{v_3} \right) = RT_2 \ln r_c$$

$$\therefore W_{\text{net}} = R \ln(r_c) \times [T_1 - T_2]$$



$$\begin{aligned}\text{Constant volume Heat addition } (Q_1) &= C_v (T_1 - T_2) \\ &= \frac{R}{\gamma - 1} (T_1 - T_2)\end{aligned}$$

$$\text{Constant temperature heat addition } Q_2 = RT_2 \ln r_c$$

$$\therefore \text{Total heat addition } Q = Q_1 + Q_2 = R \left[ T_1 \ln r_c \frac{(T_1 - T_2)}{(\gamma - 1)} \right]$$

$$\begin{aligned}\eta &= \frac{W_{\text{net}}}{Q} = \frac{\ln r_c [T_1 - T_2]}{\left[ T_1 \ln r_c - \frac{T_2 - T_1}{\gamma - 1} \right]} = \frac{(\gamma - 1) \ln r_c (T_1 - T_2)}{(\gamma - 1) T_1 \ln r_c - (T_2 - T_1)} - 1 + 1 \\ &= 1 - \left[ 1 - \frac{(\gamma - 1) \ln r_c (T_1 - T_2)}{(\gamma - 1) \ln r_c - (T_2 - T_1)} \right] \\ &= 1 - \frac{(\gamma - 1) T_1 \ln r_c - (T_2 - T_1) - (\gamma - 1) \ln r_c T_1 + (\gamma - 1) T_2 \ln r_c}{(\gamma - 1) \ln r_c - (T_2 - T_1)} \\ &= 1 - \frac{(T_1 - T_2) + (\gamma - 1) T_2 \ln r_c}{(T_1 - T_2) + (\gamma - 1) T_1 \ln r_c} \\ &= 1 - \frac{\left( \frac{T_1}{T_2} - 1 \right) + (\gamma - 1) \ln r_c}{\left( \frac{T_1}{T_2} - 1 \right) + (\gamma - 1) \frac{T_1}{T_2} \ln r_c} \quad \text{Proved}\end{aligned}$$

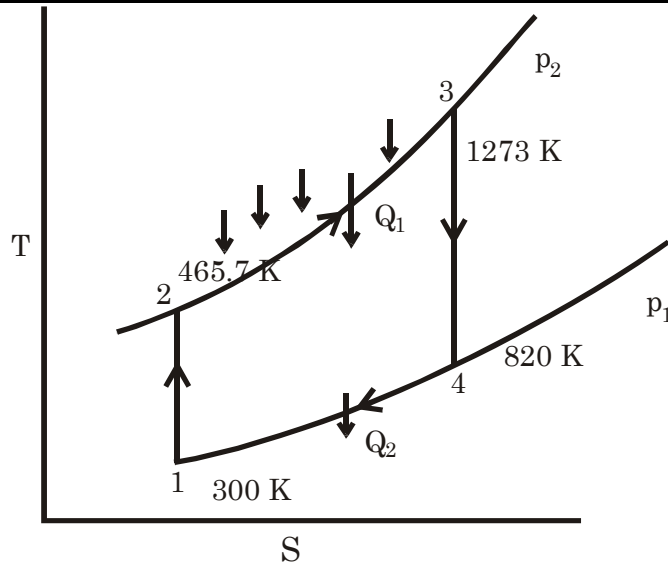
**Q13.41** Helium is used as the working fluid in an ideal Brayton cycle. Gas enters the compressor at 27°C and 20 bar and is discharged at 60 bar. The gas is heated to 1000°C before entering the turbine. The cooler returns the hot turbine exhaust to the temperature of the compressor inlet. Determine: (a) the temperatures at the end of compression and expansion, (b) the heat supplied, the heat rejected and the net work per kg of He, and (c) the cycle efficiency and the heat rate. Take  $c_p = 5.1926 \text{ kJ/kg K}$ .

(Ans. (a) 465.5, 820.2 K, (b) 4192.5, 2701.2, 1491.3 kJ/kg, (c) 0.3557, 10,121 kJ/kWh)

**Solution:** 
$$\frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{\gamma - 1}{\gamma}} = \frac{60}{20}$$

$$\begin{aligned}C_p &= 5.1926, R = 2.0786 \\ \therefore c_v &= c_p - R = 3.114\end{aligned}$$

$$\gamma = \frac{c_p}{c_v} = \frac{5.1926}{3.114} = 1.6675 \quad \therefore \frac{\gamma - 1}{\gamma} = 0.4$$



$$\therefore T_2 = T_1 \times \left(\frac{60}{20}\right)^{\frac{\gamma-1}{\gamma}} = 465.7 \text{ K}$$

$\therefore$

$$\therefore \frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{20}{60}\right)^{\frac{\gamma-1}{\gamma}} \therefore T_4 = T_3 \times \frac{1}{3^{\frac{\gamma-1}{\gamma}}} = 820 \text{ K}$$

- (a) End of compressor temperature  $T_2 = 465.7 \text{ K}$   
End of expansion temperature  $T_4 = 820 \text{ K}$

- (b) Heat supplied ( $Q_1$ ) =  $h_3 - h_2 = C_P (T_3 - T_2) = 4192 \text{ kJ/kg}$   
Heat rejected ( $Q_2$ ) =  $h_4 - h_1 = C_P (T_4 - T_1) = 2700 \text{ kJ/kg}$   
Work,  $W = Q_1 - Q_2 = 1492 \text{ kJ/kg}$

(c)  $\eta = \frac{W}{Q_1} = \frac{1492}{4192} \times 100\% = 35.6\%$

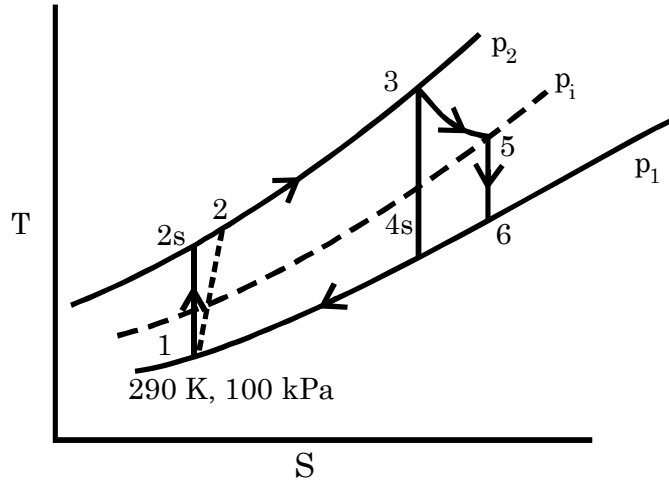
$$\text{Heat rate} = \frac{3600}{\eta} = \frac{3600}{0.356} = 10112 \text{ kJ/kWh}$$

**Q13.42**

An air standard cycle for a gas turbine jet propulsion unit, the pressure and temperature entering the compressor are 100 kPa and 290 K, respectively. The pressure ratio across the compressor is 6 to 1 and the temperature at the turbine inlet is 1400 K. On leaving the turbine the air enters the nozzle and expands to 100 kPa. Assuming that the efficiency of the compressor and turbine are both 85% and that the nozzle efficiency is 95%, determine the pressure at the nozzle inlet and the velocity of the air leaving the nozzle.

(Ans. 285 kPa, 760 m / s)

**Solution:**  $\frac{p_2}{p_1} = 6 \therefore p_2 = 600 \text{ kPa}$



$$\frac{T_{2s}}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} = 6^{\frac{1.4-1}{1.4}}$$

$$T_{2s} = 483.9 \text{ K}$$

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1}$$

$$\therefore T_2 - T_1 = \frac{T_{2s} - T_1}{\eta_c} = 228 \text{ K}$$

$$T_2 = 518 \text{ K}$$

$$T_3 = 1400 \text{ K}$$

$$W_C = C_P (T_2 - T_1) = 1.005 (518 - 290) = 229.14 \text{ kJ/kg}$$

$$\frac{T_{4s}}{T_3} = \left( \frac{p_1}{p_2} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore W_T = \frac{W_C}{\eta_T} = 269.9 \text{ kJ/kg} = C_P (T_3 - T_{4s})$$

$$\therefore T_3 - T_{4s} = 268.24 \quad \therefore T_{4s} = 1131.8 \text{ K}$$

$$\therefore \left( \frac{1131.8}{1400} \right)^{\frac{1.4}{1.4-1}} = \frac{p_1}{p_2}$$

$$\therefore p_1 = p_2 \times \left( \frac{1131.8}{1400} \right)^{\frac{1.4}{1.4-1}} = 285 \text{ kPa}$$

$$\Delta h = h_5 - h_6 = C_P (T_5 - T_6)$$

$$\frac{T_3 - T_5}{T_3 - T_{4s}} = \eta_T \quad \therefore T_3 - T_5 = 227.97 \quad \therefore T_5 = 1172 \text{ K}$$

$$\frac{T_5}{T_6} = \left( \frac{p_5}{p_6} \right)^{\frac{\gamma-1}{\gamma}} = \left( \frac{285}{100} \right)^{\frac{1.4-1}{1.4}} \Rightarrow T_6 = T_5 = 868.9 \text{ K}$$

$$\therefore \Delta h = C_P (1172 - 868.9) = 304.6 \text{ kJ/kg}$$

$$\therefore V = \sqrt{2000 \times \eta \times \Delta h} = \sqrt{2000 \times 0.95 \times 304.6} \text{ m/s} = 760.8 \text{ m/s}$$

**Q13.43** A stationary gas turbine power plant operates on the Brayton cycle and delivers 20 MW to an electric generator. The maximum temperature is 1200 K and the minimum temperature is 290 K. The minimum pressure is 95 kPa and the maximum pressure is 380 kPa. If the isentropic efficiencies of the turbine and compressor are 0.85 and 0.80 respectively, find (a) the mass flow rate of air to the compressor, (b) the volume flow rate of air to the compressor, (c) the fraction of the turbine work output needed to drive the compressor, (d) the cycle efficiency. If a regenerator of 75% effectiveness is added to the plant, what would be the changes in the cycle efficiency and the net work output?

(Ans. (a) 126.37 kg/s, (b) 110.71 m<sup>3</sup>/s, (c) 0.528,  
(d) 0.2146,  $\Delta\eta = 0.148$   $\Delta W_{net} = 0$ )

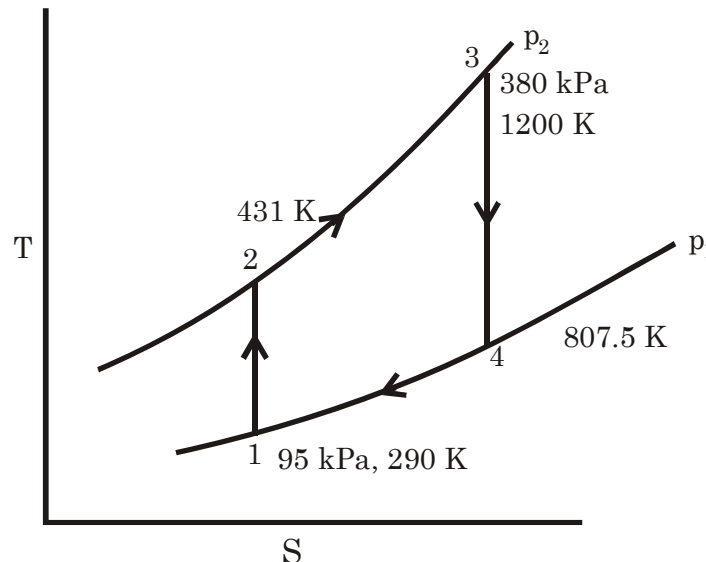
**Solution:**

$$\frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \therefore T_2 = 431 \text{ K}$$

$$\frac{T_4}{T_3} = \left( \frac{p_4}{p_3} \right)^{\frac{\gamma-1}{\gamma}} = \left( \frac{p_1}{p_2} \right)^{\frac{\gamma-1}{\gamma}} ; T_4 = 807.5 \text{ K}$$

$$\begin{aligned} \therefore W_{net} &= (h_3 - h_4) - (h_2 - h_1) \\ &= C_p [(T_3 - T_4) - (T_2 - T_1)] \\ &= 252.76 \text{ kJ/kg} \end{aligned}$$

$$\therefore \text{Mass flow rate } (\dot{m}) = \frac{20000}{252.76} = 79.13 \text{ kg/s}$$



(a) Turbine output ( $W_T$ ) =  $\dot{m} c_p (T_3 - T_4) = 31.234 \text{ MW}$

(b)  $\eta = \frac{W_C}{W_T} = \frac{T_2 - T_1}{T_3 - T_4} = 0.3592$

$$(c) \dot{m} = 79.13 \text{ kg/s}$$

$$(d) v_1 = \frac{RT_1}{p_1} = 0.8761 \text{ m}^3/\text{kg} \quad \therefore \dot{V} = \dot{m}v_1 = 69.33 \text{ m}^3/\text{s}$$

# 14. Refrigeration Cycles

## Some Important Notes

### Heat Engine, Heat Pump

#### Heat engines, Refrigerators, Heat pumps:

- A heat engine may be defined as a device that operates in a thermodynamic cycle and does a certain amount of net positive work through the transfer of heat from a **high temperature body to a low temperature body**. A steam power plant is an example of a heat engine.
- A refrigerator may be defined as a device that operates in a thermodynamic cycle and transfers a certain amount of heat from a body at a **lower temperature to a body at a higher temperature** by consuming certain amount of external work. Domestic refrigerators and room air conditioners are the examples. In a refrigerator, the required output is the heat extracted from the low temperature body.
- A heat pump is similar to a refrigerator, however, here the required output is the heat rejected to the high temperature body.

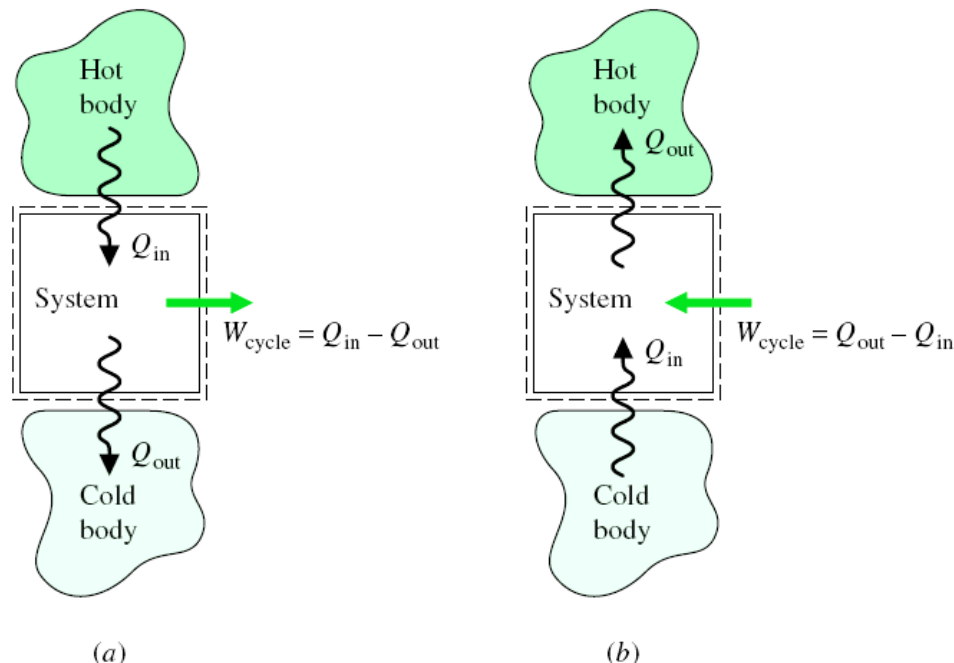
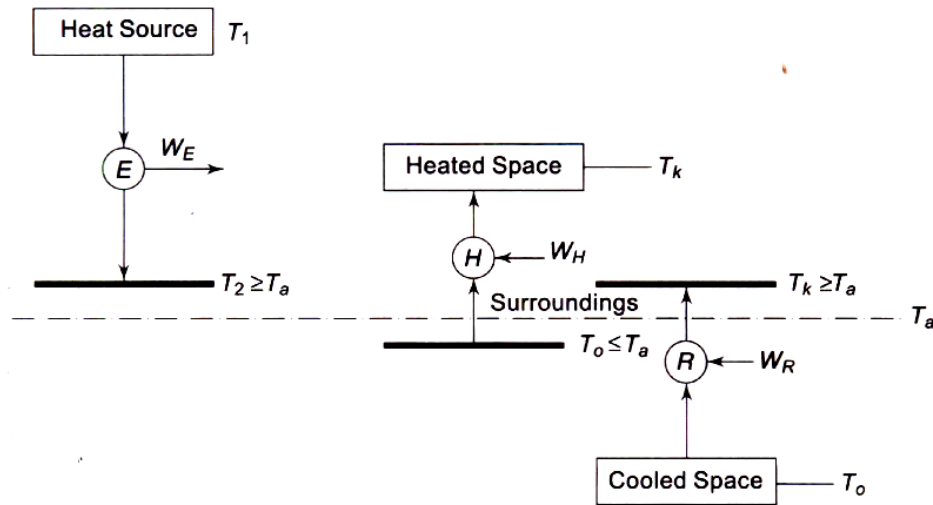


Fig. (a) Heat Engine (b) Refrigeration and heat pump cycles



**Fig. Comparison of heat engine, heat pump and refrigerating machine**

$$\text{COP}_{\text{Carnot,HP}} = \frac{Q_H}{W_{\text{cycle}}} = \frac{Q_H}{Q_H - Q_C} = \frac{T_H}{T_H - T_C}$$

$$\text{COP}_{\text{Carnot,R}} = \frac{Q_C}{W_{\text{cycle}}} = \frac{Q_C}{Q_H - Q_C} = \frac{T_C}{T_H - T_C}$$

Where

- $W_{\text{cycle}}$  = work input to the reversible heat pump and refrigerator
- $Q_H$  = heat transferred between the system and the hot reservoir
- $Q_C$  = heat transferred between the system and cold reservoir
- $T_H$  = temperature of the hot reservoir.
- $T_C$  = temperature of the cold reservoir.



Question and Solution (P K Nag)

**Q14.1** A refrigerator using R-134a operates on an ideal vapour compression cycle between 0.12 and 0.7 MPa. The mass flow of refrigerant is 0.05 kg/s. Determine

- The rate of heat removal from the refrigerated space
- The power input to the compressor
- The heat rejection to the environment
- The COP

(Ans. (a) 7.35 kW, (b) 1.85 kW, (c) 9.20 kW, (d) 3.97)

**Solution:** Try please.

**Q14.2** A Refrigerant-12 vapour compression cycle has a refrigeration load of 3 tonnes. The evaporator and condenser temperatures are  $-20^{\circ}\text{C}$  and  $40^{\circ}\text{C}$  respectively. Find

- The refrigerant flow rate in kg/s
- The volume flow rate handled by the compressor in  $\text{m}^3/\text{s}$
- The work input to the compressor in kW
- The heat rejected in the condenser in kW
- The isentropic discharge temperature.

If there is  $5^{\circ}\text{C}$  of superheating of vapour before it enters the compressor, and  $5^{\circ}\text{C}$  sub cooling of liquid before it flows through the expansion valve, determine the above quantities.

**Solution:** As  $50^{\circ}\text{C}$  temperature difference in evaporate so evaporate temperature =  $-20^{\circ}\text{C}$  and Condenser temperature is  $30^{\circ}\text{C}$ .

$$\therefore p_1 = 1.589 \text{ bar}$$

$$p_2 = 7.450 \text{ bar}$$

$$h_7 = 178.7 \text{ kJ/kg}, \quad h_3 = 64.6 \text{ kJ/kg}$$

$$h_1 = 178.7 + \frac{5}{20} (190.8 - 178.7)$$

$$\Delta h = 3.025 \text{ kJ/kg}$$

$$s_1 = 0.7088 + \frac{5}{20} (0.7546 - 0.7088)$$

$$= 0.7203 \text{ kJ/kg-K}$$

$$\therefore h_3 - h_4 = \Delta h = h_1 - h_7 = 3.025$$

$$\therefore h_4 = h_3 - \Delta h = 61.6 \text{ kJ/kg i.e.} \quad \begin{array}{ll} 25^{\circ}\text{C} & h_g = 59.7 \\ 30^{\circ}\text{C} & h_g = 64.6 \rightarrow 0.98/v_c \end{array}$$

$$\therefore \text{Degree of sub cooling} = 3.06^{\circ}\text{C}$$

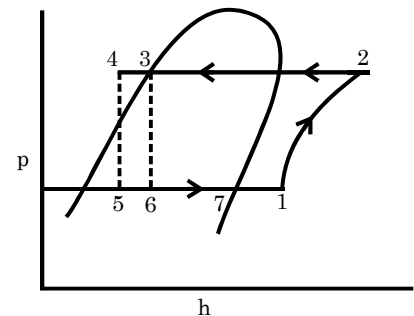
$$(a) \text{ Degree of super heat is discharge} = \frac{0.7203 - 0.6854}{0.7321 - 0.6854} \times 20 = 15^{\circ}\text{C}$$

$$\therefore \text{Discharge temperature} = 15 + 30 = 45^{\circ}\text{C}$$

$$\therefore h_2 = 199.6 + \frac{15}{20} (214.3 - 199.6) = 210.63 \text{ kJ/kg}$$

$$\therefore \text{Compressor work (W)} = h_2 - h_1 = 210.63 - 181.73 = 28.9 \text{ kJ/kg}$$

$$\text{Refrigerating effect (Q}_0\text{)} = h_7 - h_5 = h_7 - h_4 = (178.7 - 61.6) \text{ kJ/kg} = 117.1 \text{ kJ/kg}$$



[Data from CP Arora]

$$(b) \quad \therefore \quad \text{COP} = \frac{Q_o}{W} = \frac{117.1}{28.9} = 4.052$$

$$v_1 = 0.108 \text{ m}^3/\text{kg} \quad \dot{V}_1 = \dot{m}v_1 = 0.014361 \text{ m}^3/\text{s}$$

$$\frac{\pi D^2}{4} \times L \times \frac{N}{60} \times n \times \eta_{\text{vol}} = \dot{V}_1 \quad \frac{L}{D} = 1.2$$

$$L = 1.2 D$$

$$\frac{\pi \times D^2}{4} \times 1.2 D \times \frac{900}{60} \times 1 \times 0.95 = 0.014361$$

$$\therefore D = 0.1023 \text{ m} = 10.23 \text{ cm}$$

$$L = 0.1227 \text{ m} = 12.27 \text{ cm}$$

Q14.4

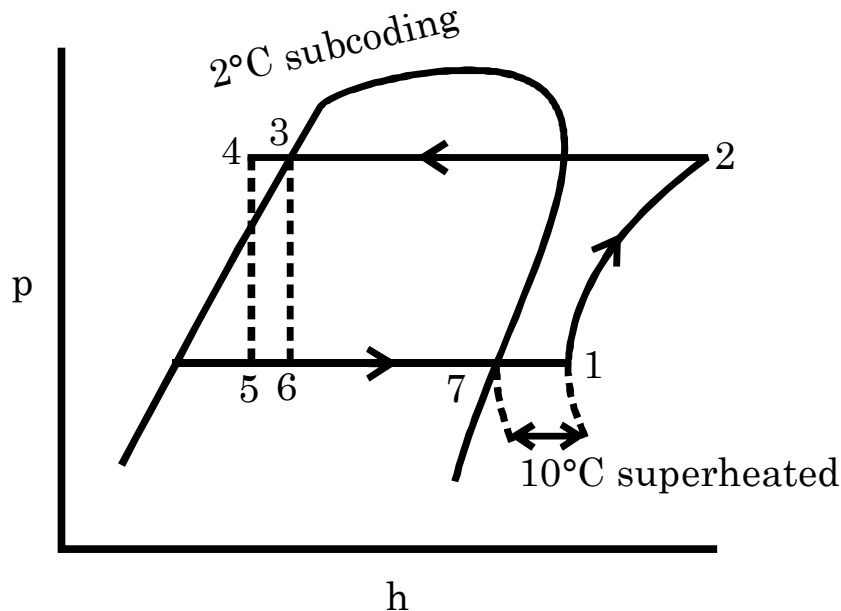
A vapour compression refrigeration system uses R-12 and operates between pressure limits of 0.745 and 0.15 MPa. The vapour entering the compressor has a temperature of  $-10^\circ\text{C}$  and the liquid leaving the condenser is at  $28^\circ\text{C}$ . A refrigerating load of 2 kW is required. Determine the COP and the swept volume of the compressor if it has a volumetric efficiency of 76% and runs at 600 rpm.

(Ans. 4.15, 243 cm<sup>3</sup>)

**Solution:**

$p_1 = 150 \text{ kPa}$ : Constant saturated temperature ( $-20^\circ\text{C}$ )

$p_2 = 745 \text{ kPa}$ : Constant saturated temperature ( $30^\circ\text{C}$ )



$$h_7 = 178.7 \text{ kJ/kg}$$

$$h_3 = 64.6 \text{ kJ/kg}$$

$$h_4 = h_{4-5} = 59.7 + \frac{3}{5} (64.6 - 59.7) = 62.64 \text{ kJ/kg} = h_5$$

$$h_1 = h_7 + \frac{10}{20} (190.8 - h_7) = 184.8 \text{ kJ/kg}$$

$$s_1 = 0.7088 + \frac{10}{20} (0.7546 - 0.7088) = 0.7317 \text{ kJ/kg-K}$$

$$h_2 = 199.6 + \left( \frac{0.7317 - 0.6854}{0.7321 - 0.6854} \right) (214.3 - 199.6) = 214.2 \text{ kJ/kg}$$

$$\therefore \text{Compressor work (W)} = h_2 - h_1 = 29.374 \text{ kJ/kg}$$

$$\text{Refrigeration effect} = (h_1 - h_5) = (184.8 - 62.64) = 122.16 \text{ kJ/kg}$$

$$\therefore \text{COP} = \frac{122.16}{29.374} = 4.16$$

$$v_1 = 0.1166 \text{ m}^3/\text{kg}$$

$$\text{Mass flow ratio } \dot{m} \times 122.16 = 2$$

$$\therefore \dot{m} = 0.016372 \text{ kg/s}$$

$$\therefore \dot{V}_1 = \dot{m} v_1 = 1.90897 \times 10^3 \text{ m}^3/\text{s} = V_s \times 0.76 \times \frac{600}{60}$$

$$\therefore V_s = 251.2 \text{ cm}^3$$

**Q14.6**

A R-12 vapour compression refrigeration system is operating at a condenser pressure of 9.6 bar and an evaporator pressure of 2.19 bar. Its refrigeration capacity is 15 tonnes. The values of enthalpy at the inlet and outlet of the evaporator are 64.6 and 195.7 kJ/kg. The specific volume at inlet to the reciprocating compressor is 0.082 m<sup>3</sup>/kg. The index of compression for the compressor is 1.13

**Determine:**

(a) The power input in kW required for the compressor

(b) The COP. Take 1 tonnes of refrigeration as equivalent to heat removal at the rate of 3.517 kW.

(Ans. (a) 11.57 kW, (b) 4.56)

**Solution:**

$$T_1 = -10^\circ\text{C}$$

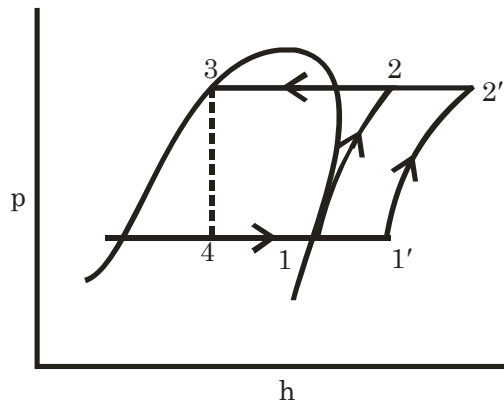
$$h_4 = 64.6 \text{ kJ/kg}$$

$$v_1 = 0.082 \text{ m}^3/\text{kg}$$

$$T_3 = 40^\circ\text{C}$$

$$h_1 = 105.7 \text{ kJ/kg}$$

$$n = 1.13$$



$$\text{Refrigeration effect } (195.7 - 64.6) \text{ kJ/kg} = 131.1 \text{ kJ/kg}$$

$$\dot{m} Q_o = 15 \times 3.517 \quad \therefore \dot{m} = 0.4024 \text{ kg/s}$$

$$\frac{v_2}{v_1} = \left( \frac{p_1}{p_2} \right)^{\frac{1}{n}} = \left( \frac{2.19}{9.6} \right)^{\frac{1}{1.13}} = v_2 = 0.022173 \text{ m}^3/\text{kg}$$

$$\therefore W_c = \frac{n}{n-1} (p_1 V_1 - p_2 V_2) = 28.93 \text{ kJ/kg}$$

$$(a) W_{\text{compressor}} = 11.64 \text{ KW}$$

$$(b) \quad \text{COP} = \frac{15 \times 3.517}{11.64} = 4532$$

**Q14.12** Determine the ideal COP of an absorption refrigerating system in which the heating, cooling, and refrigeration take place at 197°C, 17°C, and -3°C respectively.

**Solution :**  $\therefore \text{COP} = \frac{\text{Desired effort}}{\text{input}}$

$$= \frac{\text{Refrigerating effect}}{\text{heat input}}$$

$$= \frac{Q_o}{Q_h}$$

$$= \frac{Q_o}{W} \times \frac{W}{Q_h}$$

$$= (\text{COP})_R \times \eta_{\text{H.E.}}$$

For ideal process

$$(\text{COP})_R = \frac{T_o}{T_a - T_o}$$

And  $\eta_{\text{H.E.}} = \eta_{\text{Carnot}} = \left(1 - \frac{T_a}{T_h}\right)$

$$\therefore (\text{COP})_{\text{ideal}} = \frac{T_o}{T_a - T_o} \times \left(1 - \frac{T_a}{T_h}\right)$$

$$= \frac{T_o}{T_h} \times \frac{[T_h - T_a]}{[T_a - T_o]}$$

Given  $T_o = 270 \text{ K}$ ,  $T_a = 290 \text{ K}$ ,  $T_h = 470 \text{ K}$

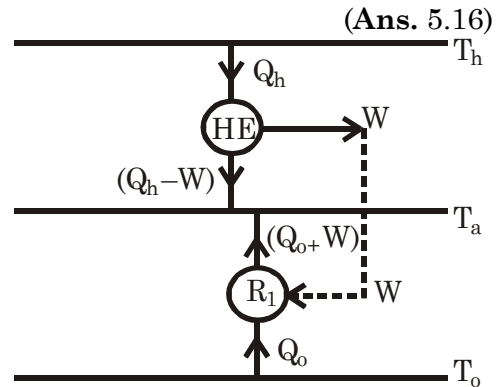
$$\therefore (\text{COP})_{\text{ideal}} = \frac{270}{470} \times \frac{[470 - 290]}{[290 - 270]} = 5.17$$

**Q14.22** Derive an expression for the COP of an ideal gas refrigeration cycle with a regenerative heat exchanger. Express the result in terms of the minimum gas temperature during heat rejection ( $T_h$ ) maximum gas temperature during heat absorption ( $T_1$ ) and pressure ratio for the cycle ( $p_2/p_1$ ).

(Ans.  $\text{COP} = \frac{T_1}{T_h r_p^{(\gamma-1)/\gamma} - T_1}$ )

**Solution:**  $\therefore \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = r_p^{\frac{\gamma-1}{\gamma}}$

$$\therefore T_2 = T_1 r_p^{\frac{\gamma-1}{\gamma}}$$



$$\frac{T_4}{T_5} = \left( \frac{p_4}{p_5} \right)^{\frac{\gamma-1}{\gamma}} = \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} = r_p^{\frac{\gamma-1}{\gamma}}$$

$$\therefore T_5 = \frac{T_4}{r_p^{\frac{\gamma-1}{\gamma}}} = \frac{T_h}{r_p^{\frac{\gamma-1}{\gamma}}}$$

For Regeneration ideal

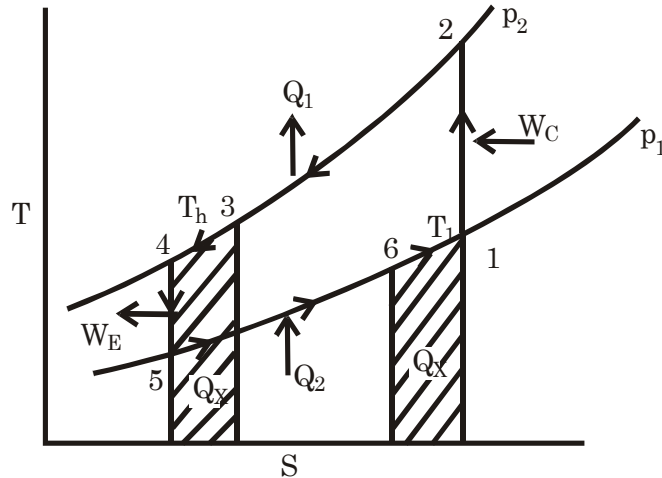
$$C_P (T_3 - T_4) = p (T_1 - T_6)$$

$$\therefore T_3 - T_h = T_1 - T_6$$

$$\begin{aligned} \therefore \text{Work input (W)} &= (h_2 - h_1) - (h_4 - h_5) \\ &= C_P [(T_2 - T_1) - (T_h - T_s)] \end{aligned}$$

$$\text{Heat rejection (Q}_1\text{)} = Q_2 + W = C_P (T_2 - T_3)$$

$$\text{Heat absorption (Q}_2\text{)} = C_P (T_6 - T_5)$$



$$\therefore \text{COP} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_6 - T_5}{(T_2 - T_3) - (T_6 - T_5)} = \frac{1}{\frac{T_2 - T_3}{T_6 - T_5} - 1}$$

$$T_2 - T_3 = T_1 r_p^{\frac{\gamma-1}{\gamma}} - T_1 = T_1 (r_p^{\frac{\gamma-1}{\gamma}} - 1)$$

$$T_6 - T_5 = T_h - \frac{T_h}{r_p^{\frac{\gamma-1}{\gamma}}} = T_h \frac{(r_p^{\frac{\gamma-1}{\gamma}} - 1)}{r_p^{\frac{\gamma-1}{\gamma}}} = \frac{T_h}{T_1 r_p^{\frac{\gamma-1}{\gamma}} - T_h} \quad \text{or} \quad \text{COP} = \frac{T_1}{T_h r_p^{(\gamma-1)/\gamma} - T_1}$$

**Q14.23**

Large quantities of electrical power can be transmitted with relatively little loss when the transmission cable is cooled to a superconducting temperature. A regenerated gas refrigeration cycle operating with helium is used to maintain an electrical cable at 15 K. If the pressure ratio is 10 and heat is rejected directly to the atmosphere at 300 K, determine the COP and the performance ratio with respect to the Carnot cycle.

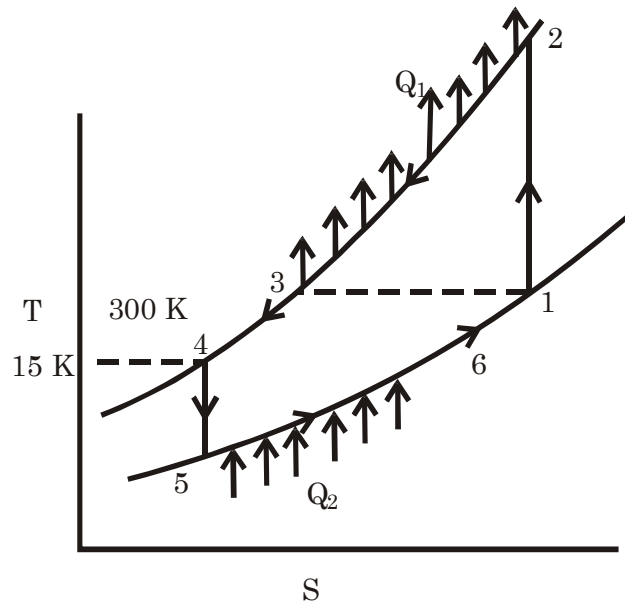
(Ans. 0.02, 0.38)

**Solution:**

$$\frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} = 10^{\frac{\gamma-1}{\gamma}}$$

$$\therefore T_2 = 300 \times 10^{\frac{1.6667 \cdot 1}{1.6667}} = 754 \text{ K}$$

$$\frac{T_5}{T_4} = \left( \frac{p_5}{p_4} \right)^{\frac{\gamma-1}{\gamma}} = \left( \frac{p}{p_2} \right)^{\frac{\gamma-1}{\gamma}} = \left( \frac{1}{10} \right)^{0.4}$$



$$\therefore T_5 = 5.9716 \text{ K}$$

Refrigerating effect ( $Q_2$ ) =  $C_P (T_6 - T_5) = 9.0284 C_P$   
 Work input ( $W$ ) =  $C_P [(T_2 - T_1) - (T_4 - T_5)] = 444.97 C_P$

$$\therefore \text{COP} = \frac{0.0284 C_P}{444.97 C_P} = 0.0203$$

$$\text{And } (\text{COP})_{\text{carnet}} = \frac{T_6}{T_6 - T_5} = \frac{15}{300 - 15} = 0.05263$$

$$\frac{\text{COP}_{\text{actual}}}{\text{COP}_{\text{carnet}}} = \frac{0.0203}{0.05263} = 0.3857$$

**Q14.25**

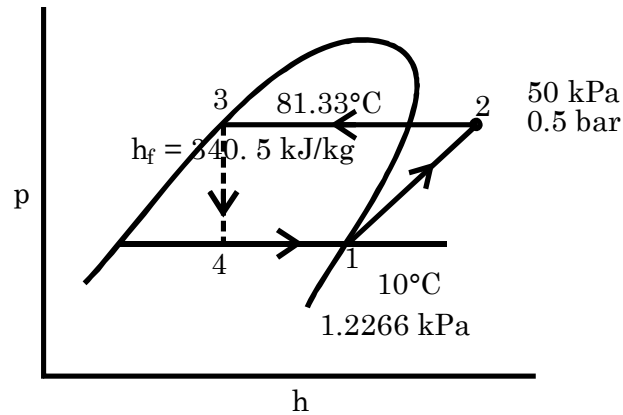
A heat pump installation is proposed for a home heating unit with an output rated at 30 kW. The evaporator temperature is 10°C and the condenser pressure is 0.5 bar. Using an ideal vapour compression cycle, estimate the power required to drive the compressor if steam/water mixture is used as the working fluid, the COP and the mass flow rate of the fluid. Assume saturated vapour at compressor inlet and saturated liquid at condenser outlet.

(Ans. 8.0 kW, 3.77, 0.001012 kg/s)

**Solution:**

$$h_1 = 2519.8 \text{ kJ/kg}$$

$$s_1 = 8.9008 \text{ kJ/kg-K}$$

$$v_1 = 106.38 \text{ m}^3/\text{kg}$$


$$400^\circ\text{C } 50 \text{ kPa } s = 8.88642, \quad h = 3278.9$$

$$500^\circ\text{C } 50 \text{ kPa } s = 9.1546, \quad h = 3488.7$$

$$\therefore h_2 = (3488.7 - 3278.9) \times \left( \frac{8.9008 - 8.8642}{9.1546 - 8.8642} \right) + 3278.9 = 3305.3 \text{ kJ/kg}$$

$$\therefore \text{Compressor work (W)} = h_2 - h_1 = (3305.3 - 2519.8) = 785.5 \text{ kJ/kg}$$

$$\text{Heating (Q)} = h_2 - h_{f3} = (3305.3 - 340.5) \text{ kJ/kg} = 2964.8 \text{ kJ/kg}$$

$$\dot{m} \times Q = 30$$

$$= \dot{m} = \frac{30}{2964.8} = 0.0101187 \text{ kg/s}$$

$$\therefore \text{COP} = \frac{2964.8}{785.5} = 3.77$$

$$\text{Compressor power} = \dot{m}W = 7.95 \text{ KW}$$

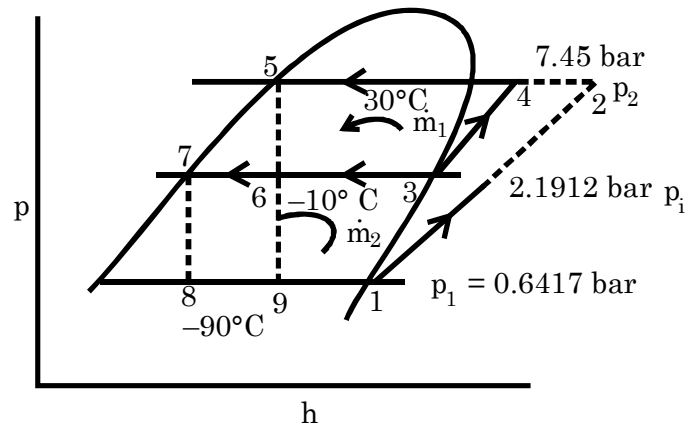
**Q14.26** A 100 tonne low temperature R-12 system is to operate on a 2-stage vapour compression refrigeration cycle with a flash chamber, with the refrigerant evaporating at  $-40^\circ\text{C}$ , an intermediate pressure of 2.1912 bar, and condensation at  $30^\circ\text{C}$ . Saturated vapour enters both the compressors and saturated liquid enters each expansion valve. Consider both stages of compression to be isentropic. Determine:

- The flow rate of refrigerant handled by each compressor
- The total power required to drive the compressor
- The piston displacement of each compressor, if the clearance is 2.5% for each machine
- The COP of the system
- What would have been the refrigerant flow rate, the total work of compression, the piston displacement in each compressor and the compressor and the COP, if the compression had occurred in a single stage? .

(Ans. (a) 2.464, 3.387 kg/s, (b) 123 kW, (c) 0.6274, 0.314 m<sup>3</sup>/s, (d) 2.86, (e) 3.349 kg/s, 144.54 kW, 1.0236 m<sup>3</sup>/s, 2.433)

**Solution:**

$h_1 = 169 \text{ kJ/kg}$   
 $h_3 = 183.2 \text{ kJ/kg}$   
 $h_5 = 64.6 \text{ kJ/kg} = h_6$   
 $h_7 = h_8 = 26.9 \text{ kJ/kg}$



$$S_1 = S_2 = 0.7274 \text{ kJ/kg} - \text{K}$$

$$S_3 = S_4 = 0.7020 \text{ kJ/kg} - \text{K}$$

∴ From P.H chart of R12

$$h_2 = 190 \text{ kJ/kg}$$

$$h_4 = 206 \text{ kJ/kg}$$

$$\dot{m}_2 h_2 + \dot{m}_1 h_5 = \dot{m}_2 h_7 + \dot{m}_1 h_3$$

$$\therefore \dot{m}_2 (h_2 - h_7) = \dot{m}_1 (h_3 - h_5)$$

$$\therefore \frac{\dot{m}_1}{\dot{m}_2} = \frac{h_2 - h_7}{h_3 - h_5} = \frac{190 - 26.9}{183.2 - 64.6} = 1.3752$$

$$\dot{m}_2 (h_1 - h_8) = \frac{100 \times 14000}{3600}$$

$$(a) \quad \therefore \dot{m}_2 = 2.7367 \text{ kg/s}$$

$$\dot{m}_1 = \dot{m}_2 \times 1.3752 = 3.7635 \text{ kg/s}$$

$$(b) \text{ Power of compressor (P)} = \dot{m}_2 (h_2 - h_1) + \dot{m}_1 (h_4 - h_3) \\ = 14328 \text{ kW}$$

$$(d) \text{ COP} = \frac{\text{Refrigeration efficiency}}{\text{Compressor}} = \frac{100 \times 14000}{3600 \times 143.28} = 2.7142$$

(e) For single storage

From R12 chart  $h_{a'} = 2154 \text{ kJ/kg}$ ,  $h_g = h_s = 64.6 \text{ kJ/kg}$

$$\therefore \dot{m} (h_1 - h_9) = \frac{100 \times 14000}{3600} \Rightarrow \dot{m} = 3.725 \text{ kg/s}$$

$$\text{Compressor power (P)} = \dot{m} (h_{a'} - h_1) = 3.725 \times 46 \\ = 171.35 \text{ kW}$$



$$\text{COP} = \frac{\frac{100 \times 14000}{3600}}{171.35} = 2.27$$